Measurement incompatibility and steering are necessary and sufficient for contextuality

A one-to-one mapping from nonclassical entities to nonclassical correlations.

In collaboration with Roope Uola

arXiv: 1905.03614

T1

From Bell nonlocality to nonclassical features

	$\not\Leftarrow$	Entanglement
Bell nonlocality	$\Rightarrow$	Measurement incompatibility
	$\not\Leftarrow$	Steerability

<u>Joint measurability</u>

$$M_{a|x} = \sum_{\lambda} p(a|x,\lambda) G_{\lambda}$$

<u>Unsteerability</u>

