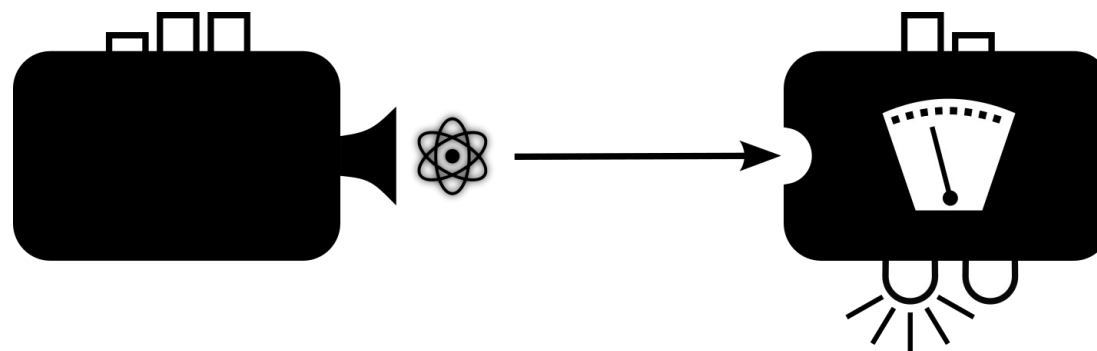


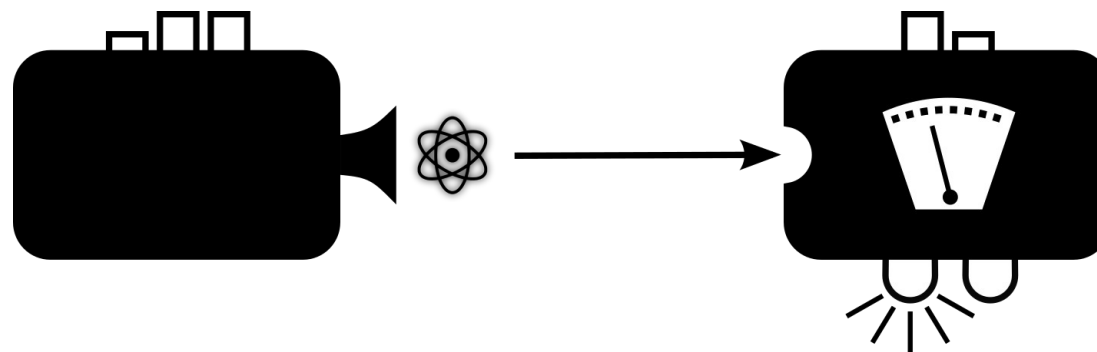
QUANTUM CORRELATIONS UNDER RESTRICTED TRANSFER OF INFORMATION



Jonatan Bohr Brask
DTU Physics



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A. Tavakoli, E. Zambrini Cruzeiro,
JBB, N. Gisin, N. Brunner

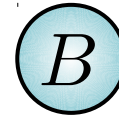
arXiv:1909.05656



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In any physical theory with a notion of distinct observables, we can look at correlations....

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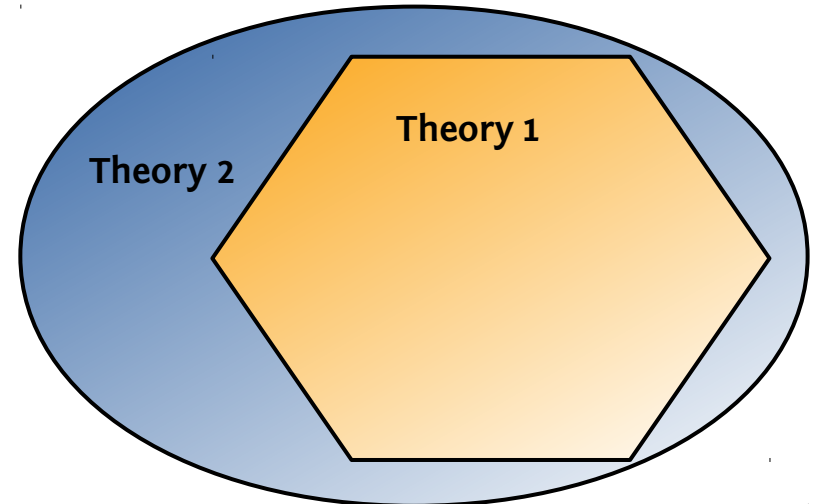


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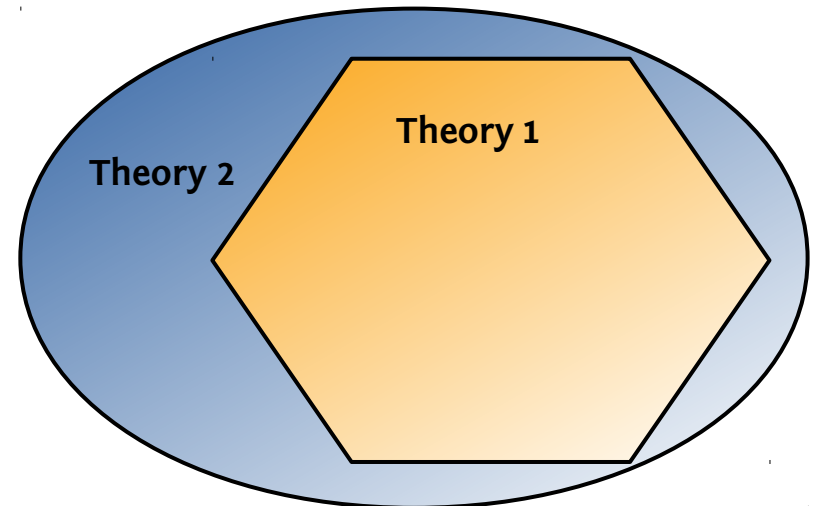
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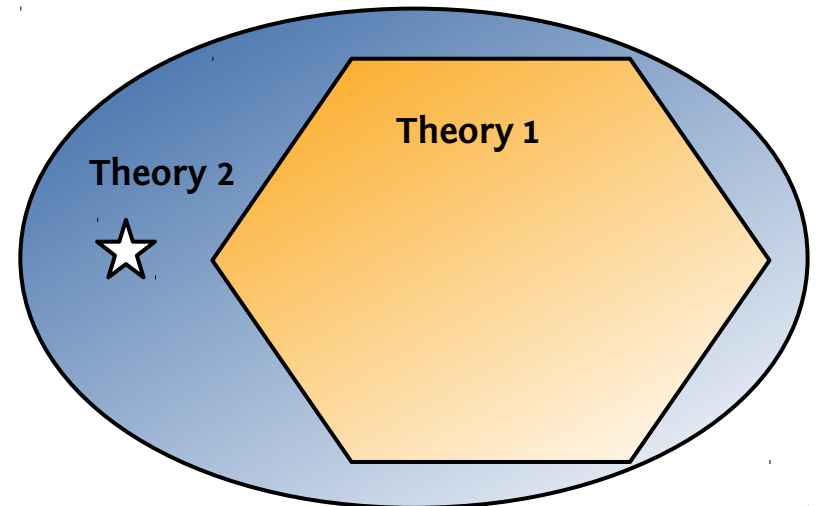
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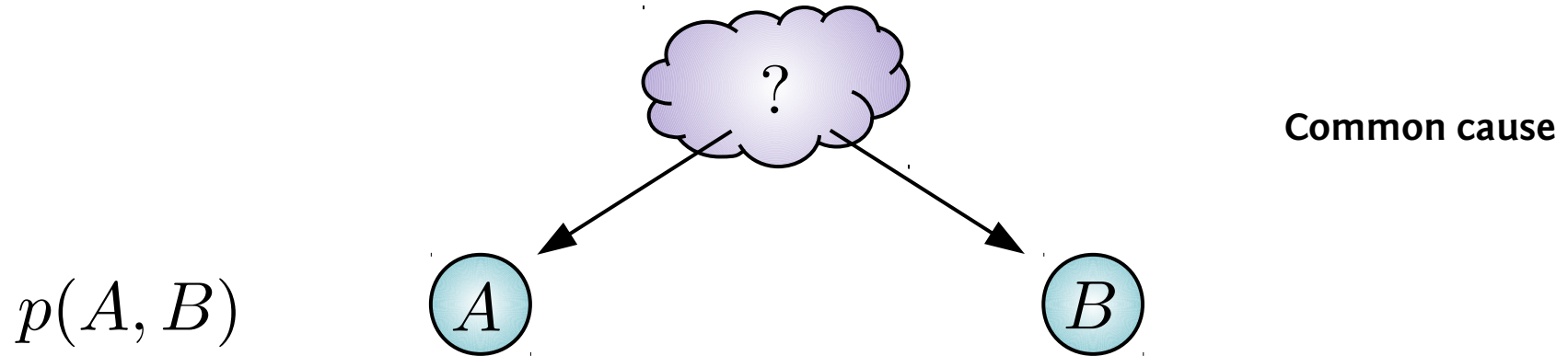
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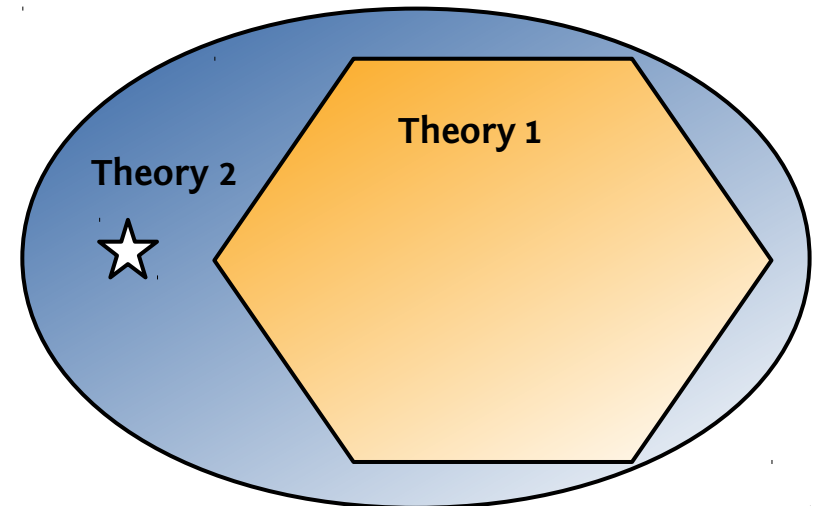
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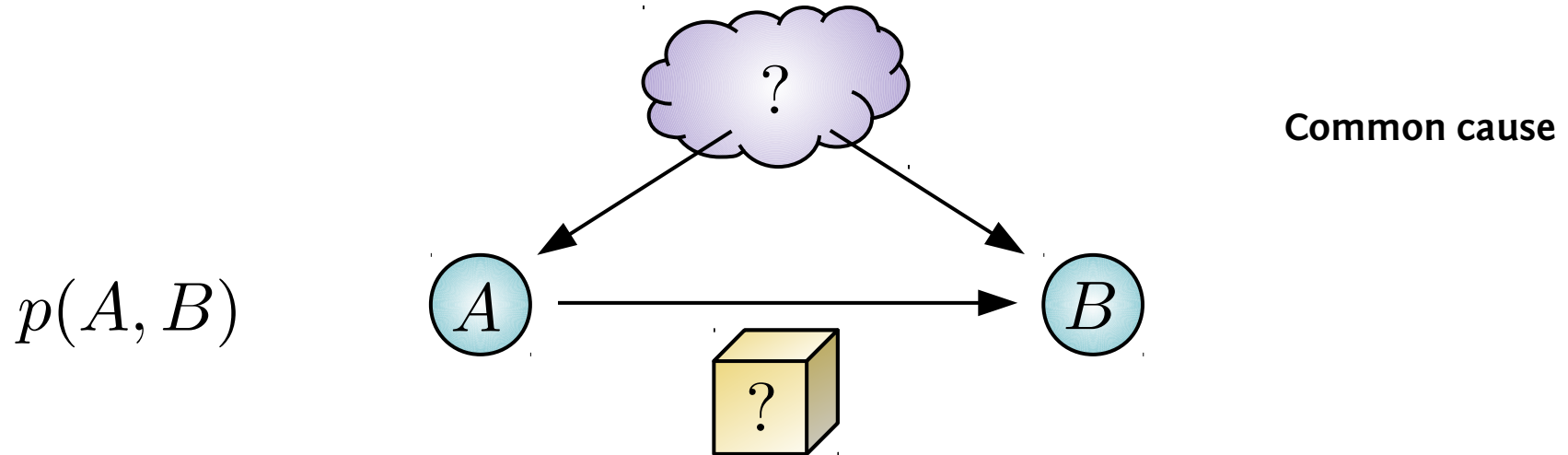
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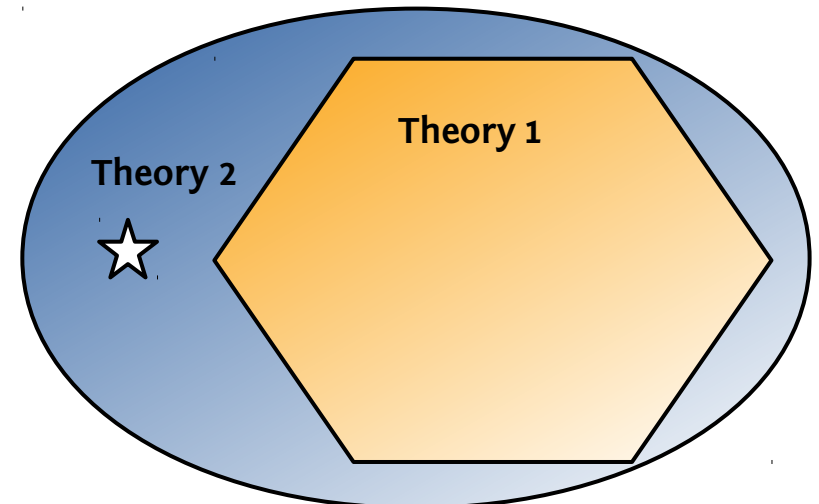
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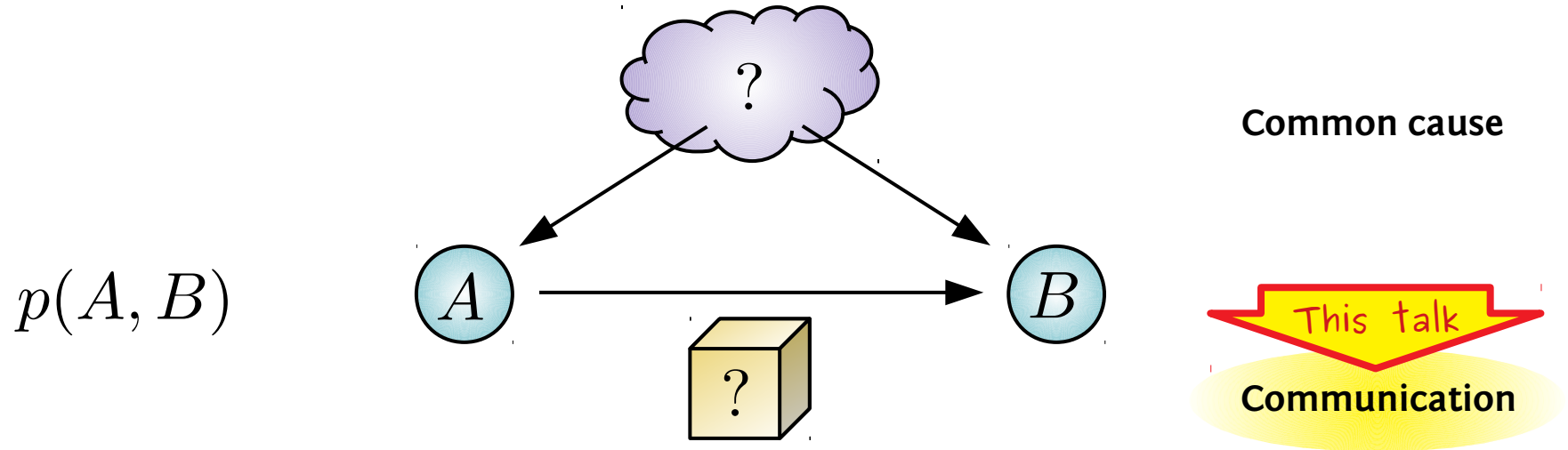
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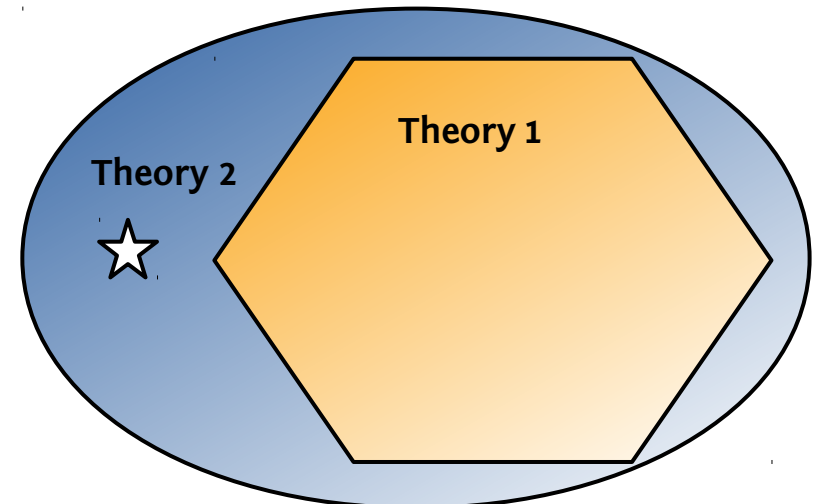
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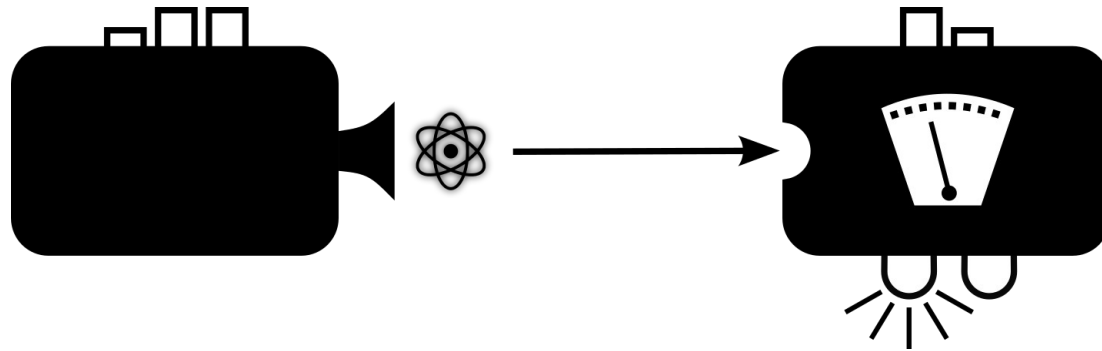
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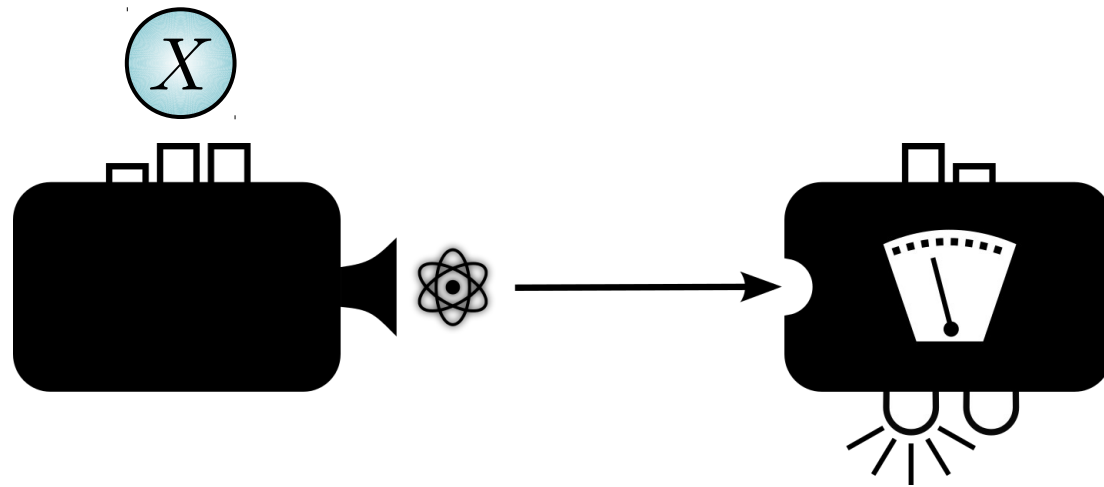
COMMUNICATION AND CORRELATIONS – PREPARE-AND-MEASURE SCENARIOS

Can study relation between communication and correlations in prepare-and-measure setups.



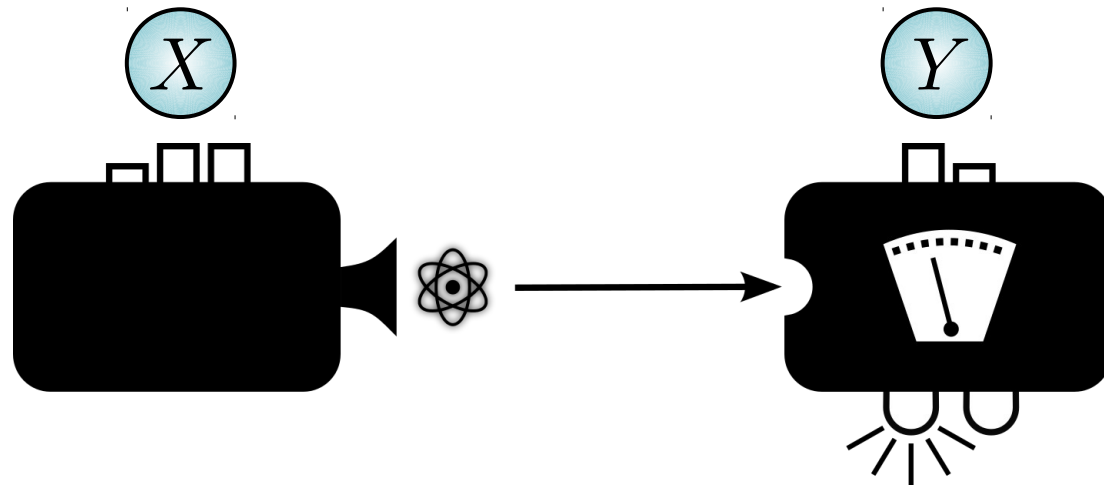
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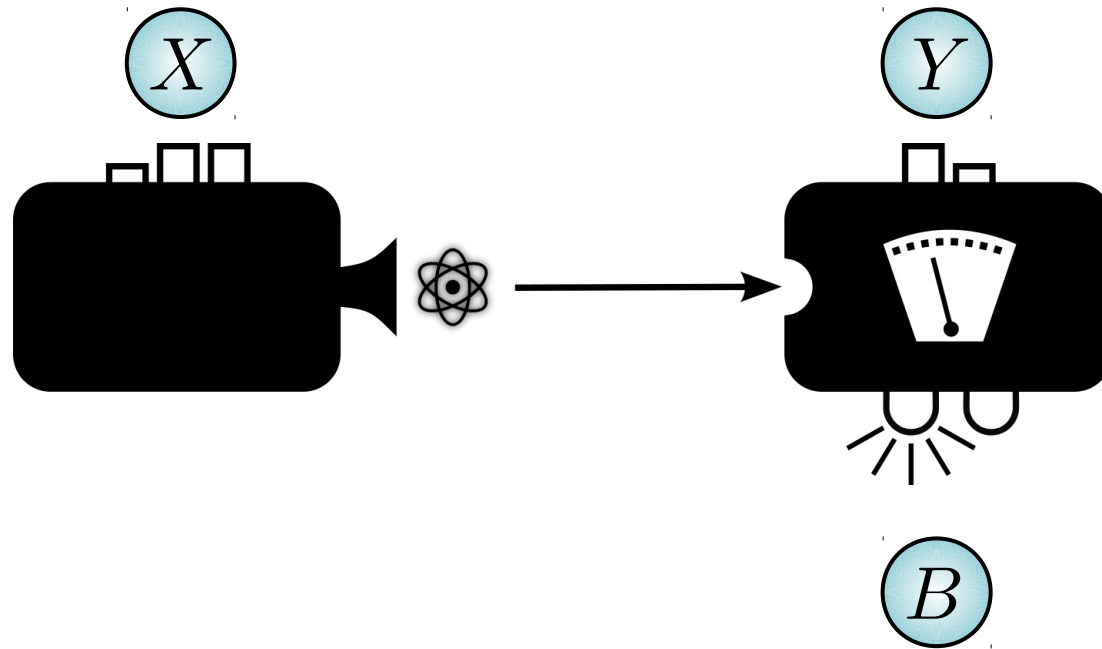
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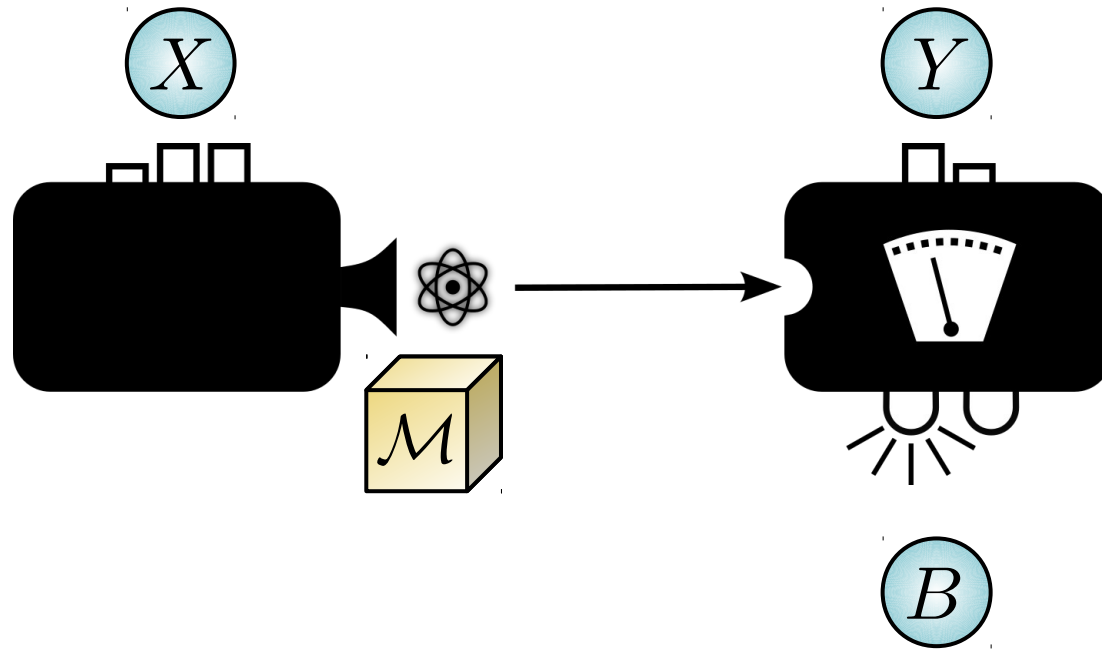
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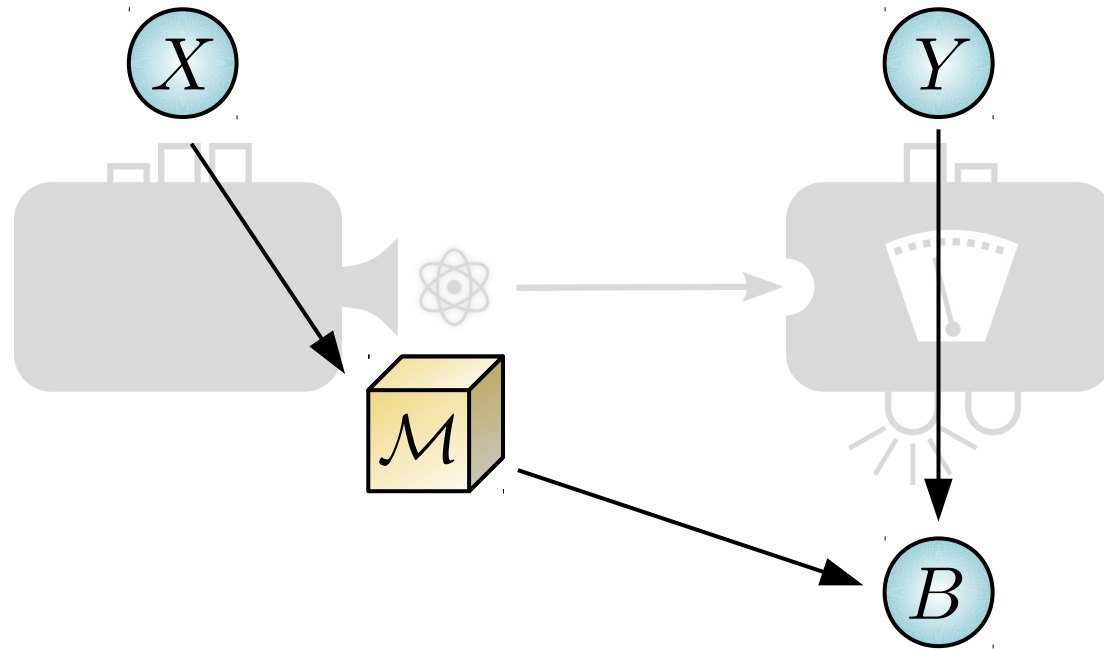
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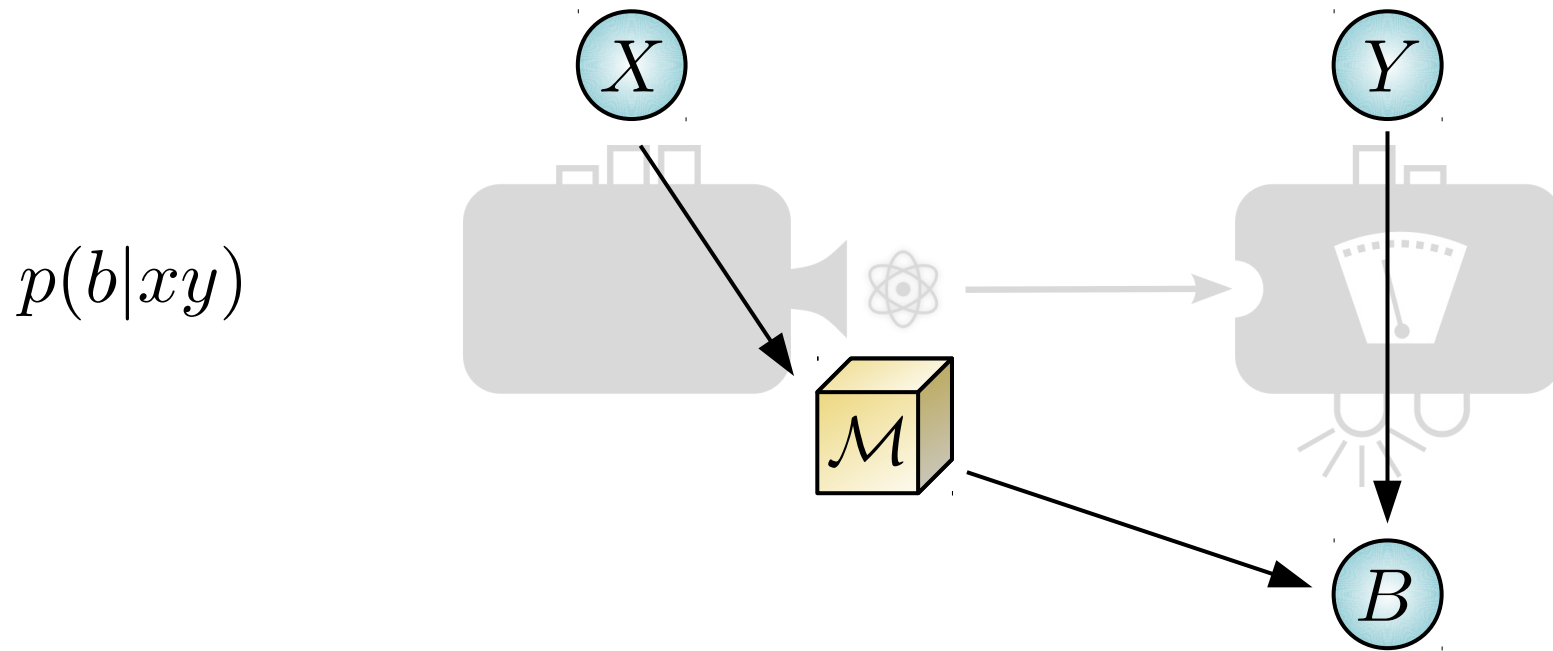
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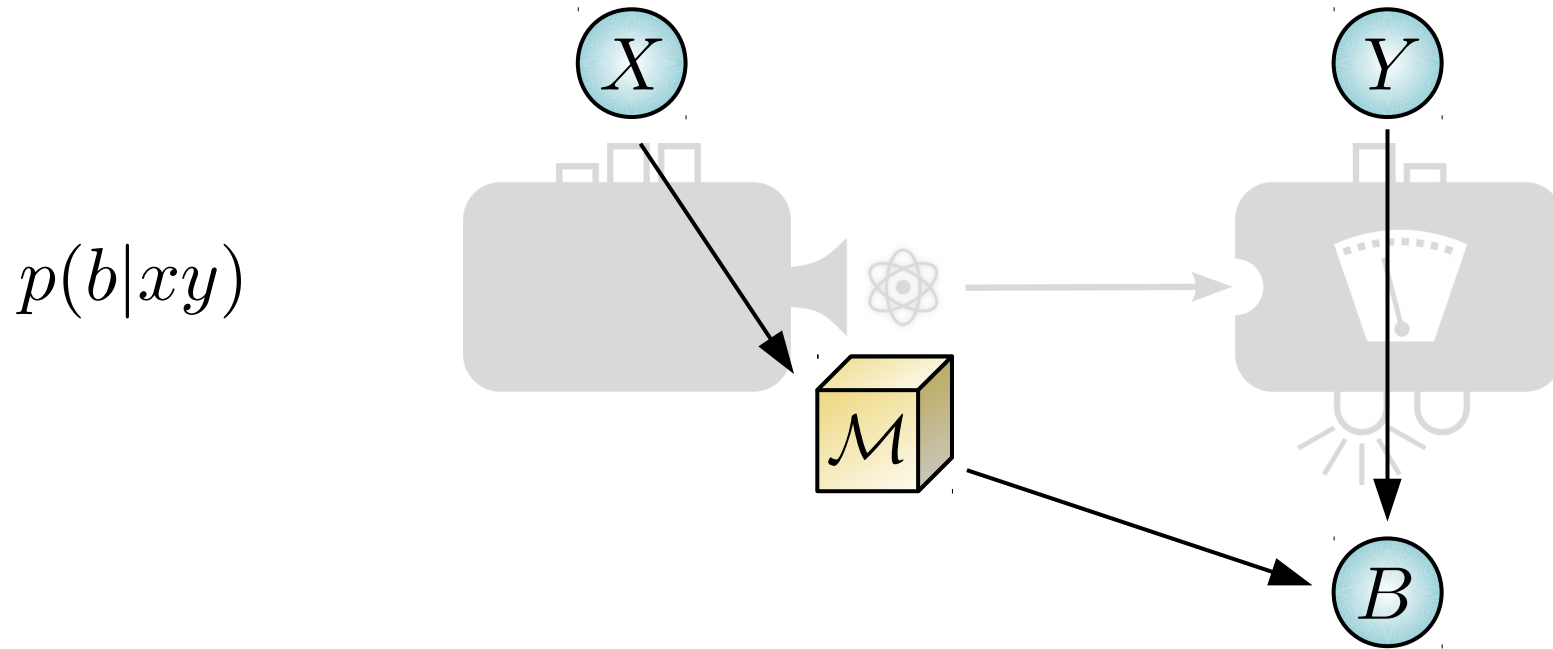
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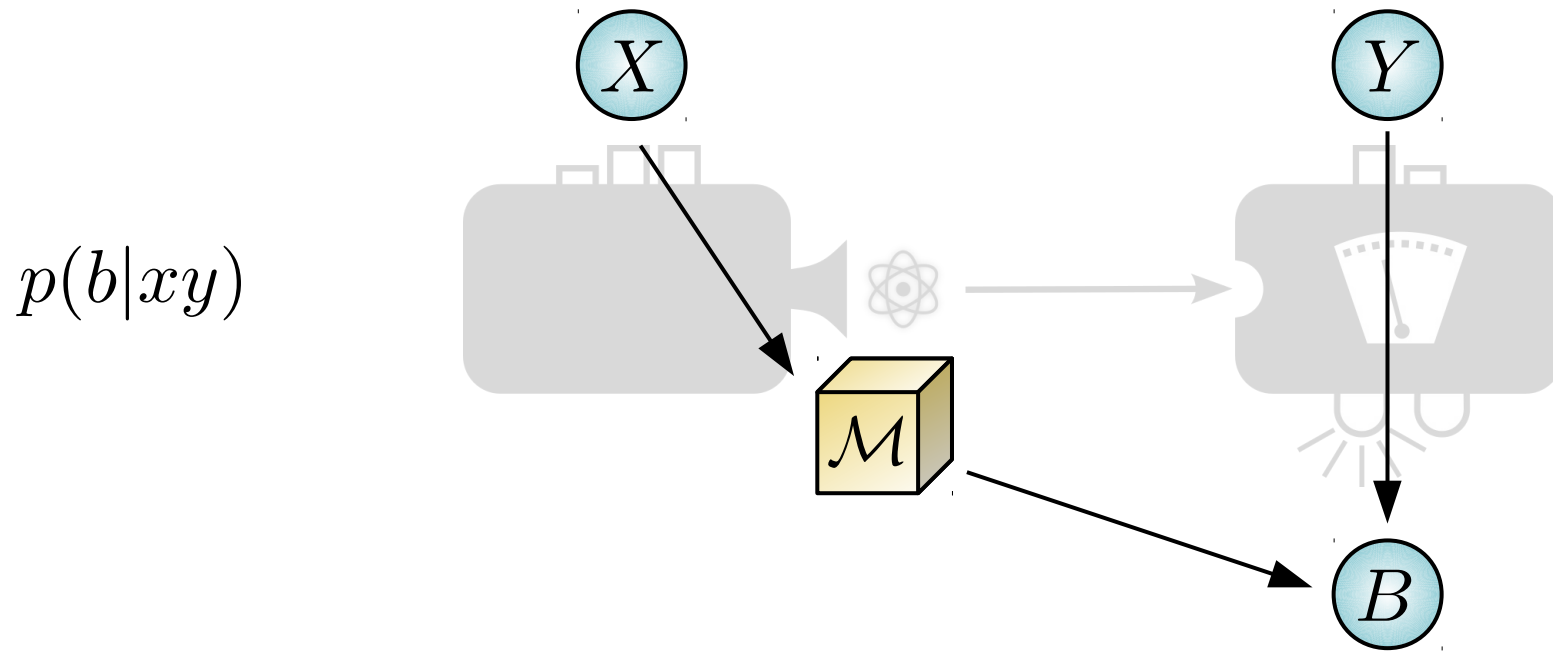
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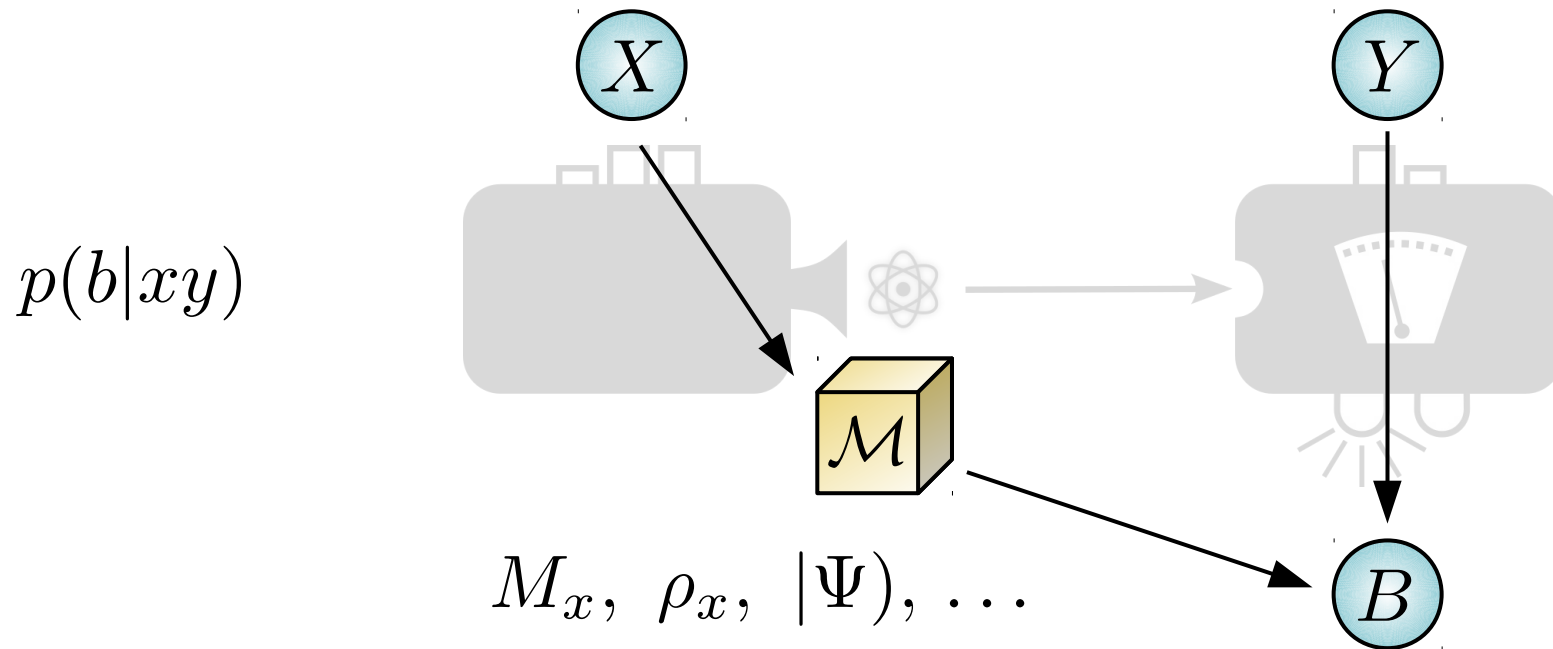


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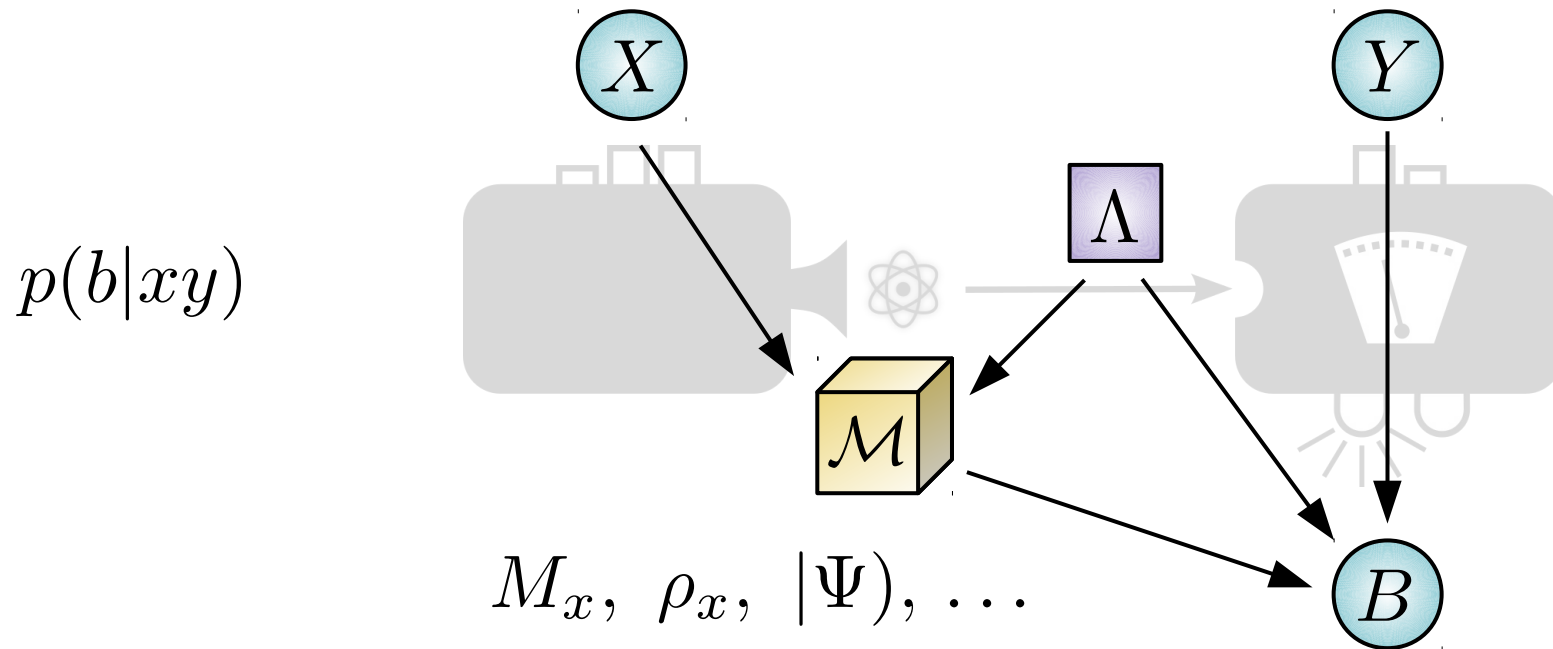


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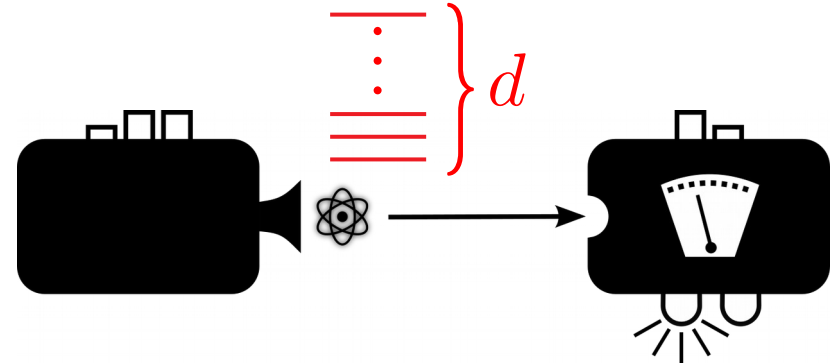
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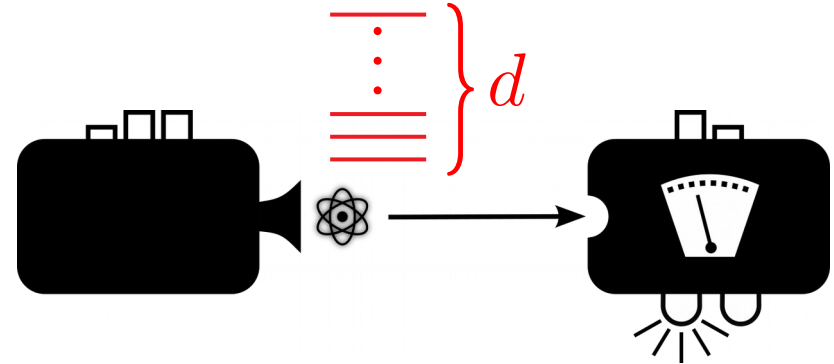


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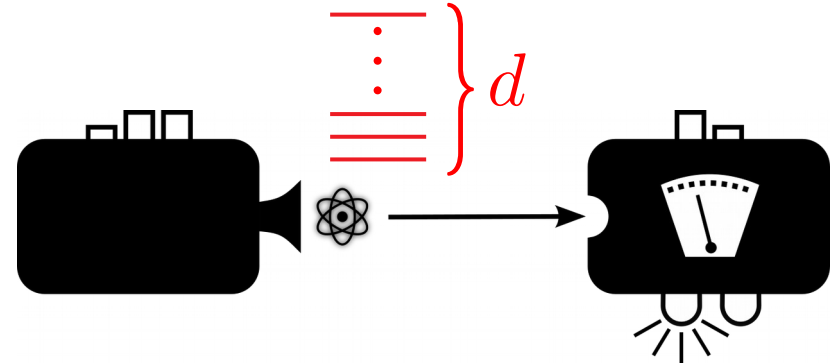


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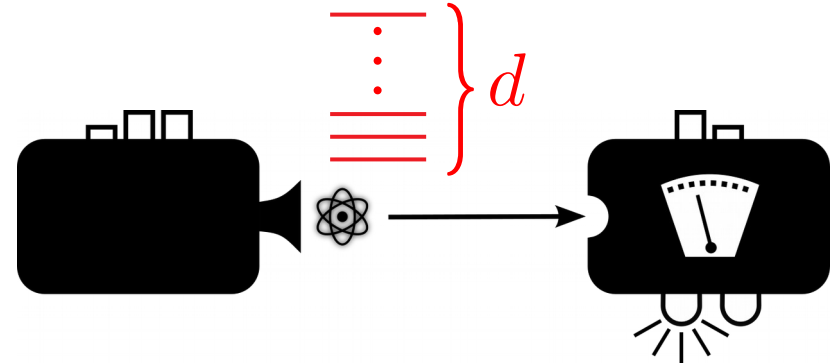
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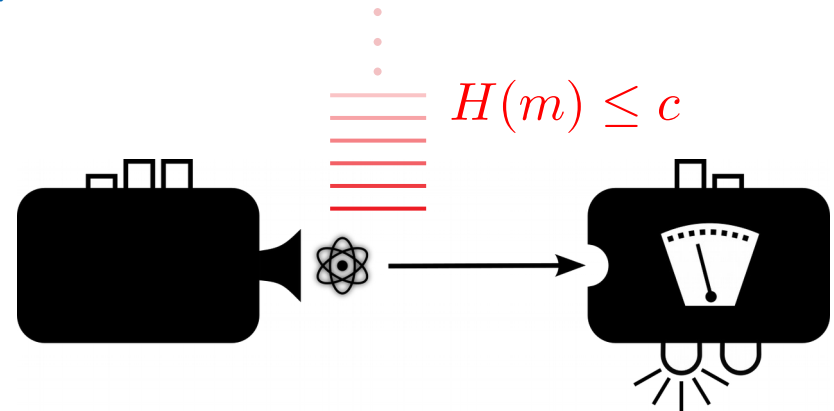
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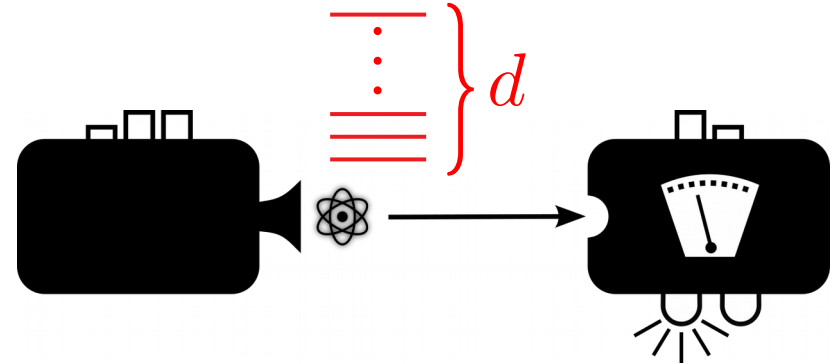


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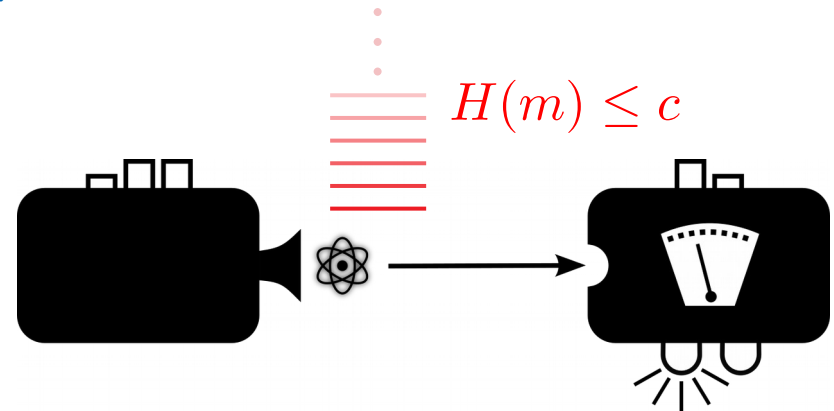
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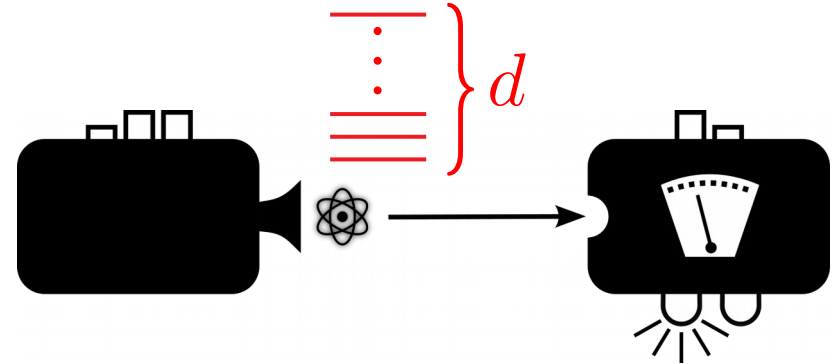


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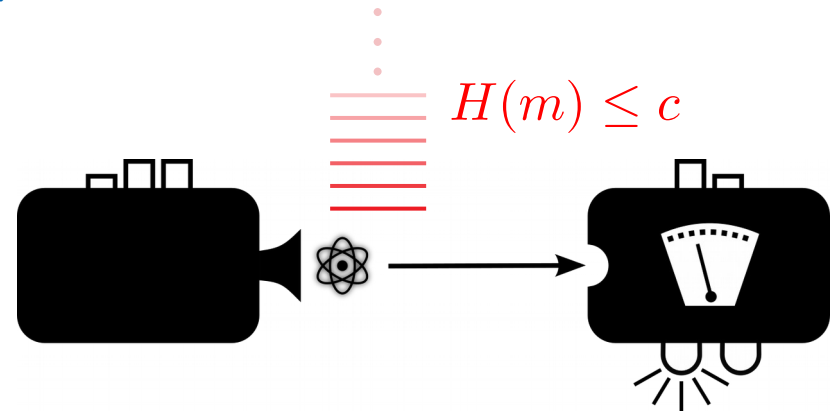
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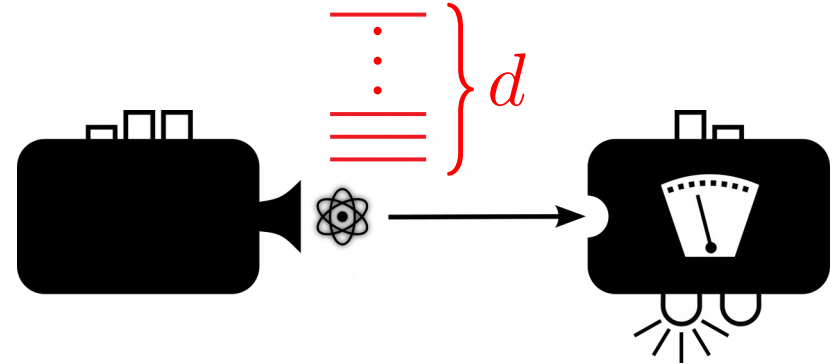


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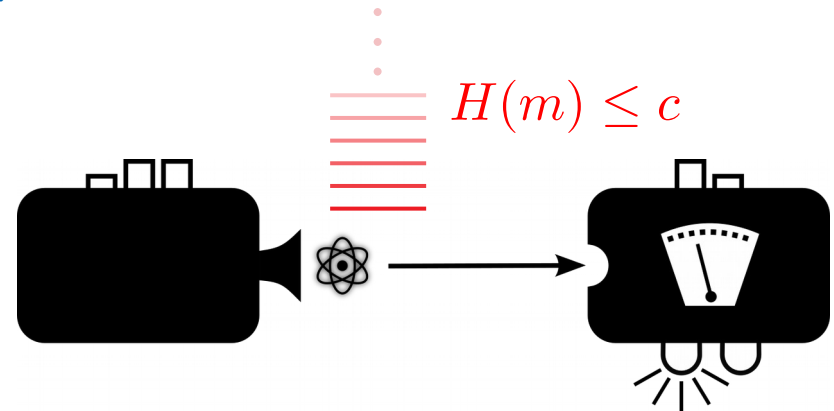
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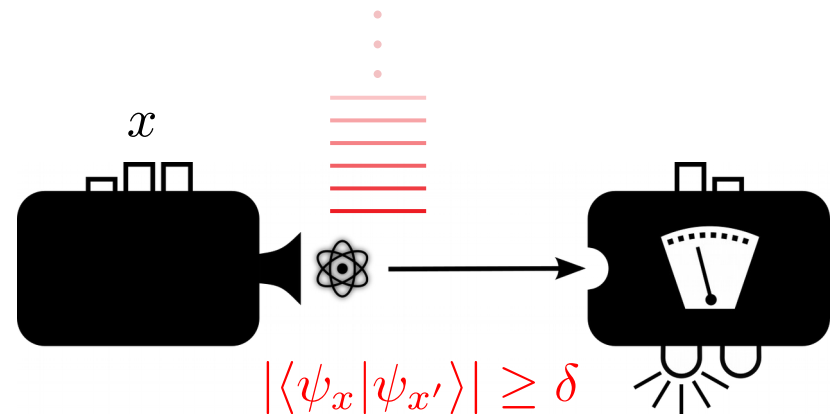
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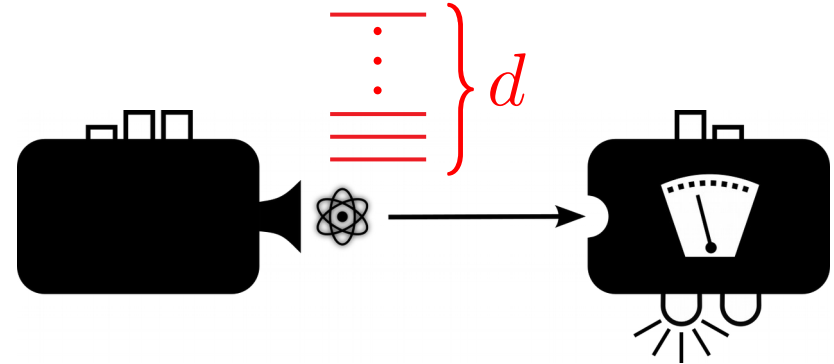
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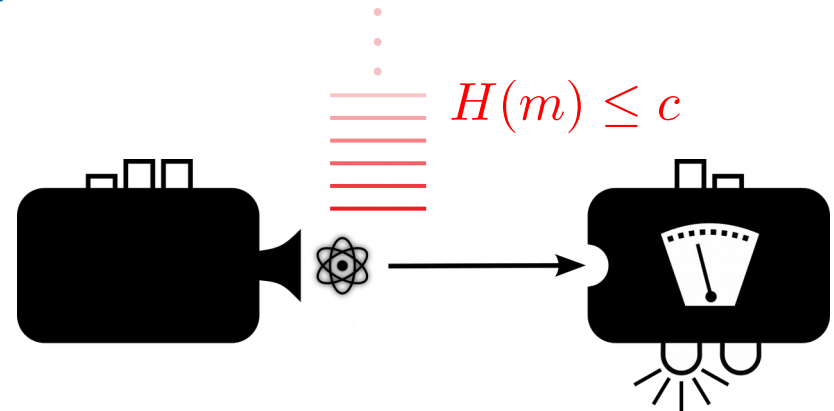
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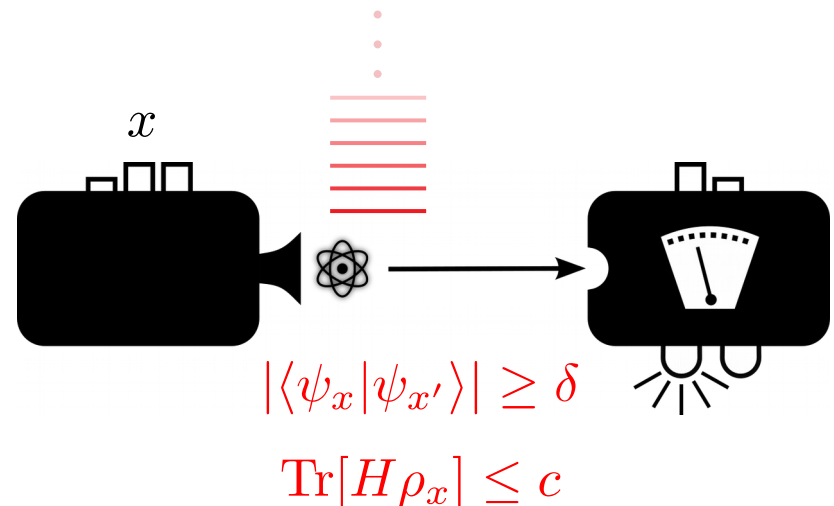
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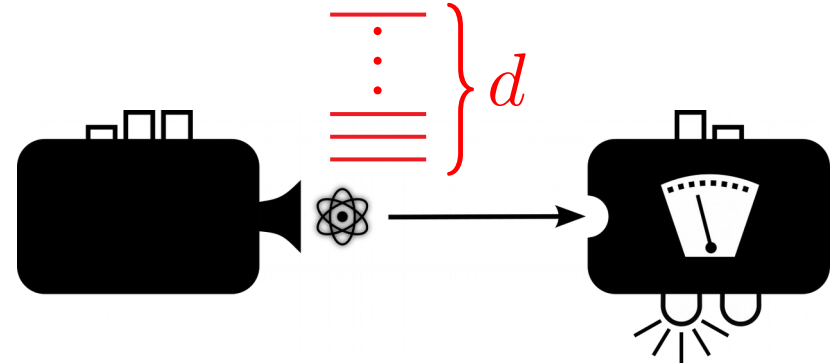


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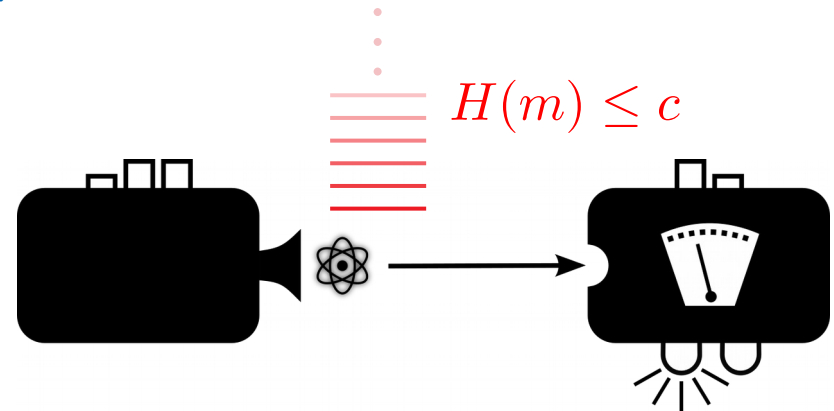
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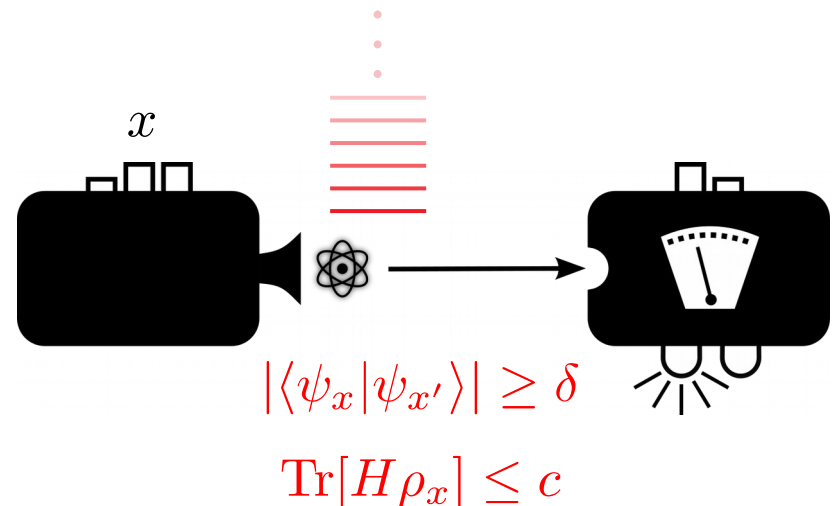
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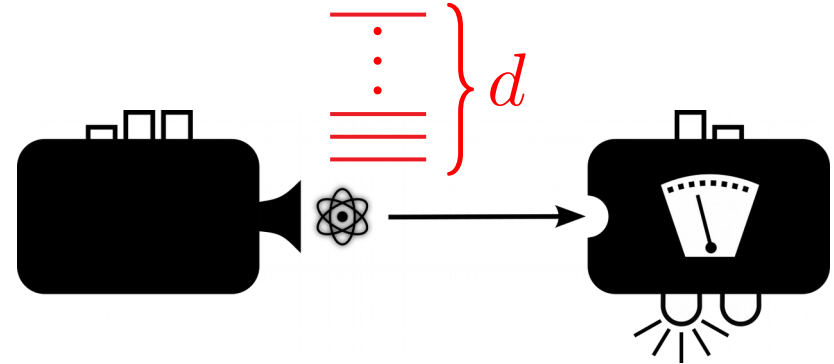
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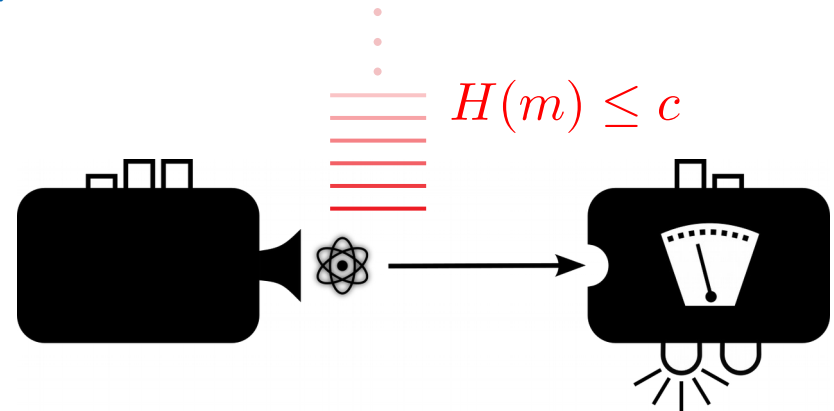
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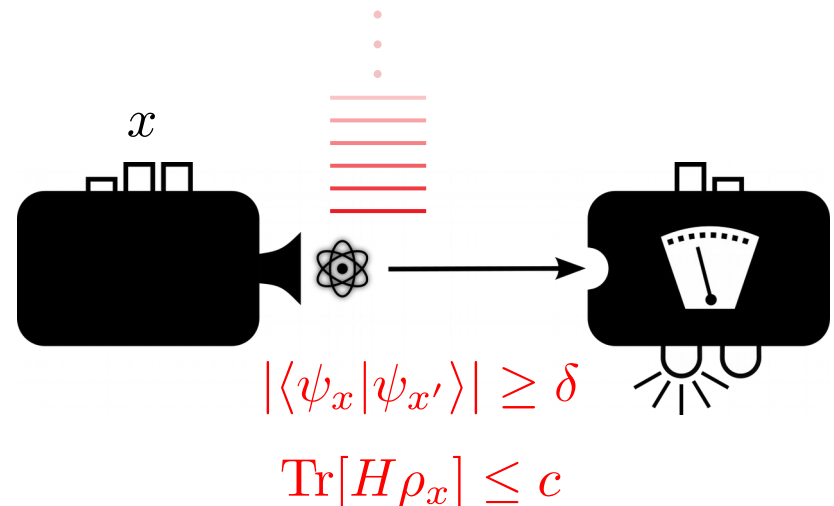
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- Advantageous for optical implementations of “grey-box” (semi-device-independent) QIP applications (e.g. QRNG).

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Directly limit information about input which can be recovered from message.

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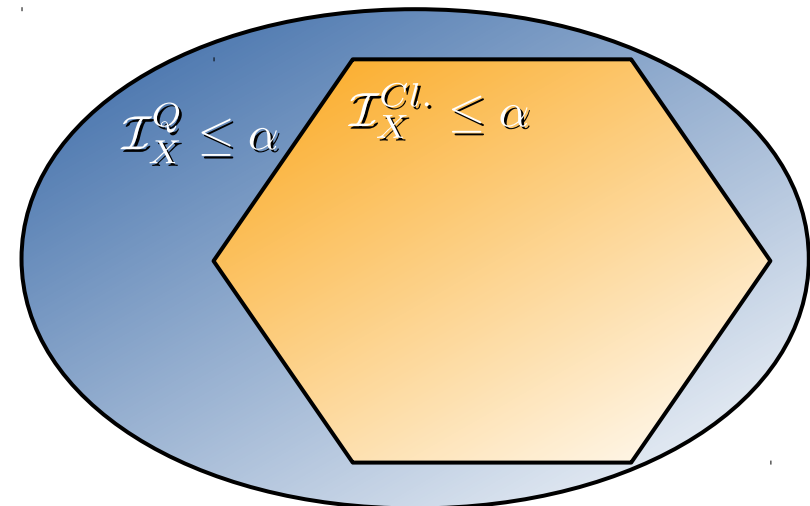
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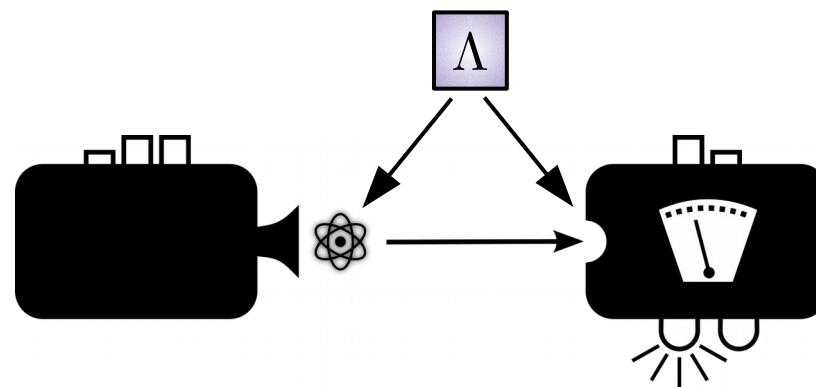
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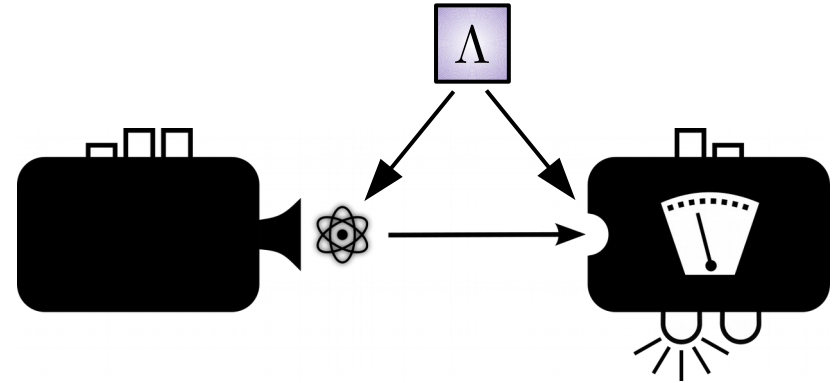
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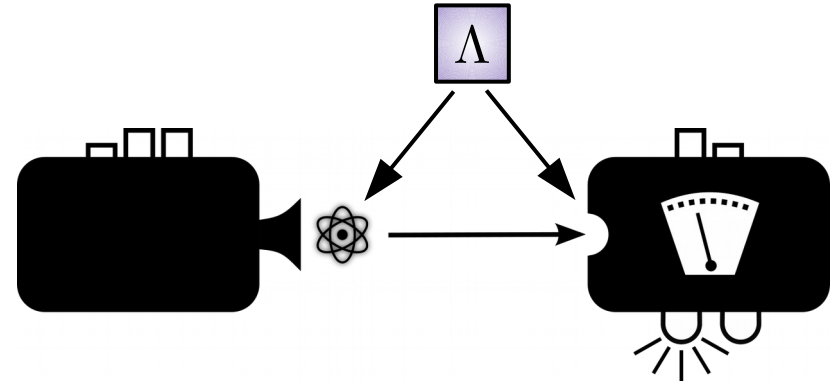
$$E.g. : P_g(X|\mathcal{E}_{\lambda}) = \max_{N_x^{\lambda}} \sum_x p_X(x) [N_x^{\lambda} \rho_x^{\lambda}]$$



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$$E.g. : P_g(X|\mathcal{E}_{\lambda}) = \max_{N_x^{\lambda}} \sum_x p_X(x) [N_x^{\lambda} \rho_x^{\lambda}]$$

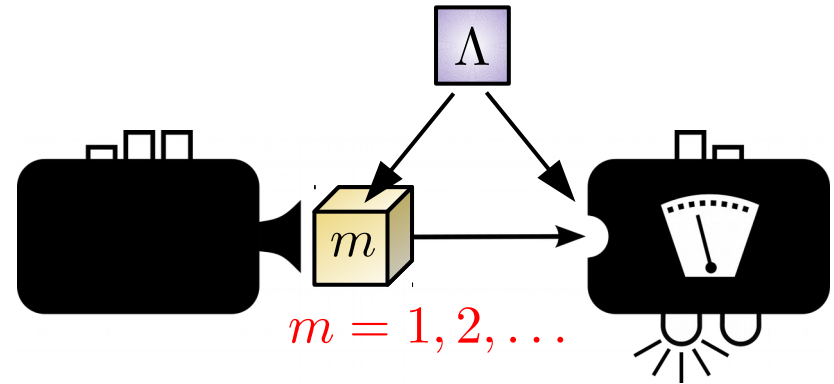


CLASSICAL CORRELATIONS UNDER RESTRICTED INFORMATION

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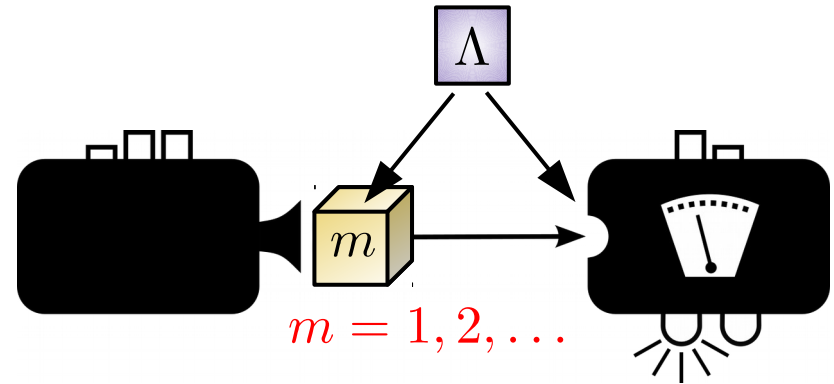
CLASSICAL CORRELATIONS UNDER RESTRICTED INFORMATION

$$p(b|x, y) = \sum_{\lambda} p(\lambda) \sum_{m=1}^d p_A(m|x, \lambda) p_B(b|m, y, \lambda)$$

We allow for shared randomness.

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CLASSICAL CORRELATIONS UNDER RESTRICTED INFORMATION

can restrict to $d = \# \text{ inputs}$

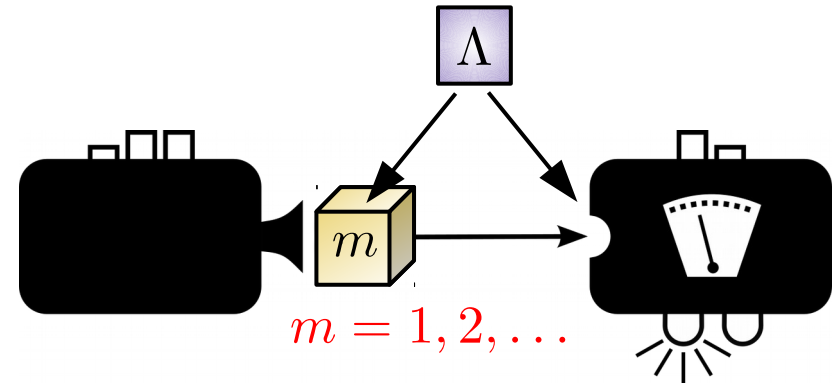
can take these deterministic

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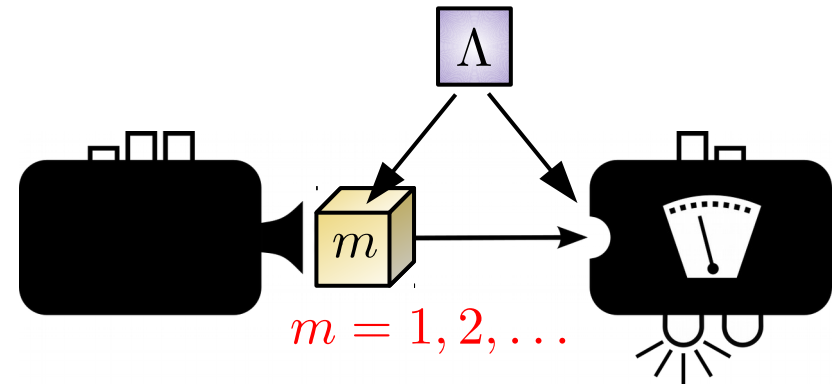


$$\mathcal{I}_X \leq \alpha \quad \rightarrow \quad \sum_{\lambda} p(\lambda) P_g^{\lambda} \leq 2^{\alpha - H_{\min}(X)}$$

We allow for shared randomness.

$$P_g(X|\mathcal{E}) = \sum_{\lambda} P_g(X|\mathcal{E}_{\lambda})$$

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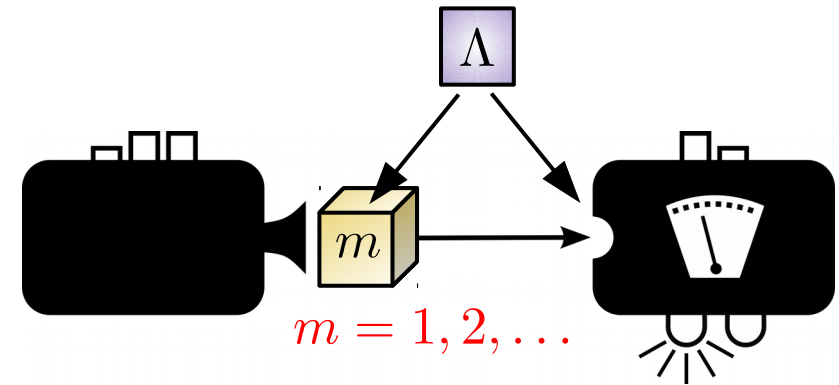
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linear in dist. over λ

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CLASSICAL CORRELATIONS UNDER RESTRICTED INFORMATION

can restrict to $d = \# \text{ inputs}$

can take these deterministic

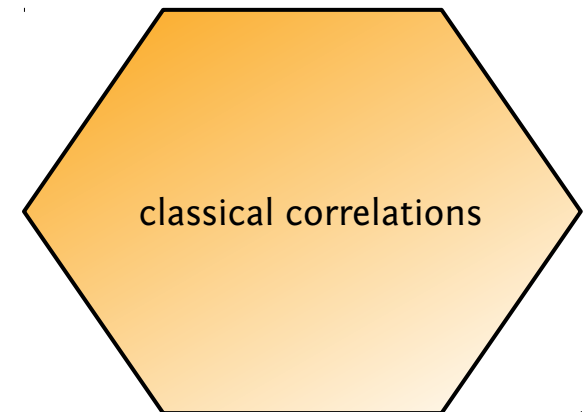
$$p(b|x, y) = \sum_{\lambda} p(\lambda) \sum_{m=1}^d p_A(m|x, \lambda) p_B(b|m, y, \lambda)$$



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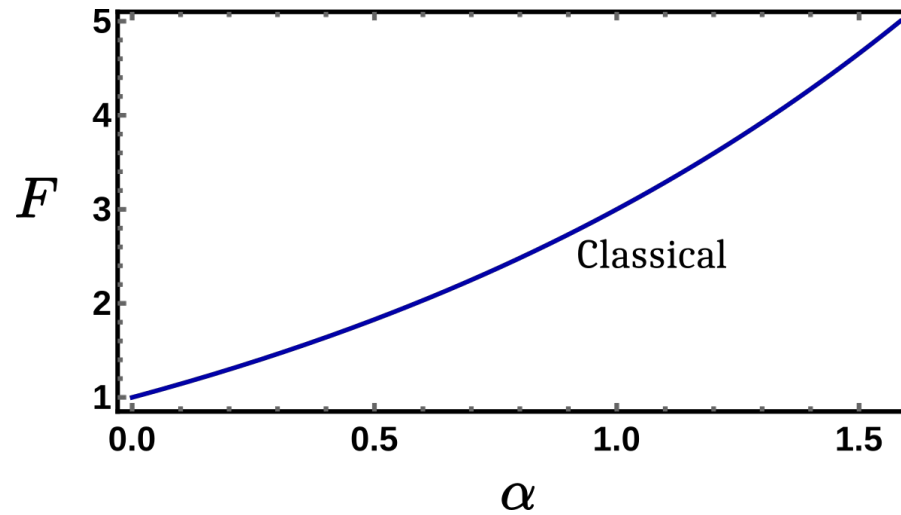
linear in dist. over λ

Classical set is a convex polytope



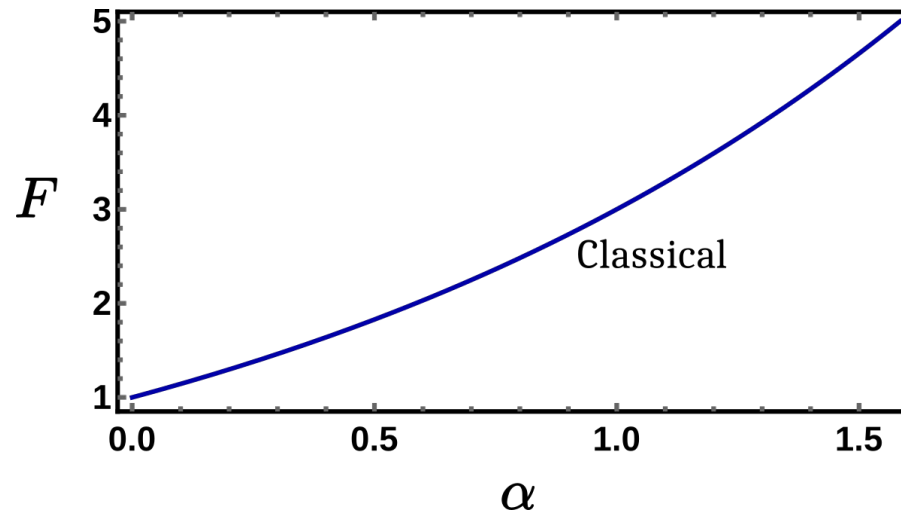
Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

$$F = -(E_{11} + E_{12} + E_{21}) + (E_{22} + E_{31}) \leq 2^{\alpha+1} - 1 \quad E_{xy} = p(0|xy) - p(1|xy)$$



Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

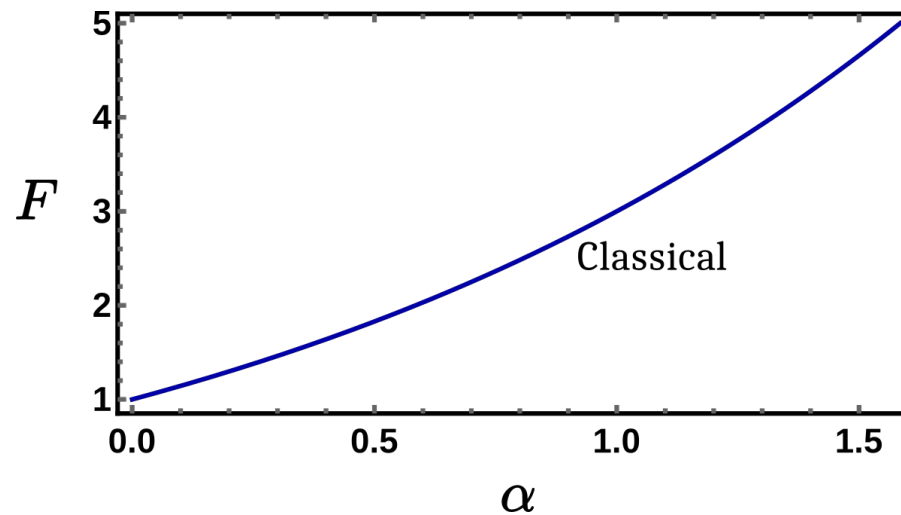
$$F = -(E_{11} + E_{12} + E_{21}) + (E_{22} + E_{31}) \leq 2^{\alpha+1} - 1 \quad E_{xy} = p(0|xy) - p(1|xy)$$



QUANTUM VIOLATION

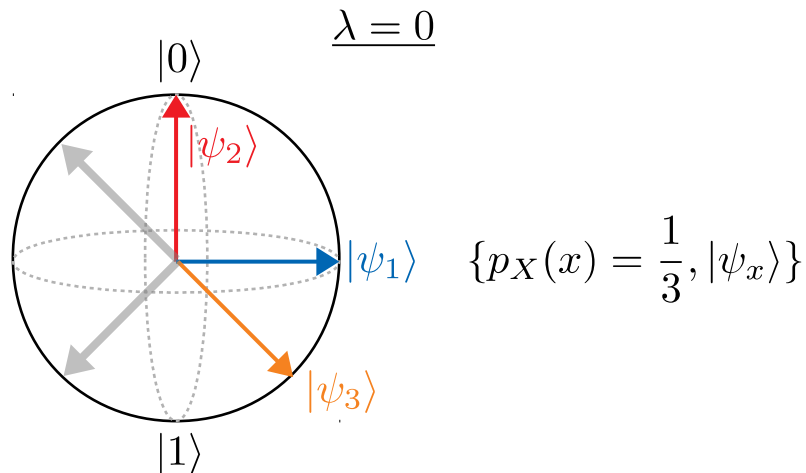
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QUANTUM VIOLATION

$$p(\lambda = 0) = q$$



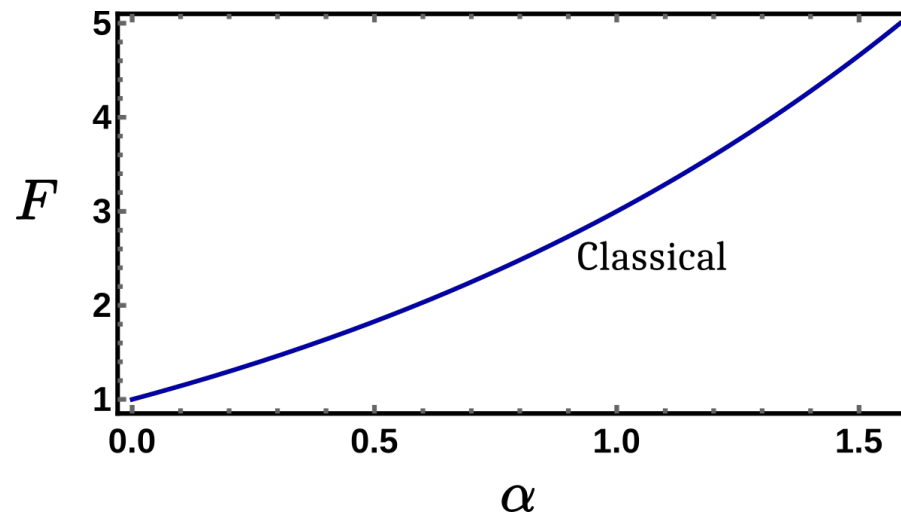
$$\lambda = 1$$

No communication.

$b = 1$ always

Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

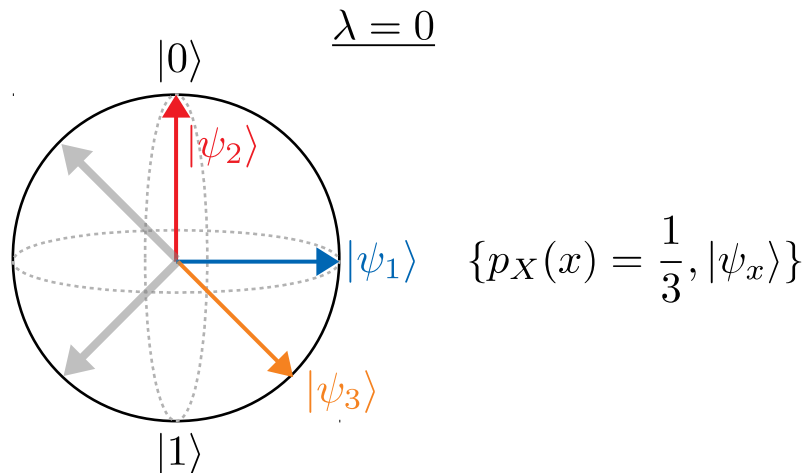
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QUANTUM VIOLATION

$$p(\lambda = 0) = q$$

$$\mathcal{I}_X = \log_2(1 + q)$$



$$\underline{\lambda = 0}$$

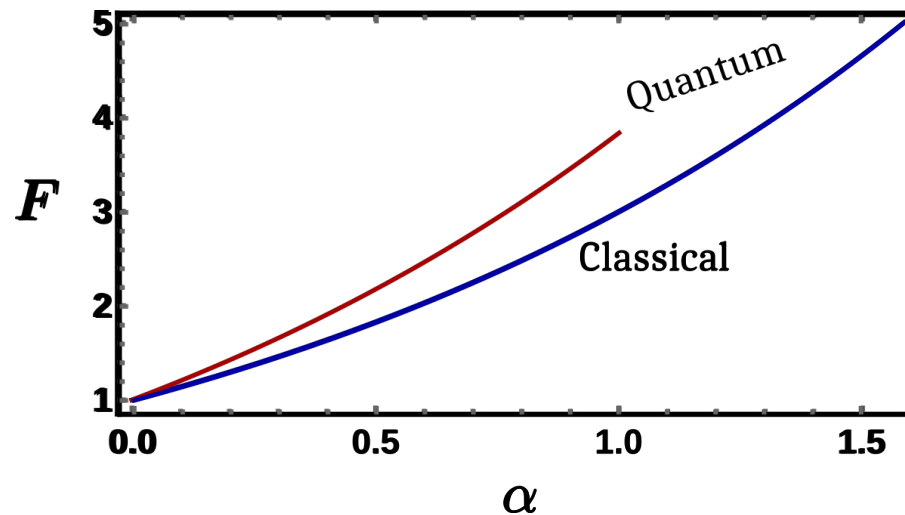
$$\underline{\lambda = 1}$$

No communication.

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Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

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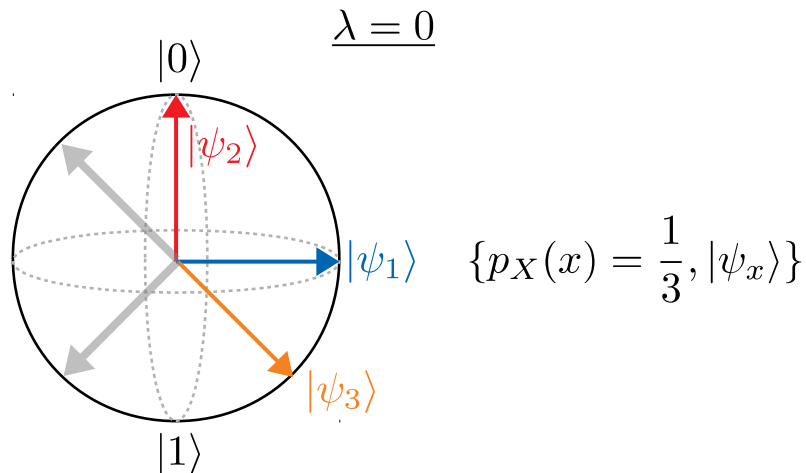


$$1 + 2\sqrt{2}q$$

QUANTUM VIOLATION

$$p(\lambda = 0) = q$$

$$\mathcal{I}_X = \log_2(1 + q)$$



$$\underline{\lambda = 0}$$

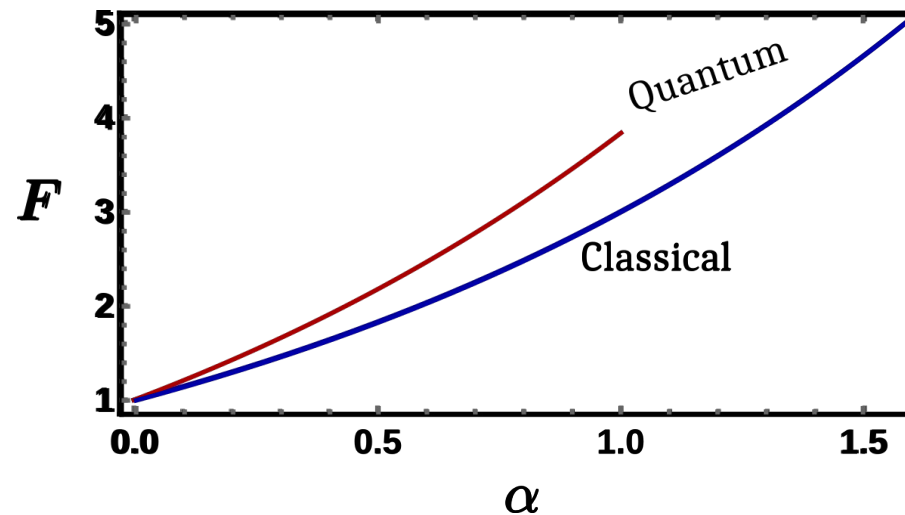
$$\underline{\lambda = 1}$$

No communication.

$b = 1$ always

Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

$$F = -(E_{11} + E_{12} + E_{21}) + (E_{22} + E_{31}) \leq 2^{\alpha+1} - 1 \quad E_{xy} = p(0|xy) - p(1|xy)$$



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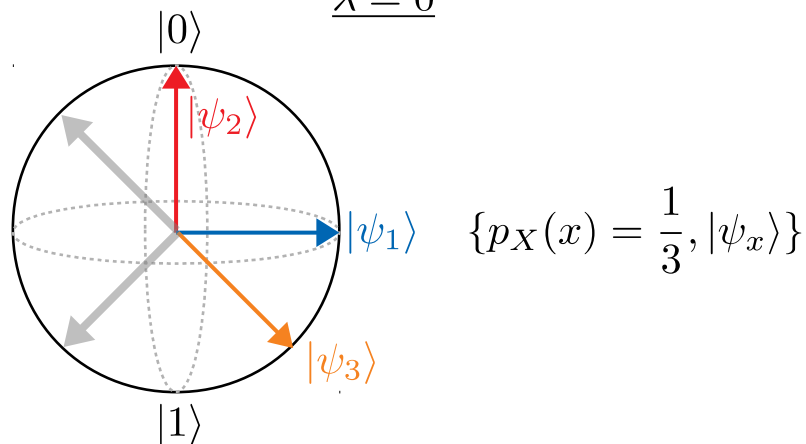
QUANTUM VIOLATION

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$$\underline{\lambda = 1}$$

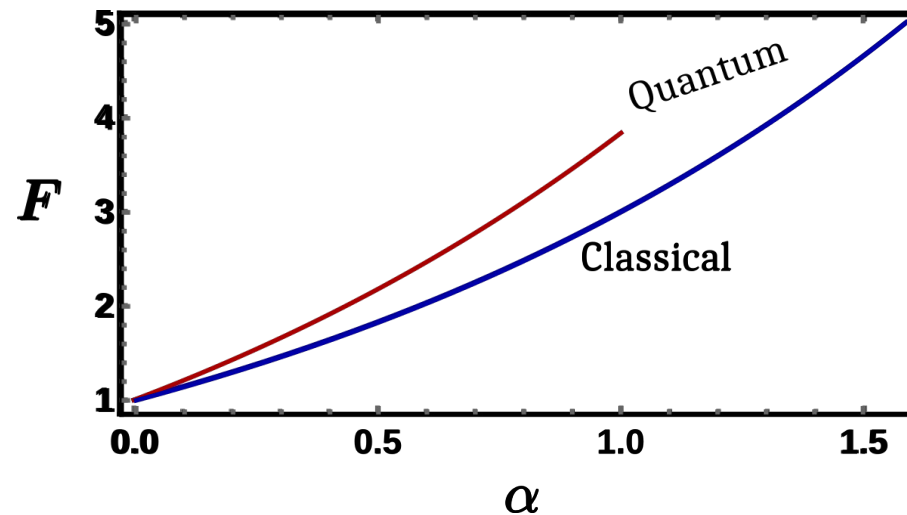
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Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

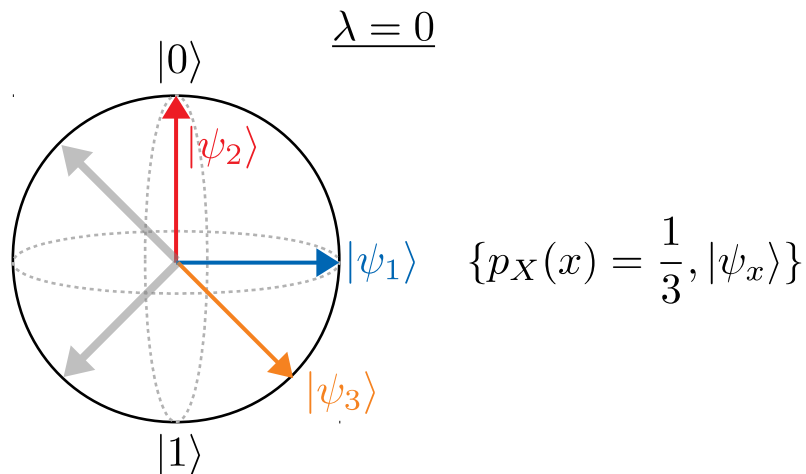
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QUANTUM VIOLATION

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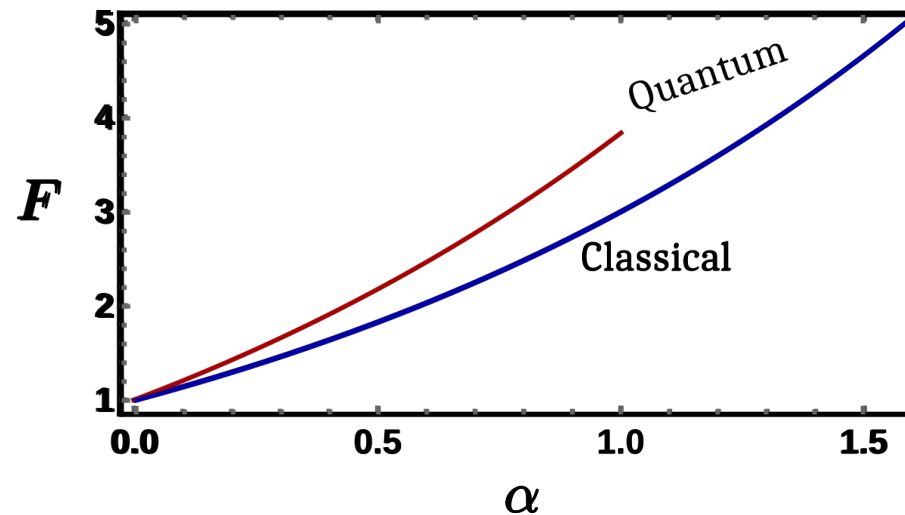


$$\underline{\lambda = 0}$$

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Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

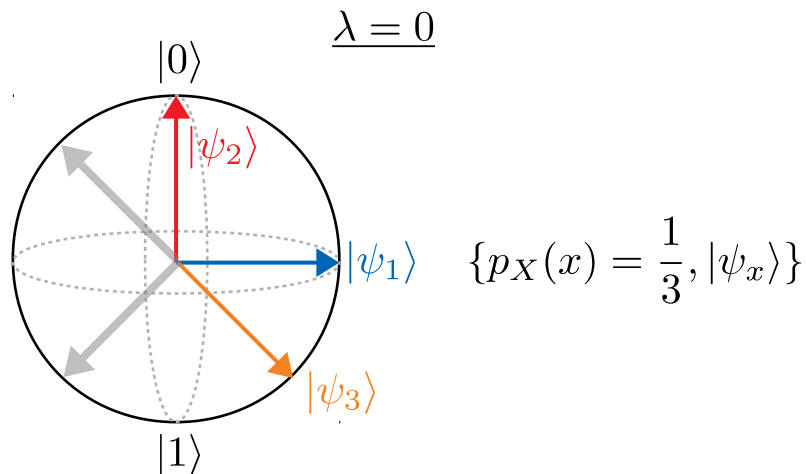
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QUANTUM VIOLATION

$$p(\lambda = 0) = q$$



$$\underline{\lambda = 0}$$

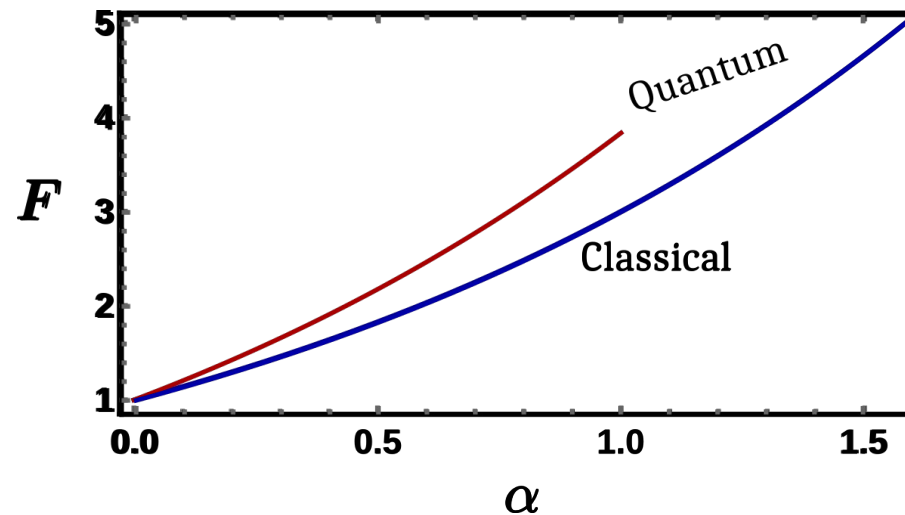
$$\underline{\lambda = 1}$$

Send input

$$F = 5$$

Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

$$F = -(E_{11} + E_{12} + E_{21}) + (E_{22} + E_{31}) \leq 2^{\alpha+1} - 1 \quad E_{xy} = p(0|xy) - p(1|xy)$$

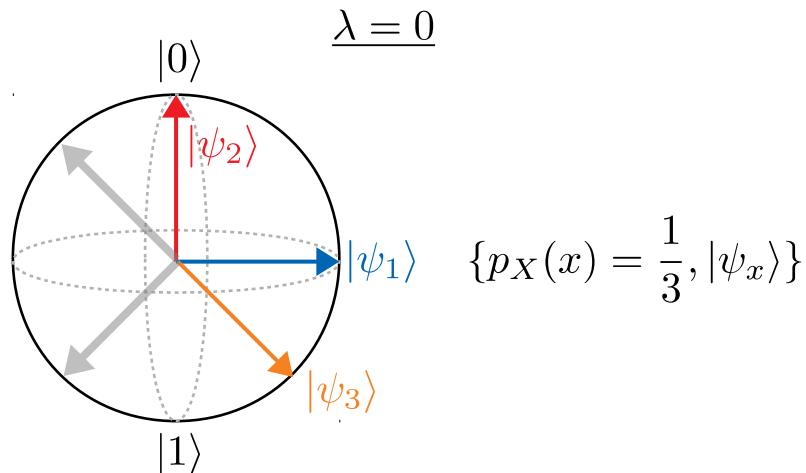


$$1 + 2\sqrt{2}q$$

QUANTUM VIOLATION

$$p(\lambda = 0) = q$$

$$\mathcal{I}_X = \log_2(3 - q)$$



$$\underline{\lambda = 0}$$

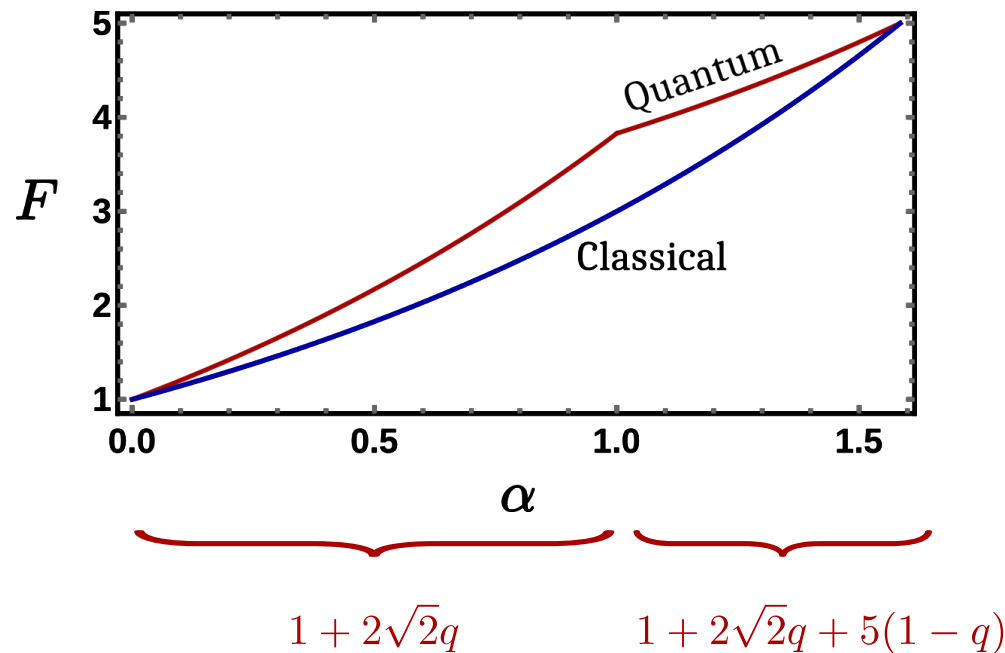
$$\underline{\lambda = 1}$$

Send input

$$F = 5$$

Facet inequality / “information witness” - for 3 inputs (Alice), 2 inputs, 2 outputs (Bob)

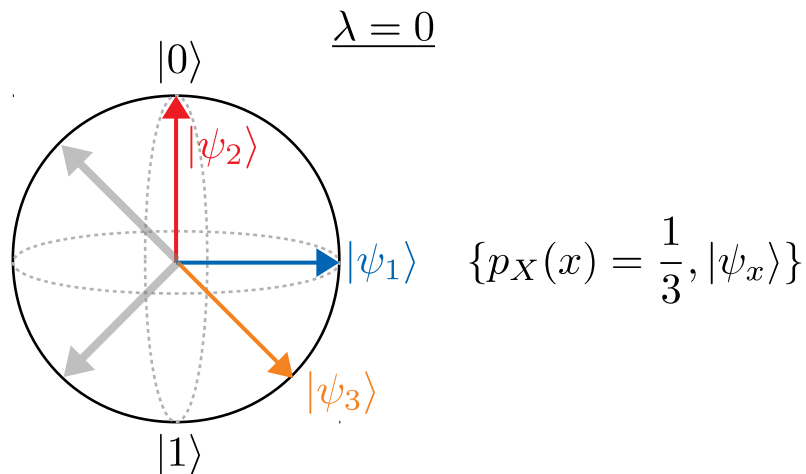
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QUANTUM VIOLATION

$$p(\lambda = 0) = q$$

$$\mathcal{I}_X = \log_2(3 - q)$$



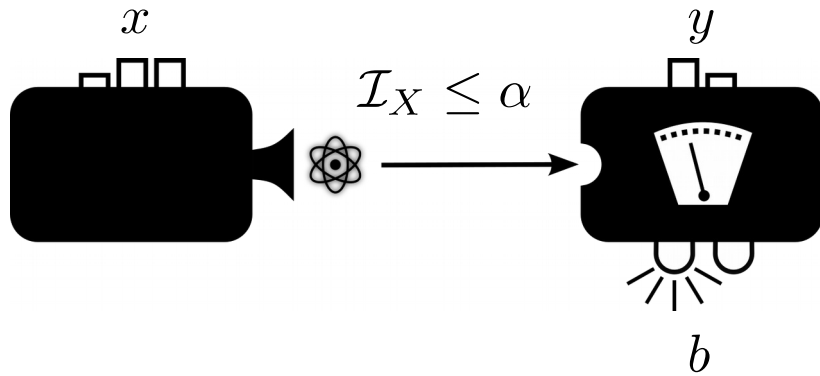
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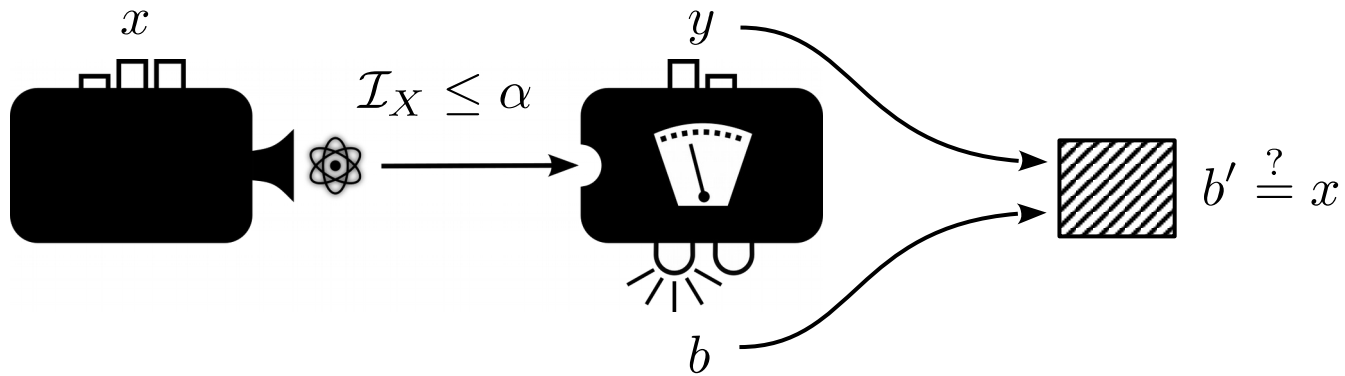
THEORY-INDEPENDENT BOUND

Post-processing of output \rightarrow lower bound on information given observed data



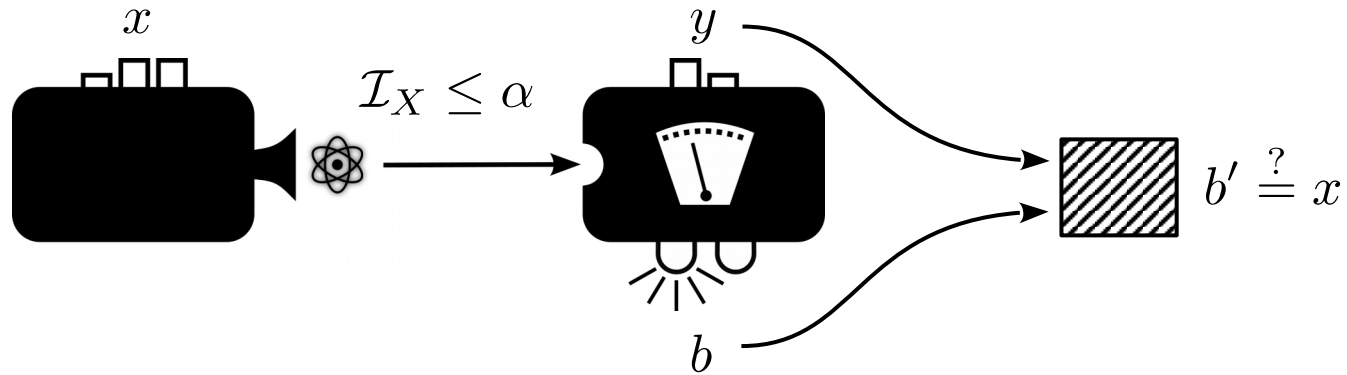
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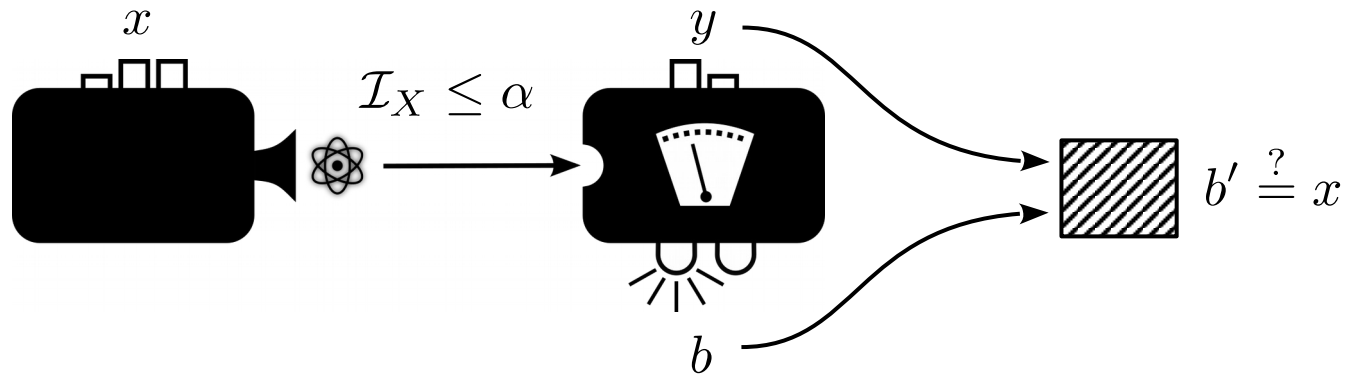


How well can we recover
the input given the data?

$$p(b|xy)$$

THEORY-INDEPENDENT BOUND

Post-processing of output \rightarrow lower bound on information given observed data



How well can we recover
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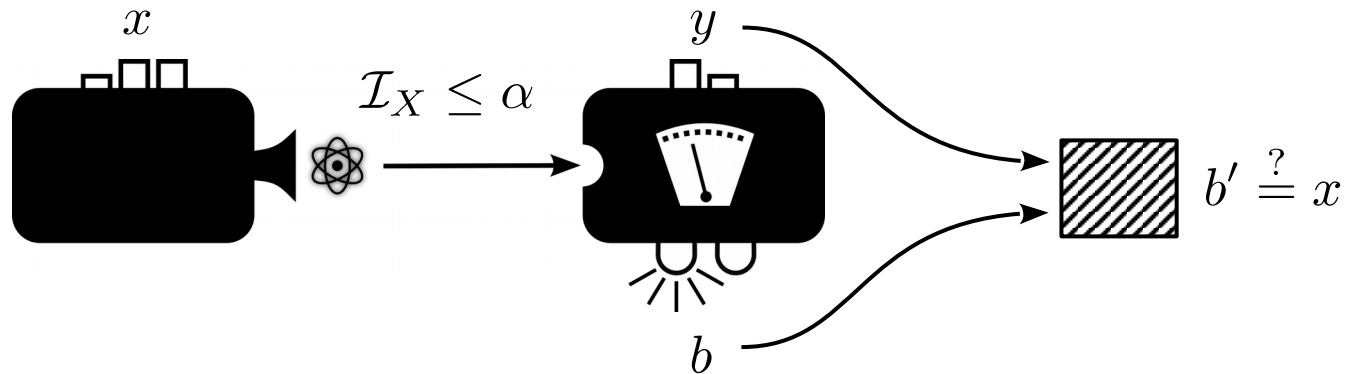
$$p(b|xy)$$

For any input for Bob and any (classical) post-processing

$$\sum_{x,b} p_X(x) p(b|xy) p(b' = x|y, b) \leq 2^{\alpha - H_{\min}(X)}$$

THEORY-INDEPENDENT BOUND

Post-processing of output \rightarrow lower bound on information given observed data



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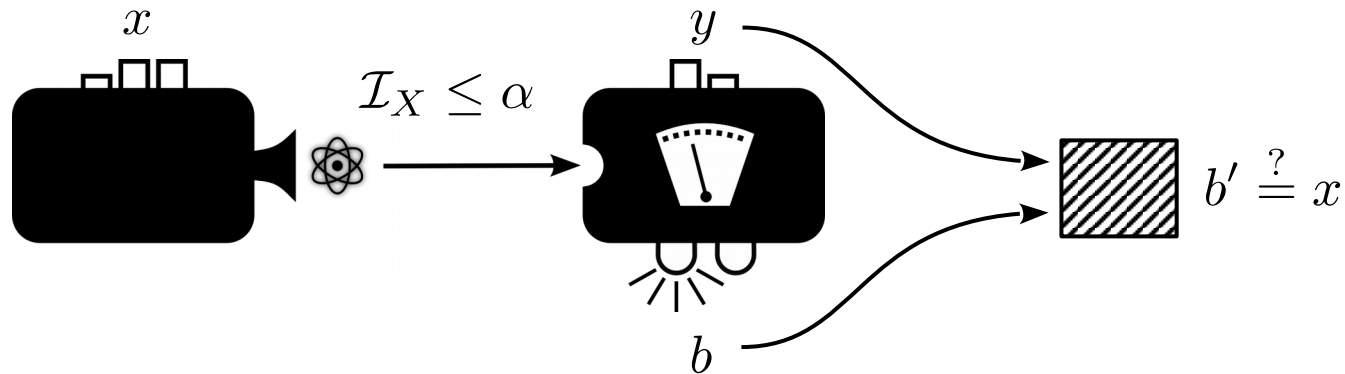
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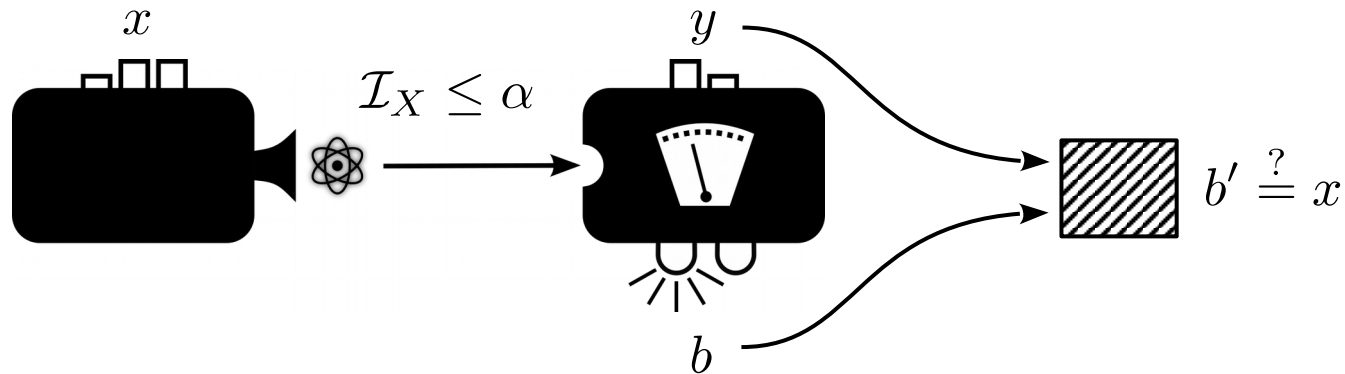
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observed data

post-processing

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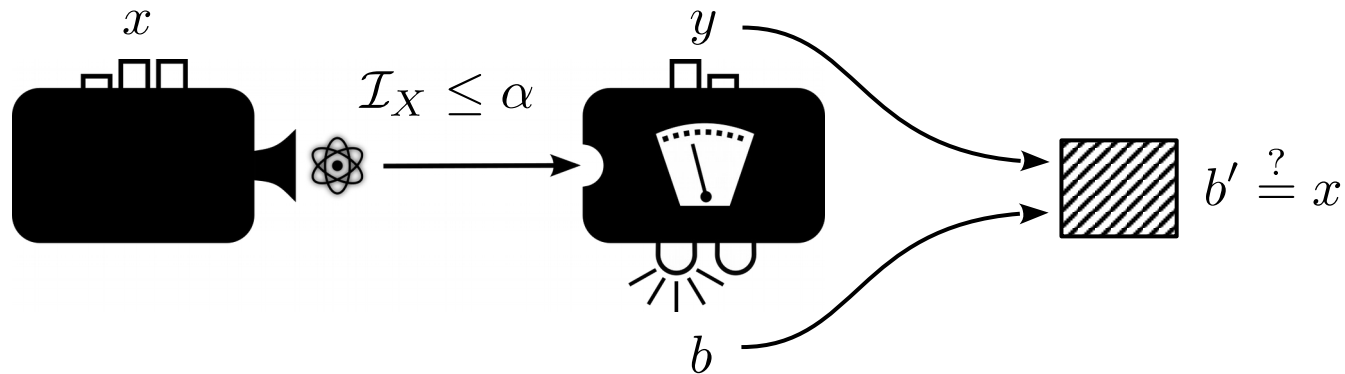
observed data

information in message

post-processing

THEORY-INDEPENDENT BOUND

Post-processing of output \rightarrow lower bound on information given observed data



How well can we recover the input given the data?

$$p(b|xy)$$

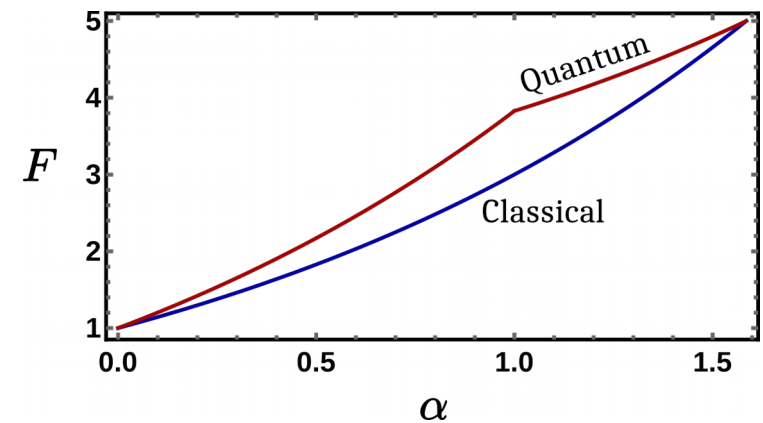
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observed data

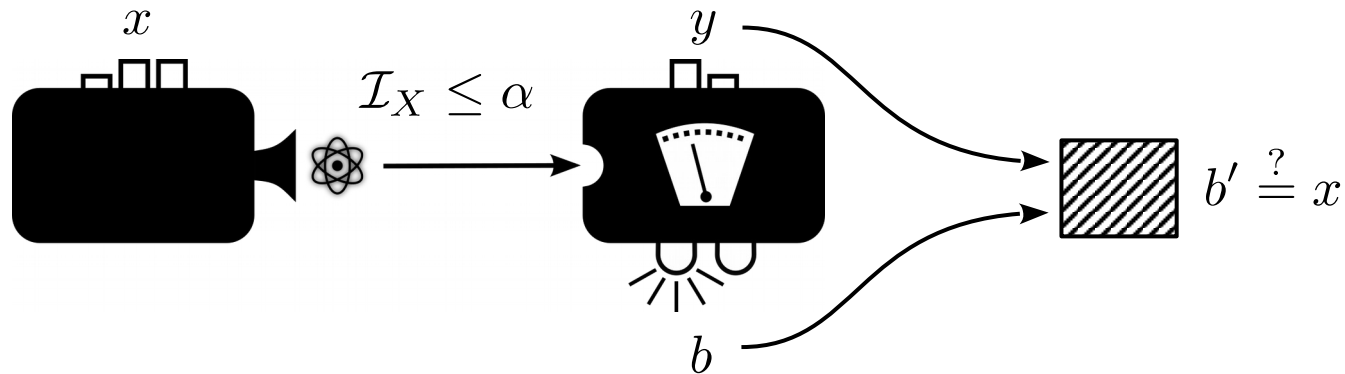
information in message

post-processing



THEORY-INDEPENDENT BOUND

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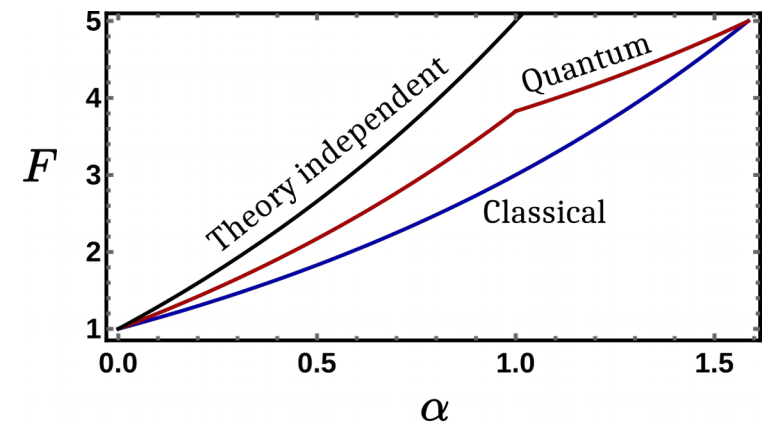
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INFORMATION VS. DIMENSION

Correlation achievable with qudits (dim. d)



Achievable with $\mathcal{I}_X \leq \log_2(d)$

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INFORMATION VS. DIMENSION

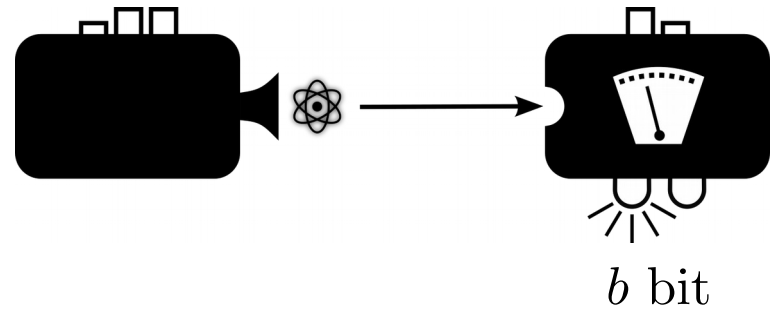
Correlation achievable with qudits (dim. d)



Achievable with $\mathcal{I}_X \leq \log_2(d)$

Example : random access code

(x_1, x_2, x_3, x_4) bits



INFORMATION VS. DIMENSION

Correlation achievable with qudits (dim. d)

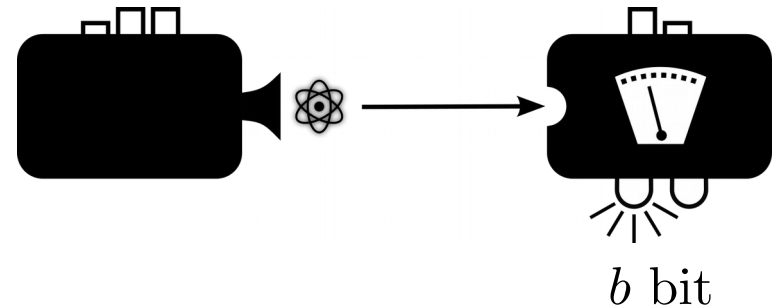


Achievable with $\mathcal{I}_X \leq \log_2(d)$

Example : random access code

$$F_{RAC} = \frac{1}{64} \sum_{x,y} p(b = x_y | x, y)$$

(x_1, x_2, x_3, x_4) bits



INFORMATION VS. DIMENSION

Correlation achievable with qudits (dim. d)

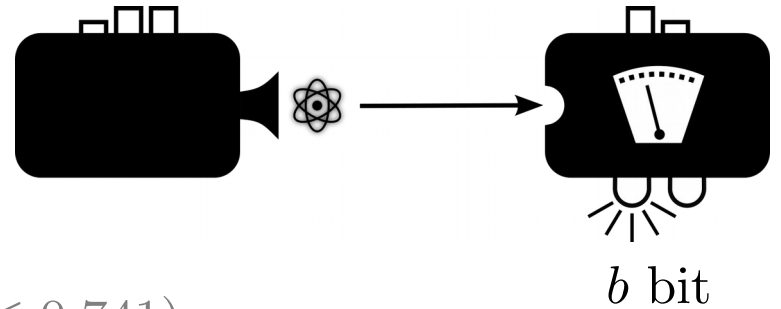


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(x_1, x_2, x_3, x_4) bits



For qubits: $F_{RAC} < 3/4$ (and probably $F_{RAC} \leq 0.741$)

INFORMATION VS. DIMENSION

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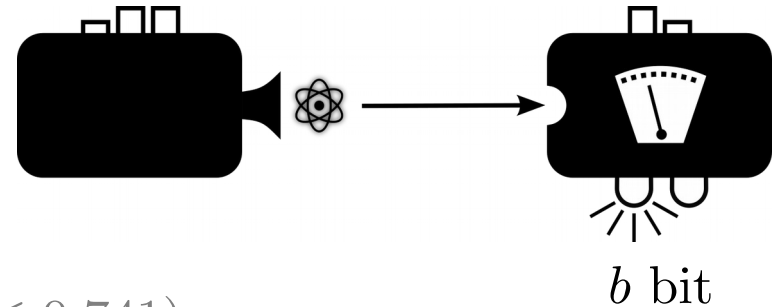


Achievable with $\mathcal{I}_X \leq \log_2(d)$

Example : random access code

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(x_1, x_2, x_3, x_4) bits



For qubits: $F_{RAC} < 3/4$ (and probably $F_{RAC} \leq 0.741$)

For bounded information: $F_{RAC} = 3/4$ with $\mathcal{I}_X = 1$

INFORMATION VS. DIMENSION

Correlation achievable with qudits (dim. d)

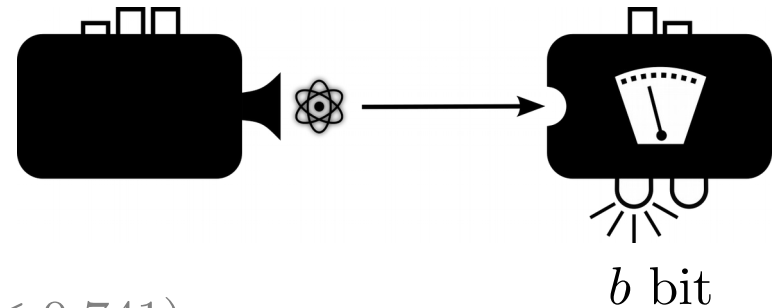


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Example : random access code

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For bounded information: $F_{RAC} = 3/4$ with $\mathcal{I}_X = 1$

States

$$\rho_x = \frac{1}{8} (2\mathbb{1} \otimes \mathbb{1} - (-1)^{x_4} \mathbb{1} \otimes \sigma_y - (-1)^{x_1} \sigma_x \otimes \sigma_x - (-1)^{x_2} \sigma_y \otimes \sigma_x - (-1)^{x_3} \sigma_z \otimes \sigma_x)$$

INFORMATION VS. DIMENSION

Correlation achievable with qudits (dim. d)

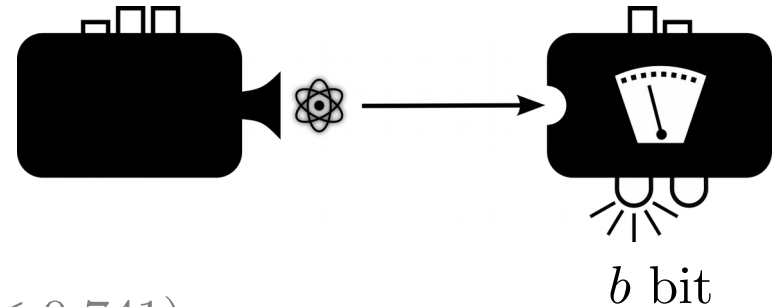


Achievable with $\mathcal{I}_X \leq \log_2(d)$

Example : random access code

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(x_1, x_2, x_3, x_4) bits

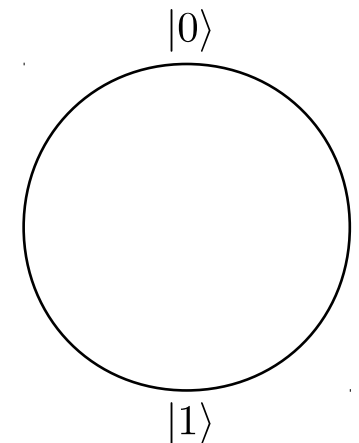


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INFORMATION VS. DIMENSION

Correlation achievable with qudits (dim. d)

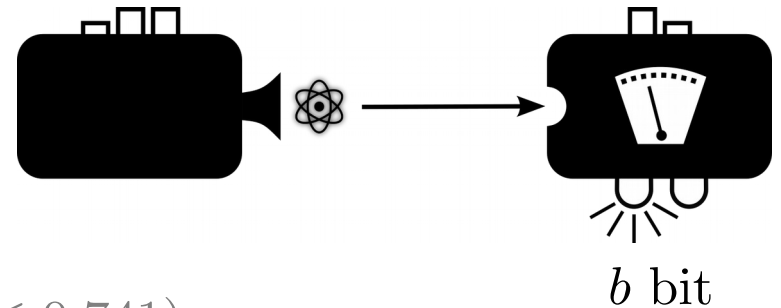


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(x_1, x_2, x_3, x_4) bits

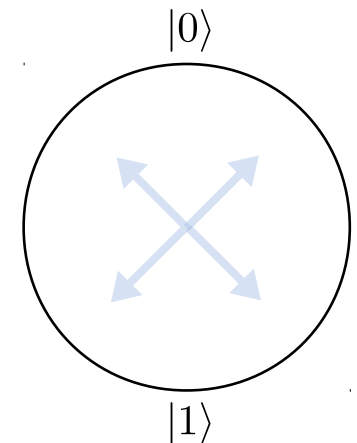


For qubits: $F_{RAC} < 3/4$ (and probably $F_{RAC} \leq 0.741$)

For bounded information: $F_{RAC} = 3/4$ with $\mathcal{I}_X = 1$

States

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INFORMATION VS. DIMENSION

Correlation achievable with qudits (dim. d)

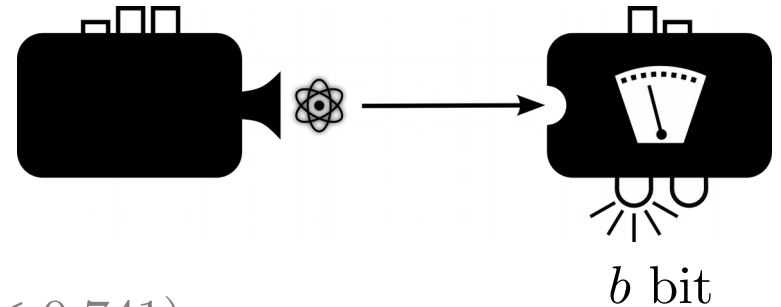


Achievable with $\mathcal{I}_X \leq \log_2(d)$

Example : random access code

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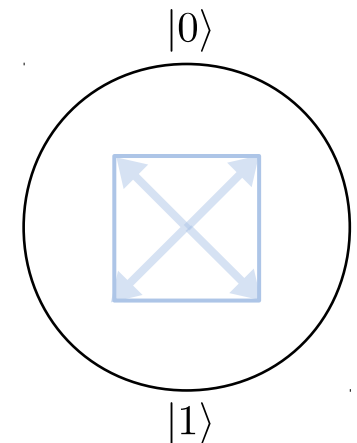


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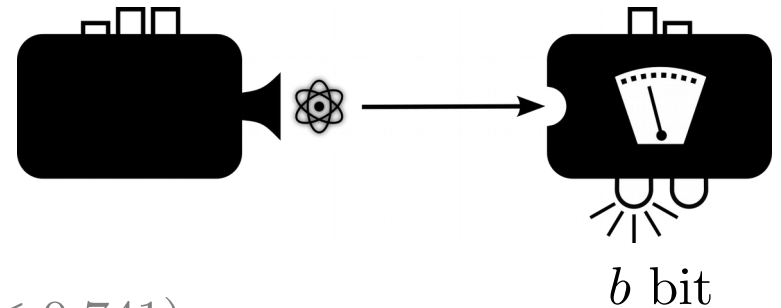


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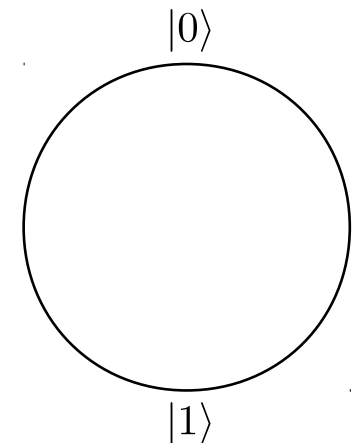


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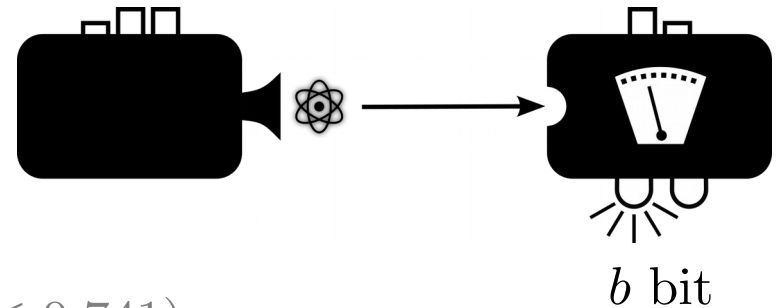


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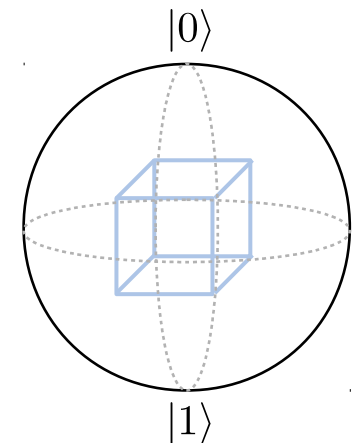


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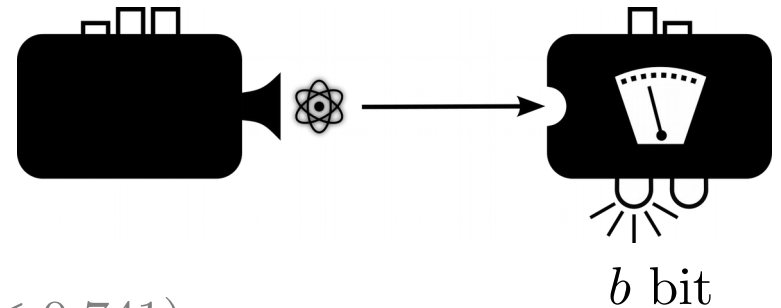


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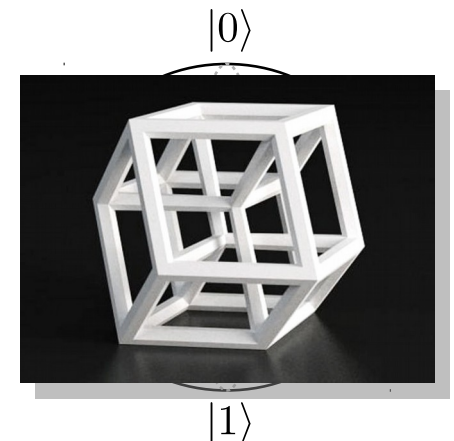


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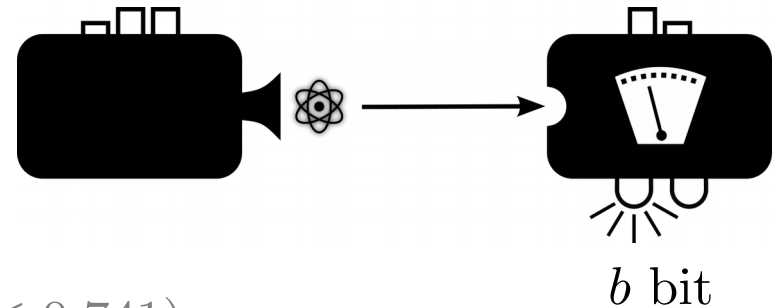


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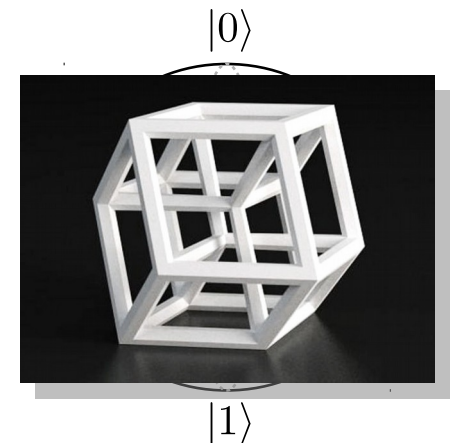
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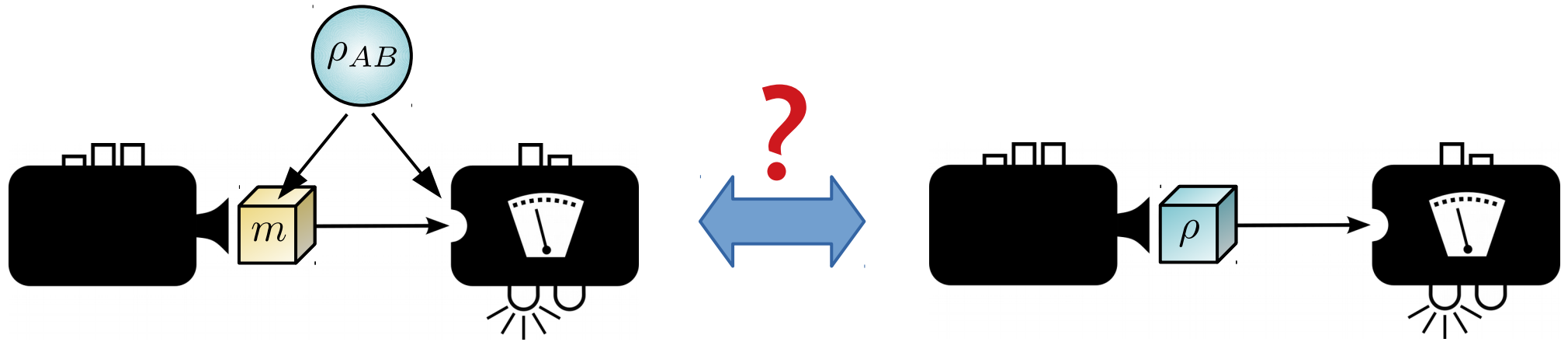
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Measurements

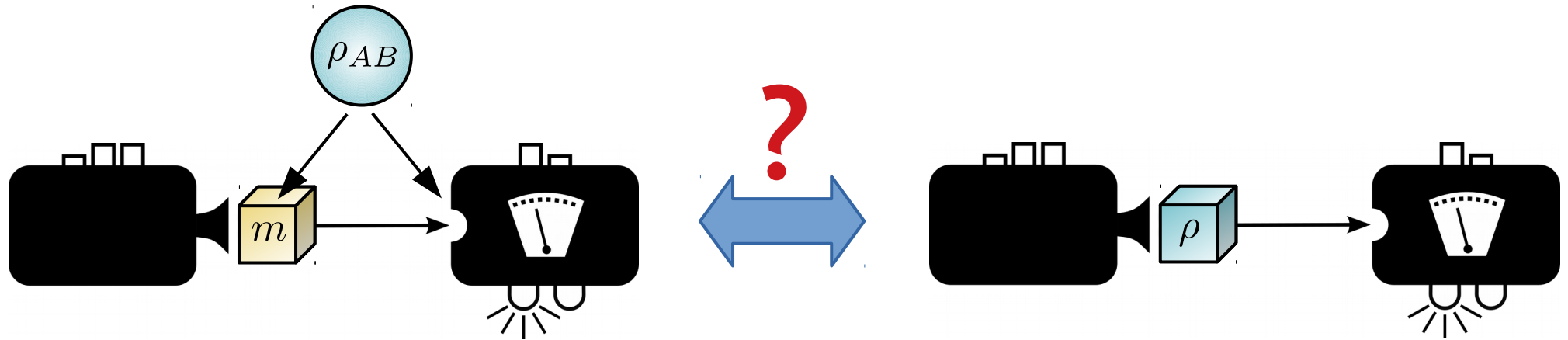
$$\begin{array}{ll} \sigma_x \otimes \sigma_x & \sigma_z \otimes \sigma_x \\ \sigma_y \otimes \sigma_x & \mathbb{1} \otimes \sigma_y \end{array}$$



ENTANGLEMENT-ASSISTED CLASSICAL COMM. VS. QUANTUM COMM.



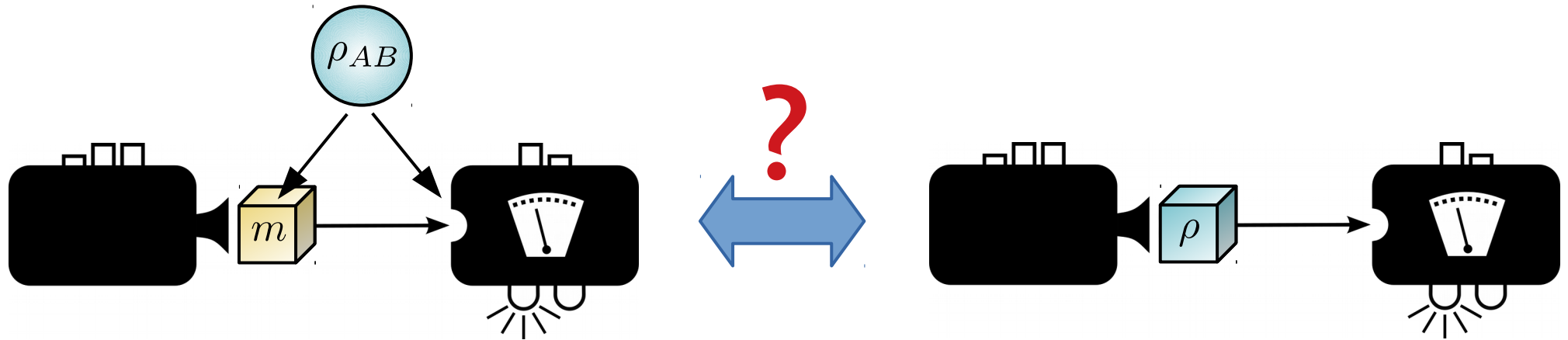
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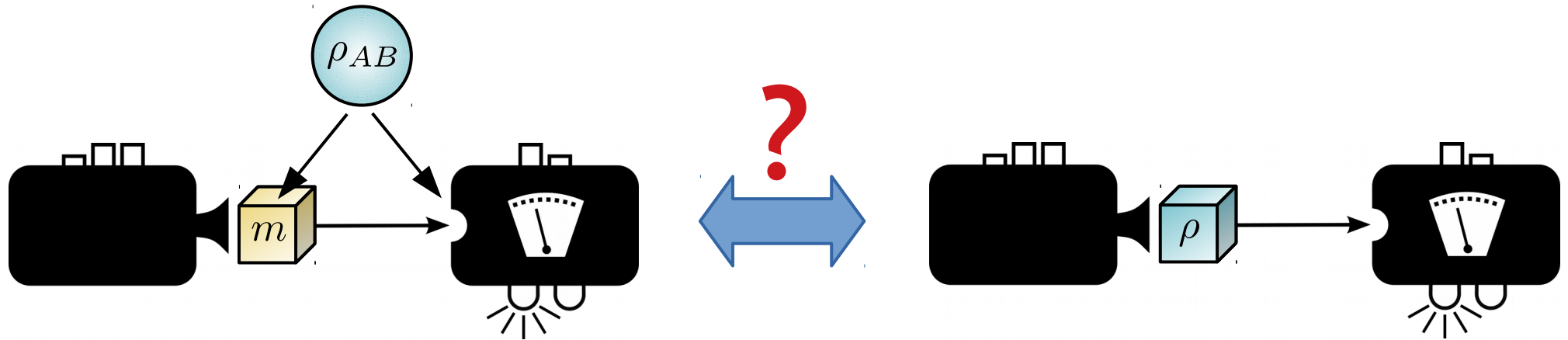


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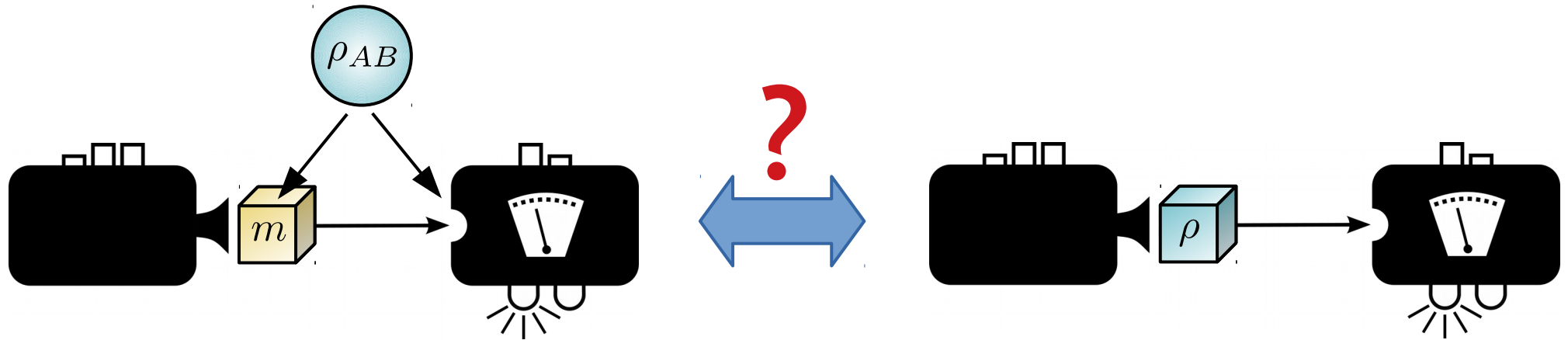
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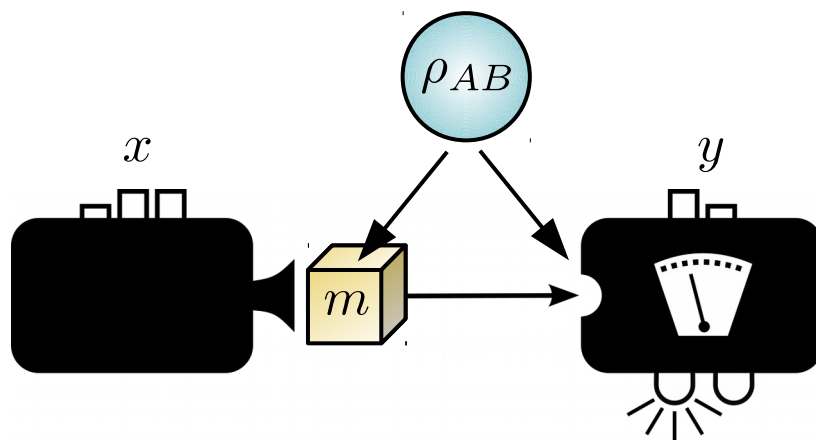
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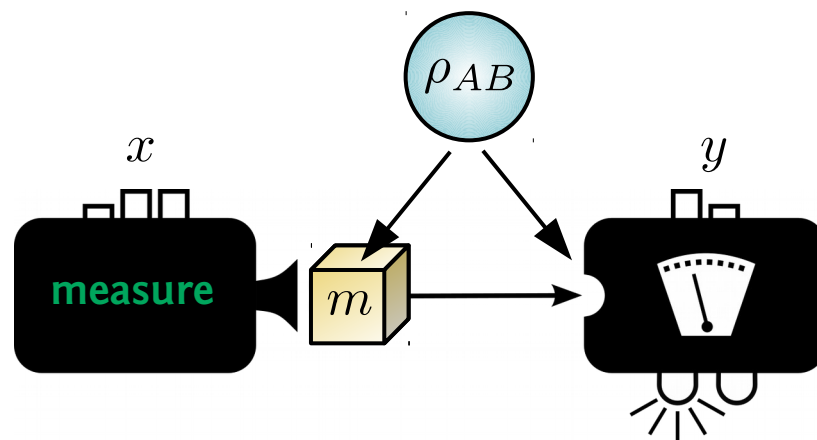
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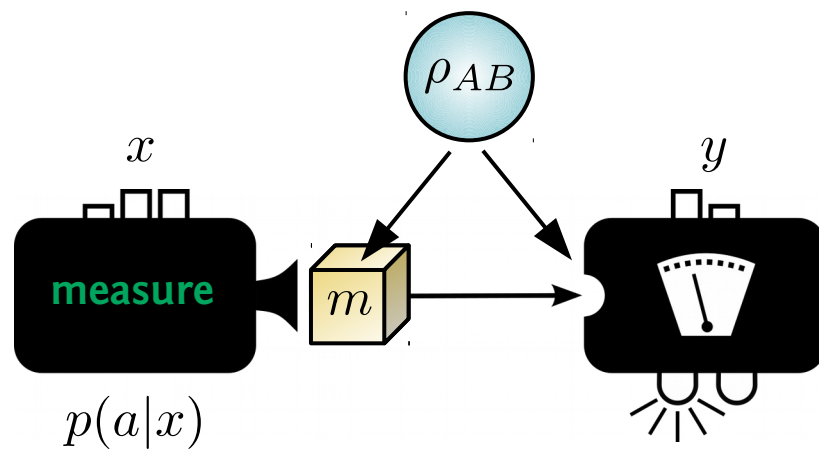
Entanglement-assisted classical communication



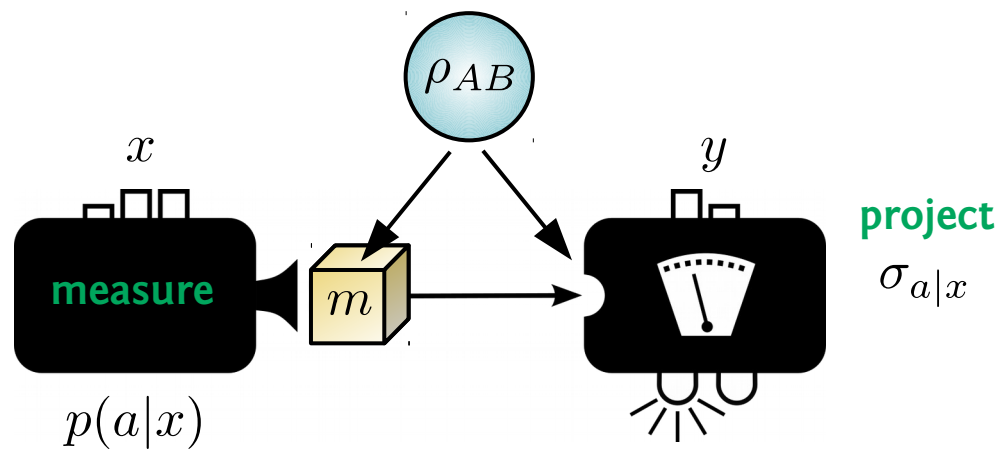
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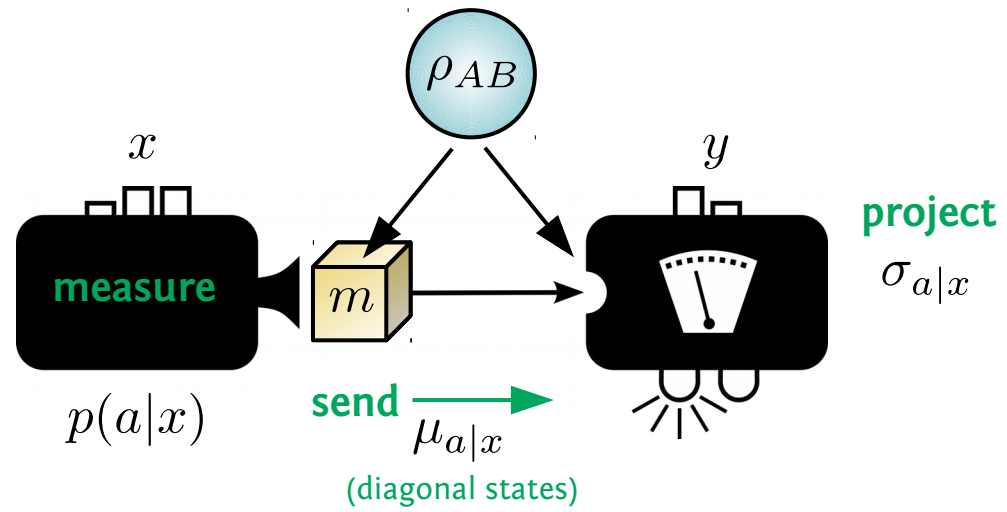
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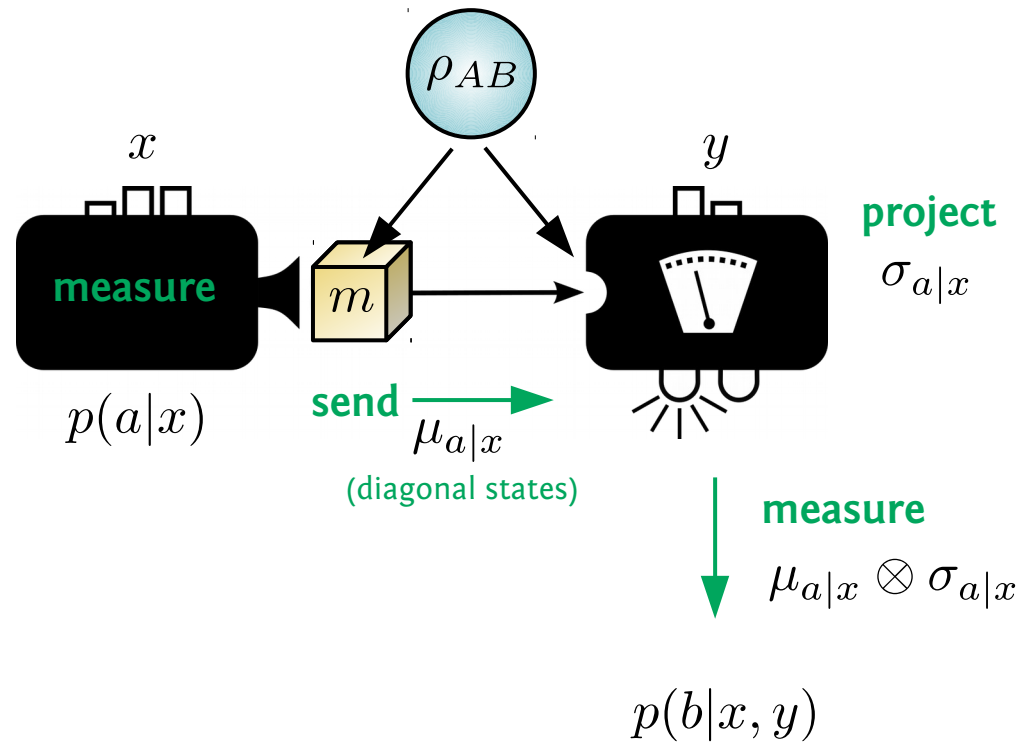
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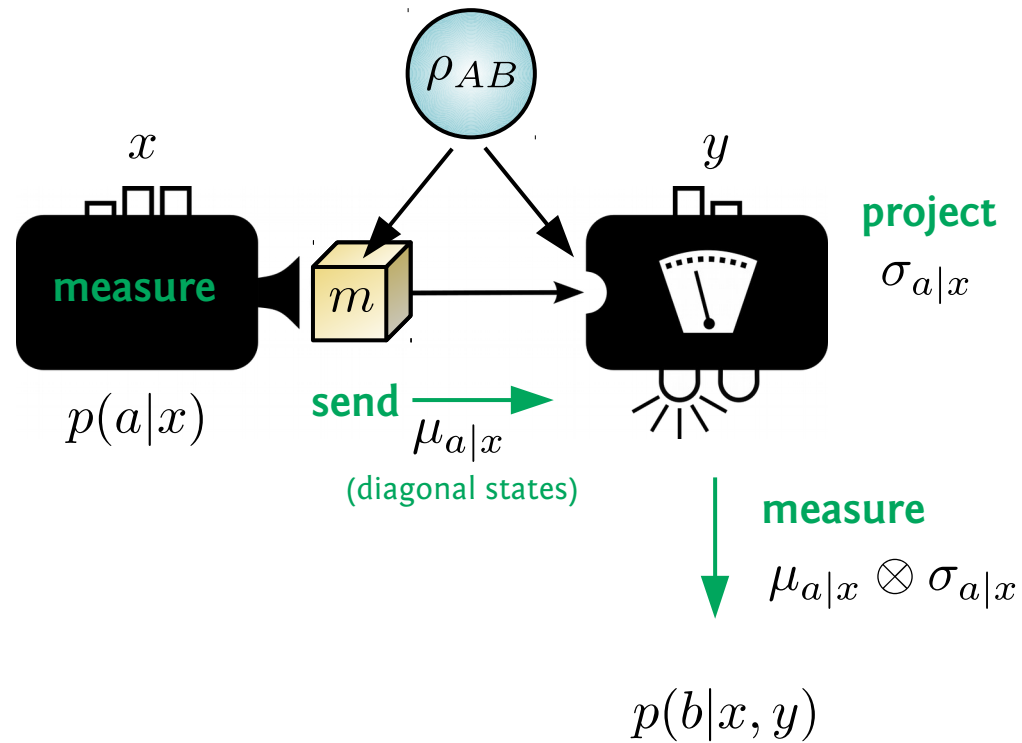
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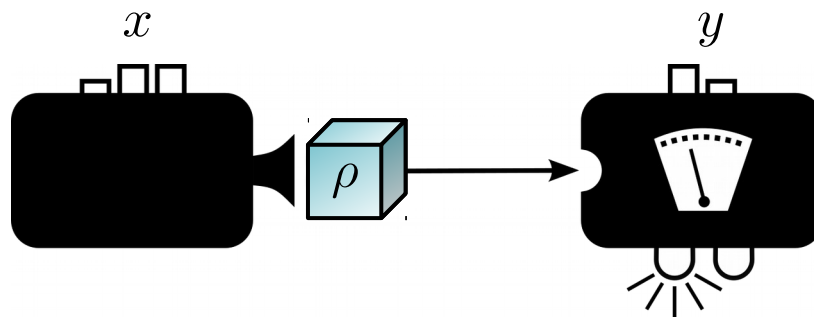
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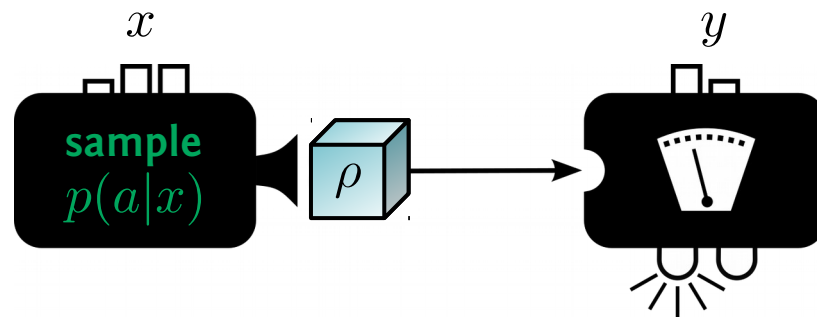
Information cost:

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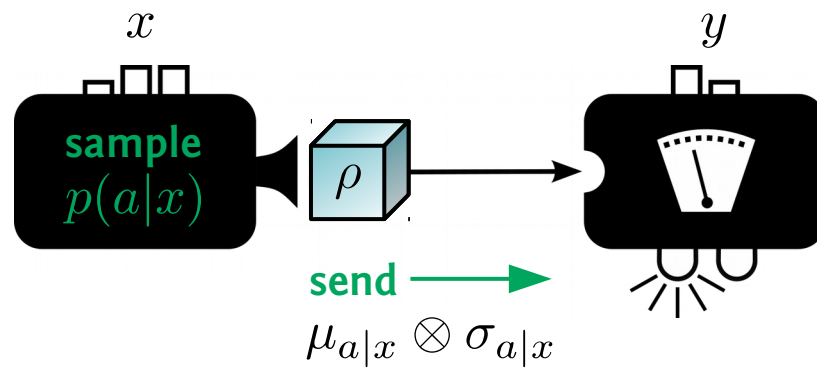
Quantum communication



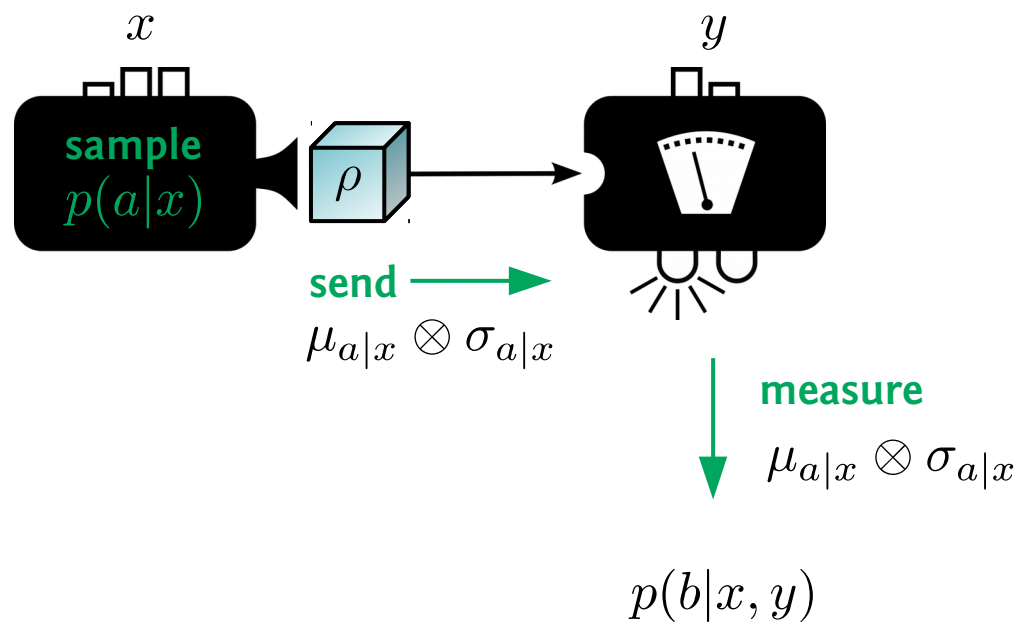
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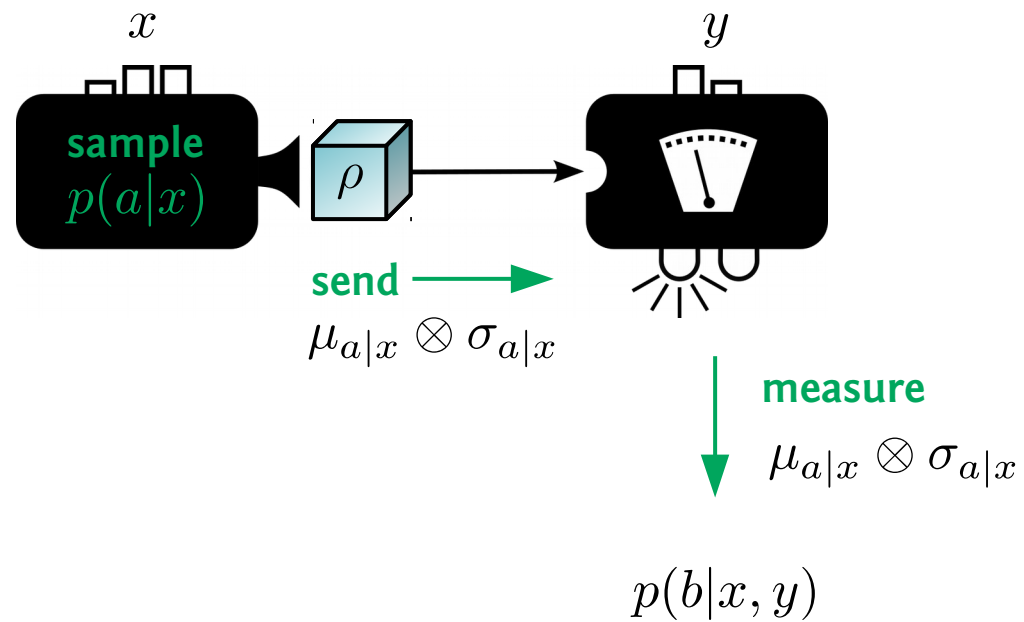
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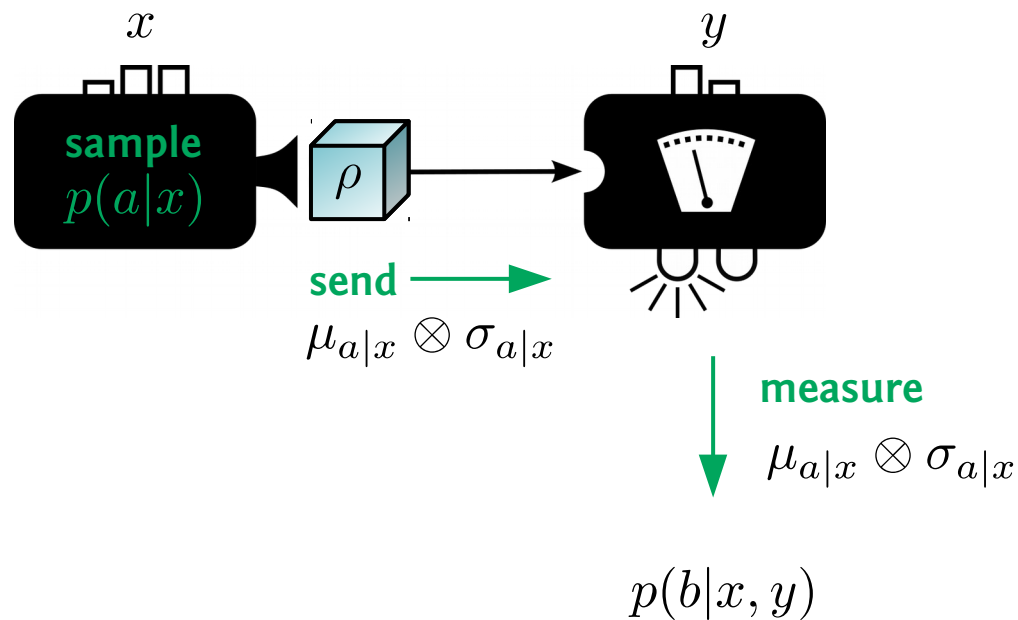


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Reproduces the same correlation as in the entanglement-assisted case.

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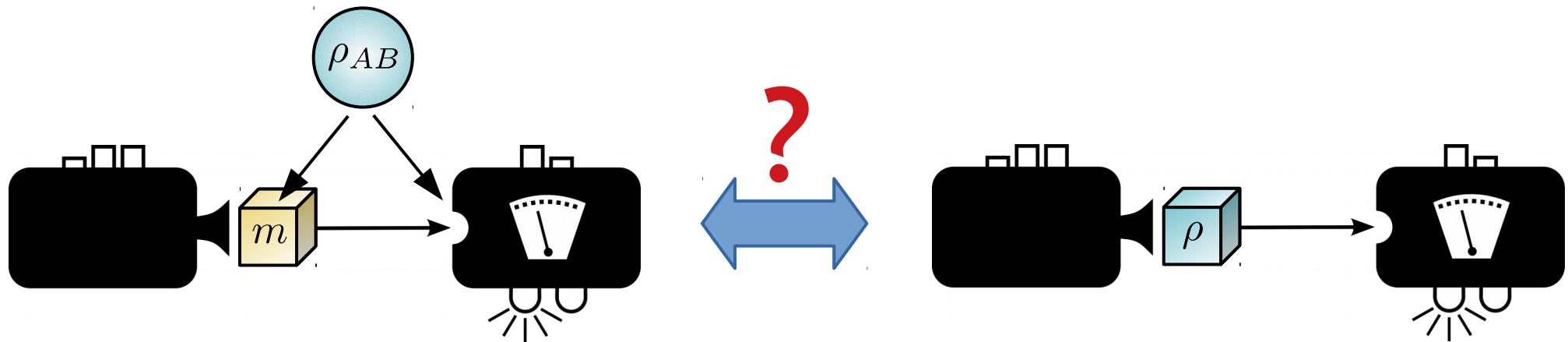


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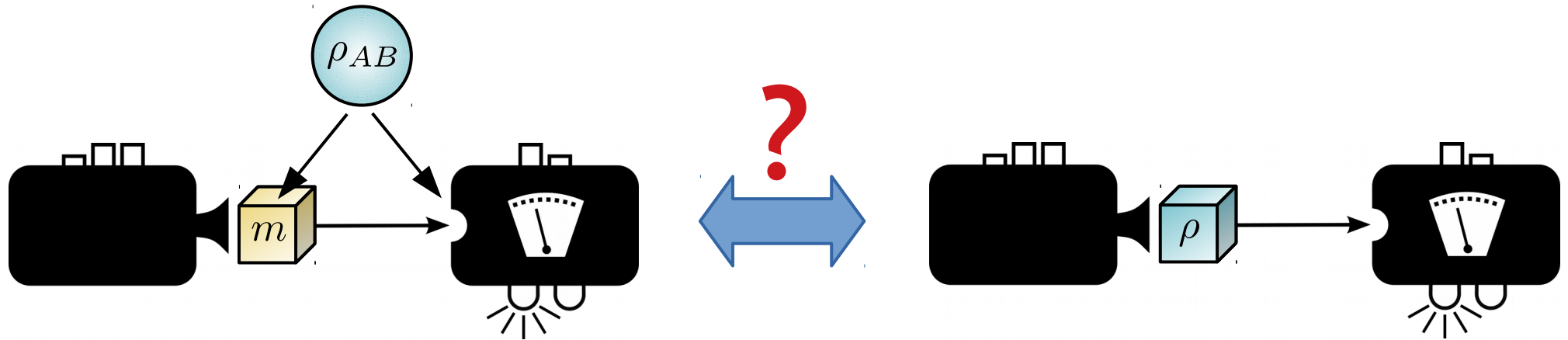
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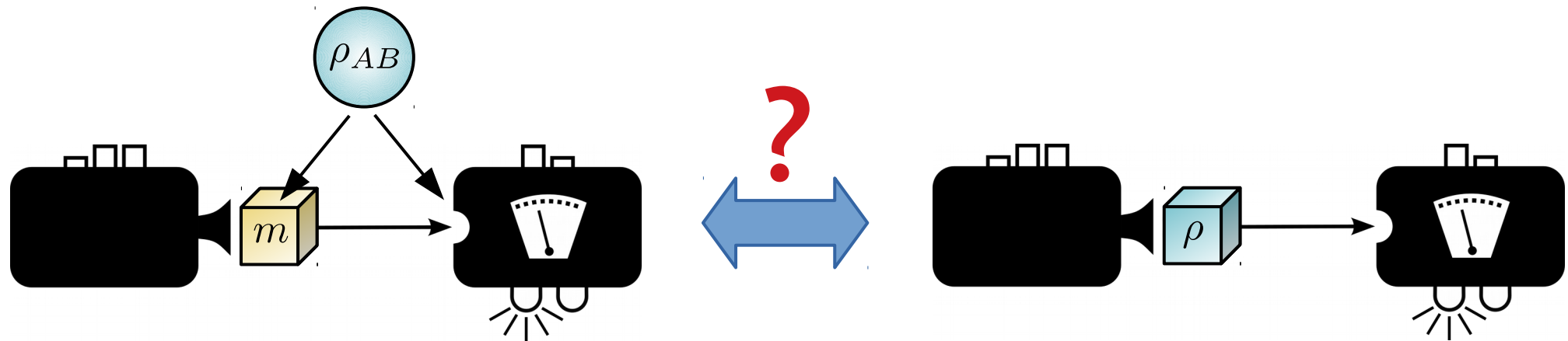
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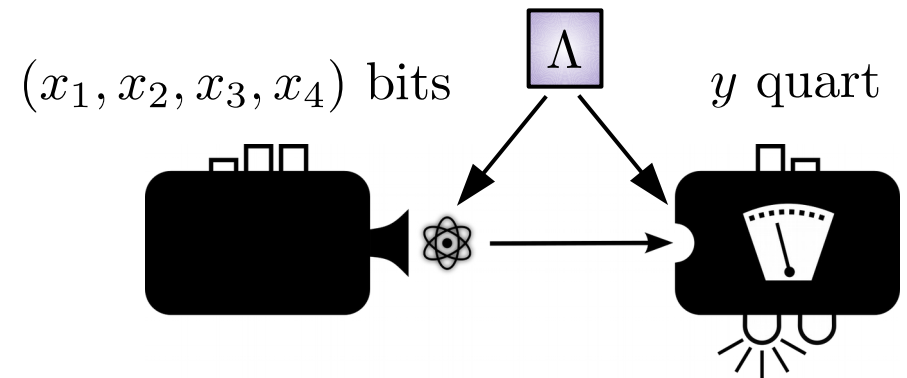
$$\text{because } \text{Tr}[(\mu_{a|x} \otimes \sigma_{a|x}) N_x] \leq \text{Tr}[\mu_{a|x} \text{Tr}_B[N_x]]$$

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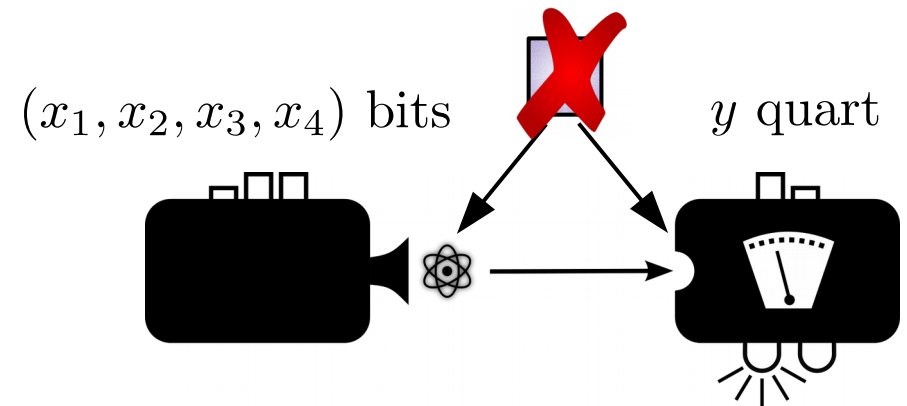
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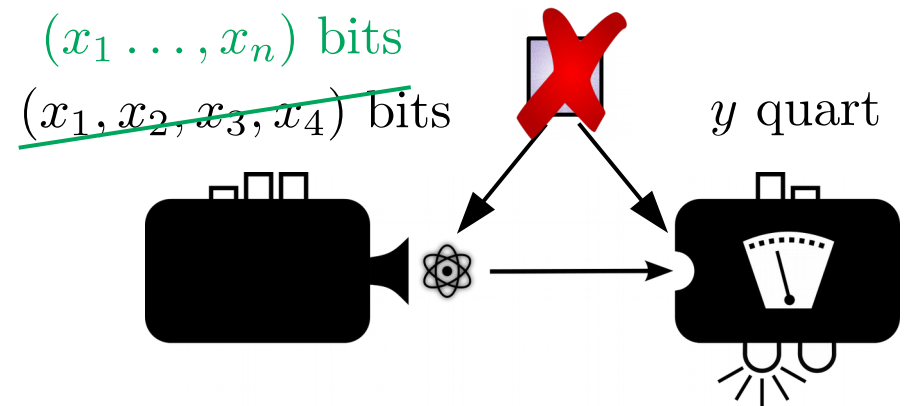
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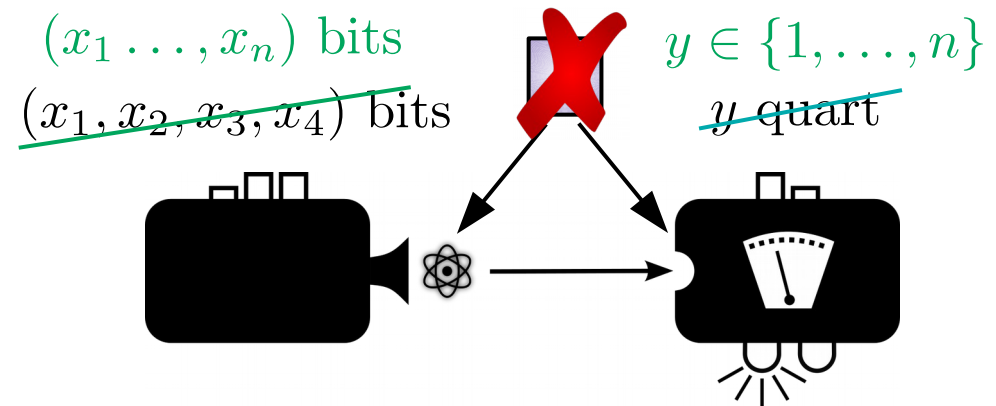
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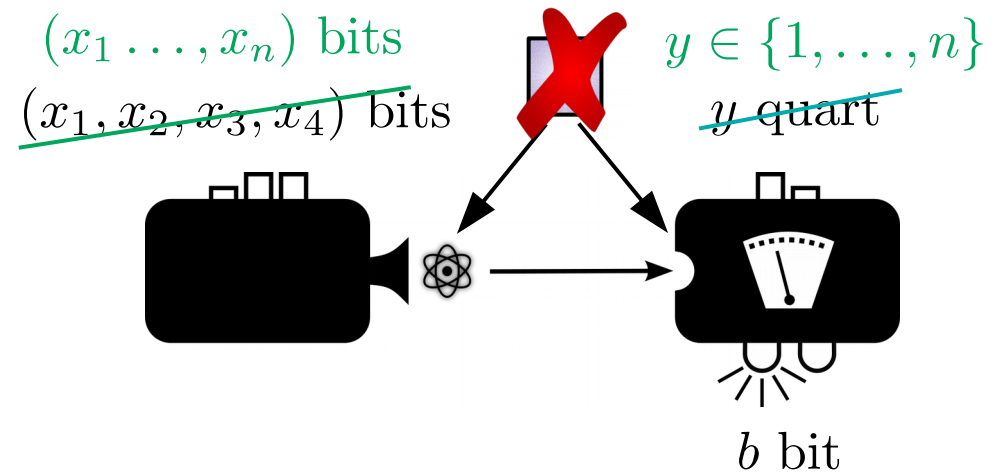
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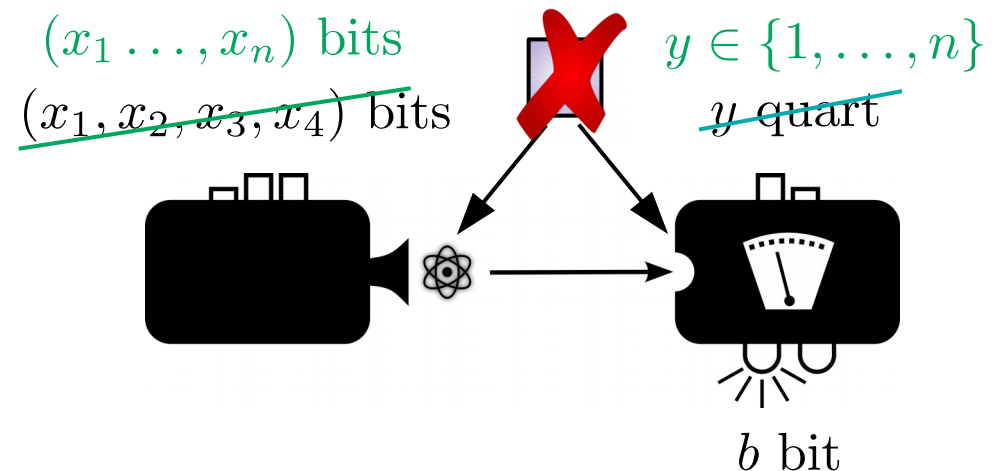
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Bell inequalities + q. comm > ent.-assisted. class. comm.



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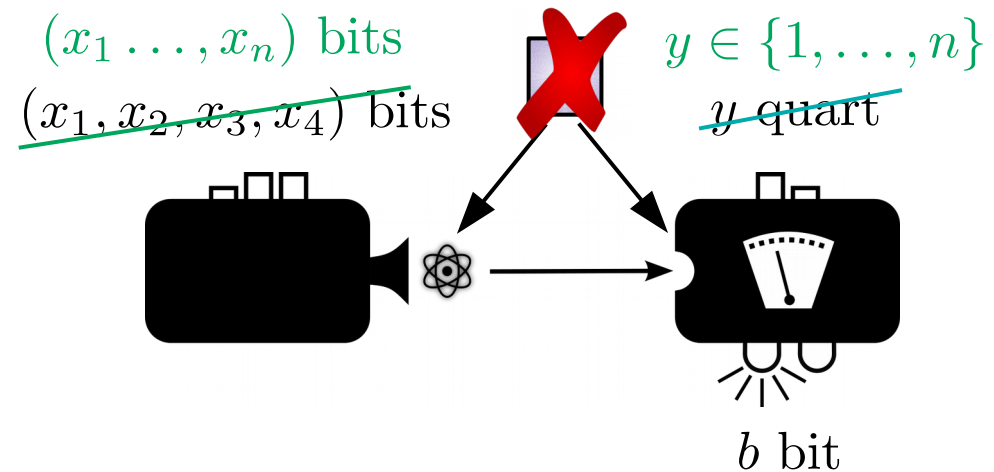
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} Unbounded separation

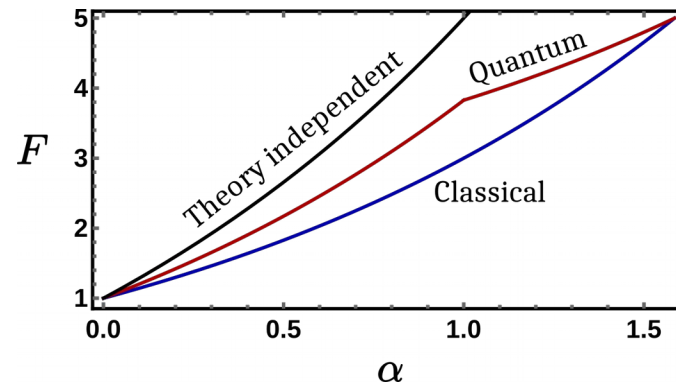
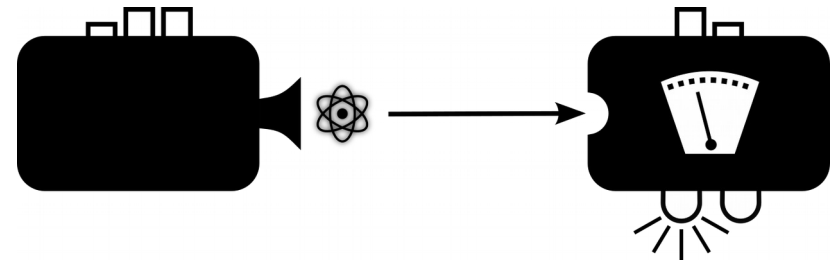
SUMMARY

arXiv:1909.05656

Bounding information – ability to guess the input from the message.

Alternative to bounding dimension, entropy, overlap, energy,...

- Separate classical from quantum correlations.
- Device-independent bound on the information.



- Stronger correlations with same/less information as dimension-bounded schemes.
- Restore hierarchy of quantum communication vs. entanglement-assisted classical communication.

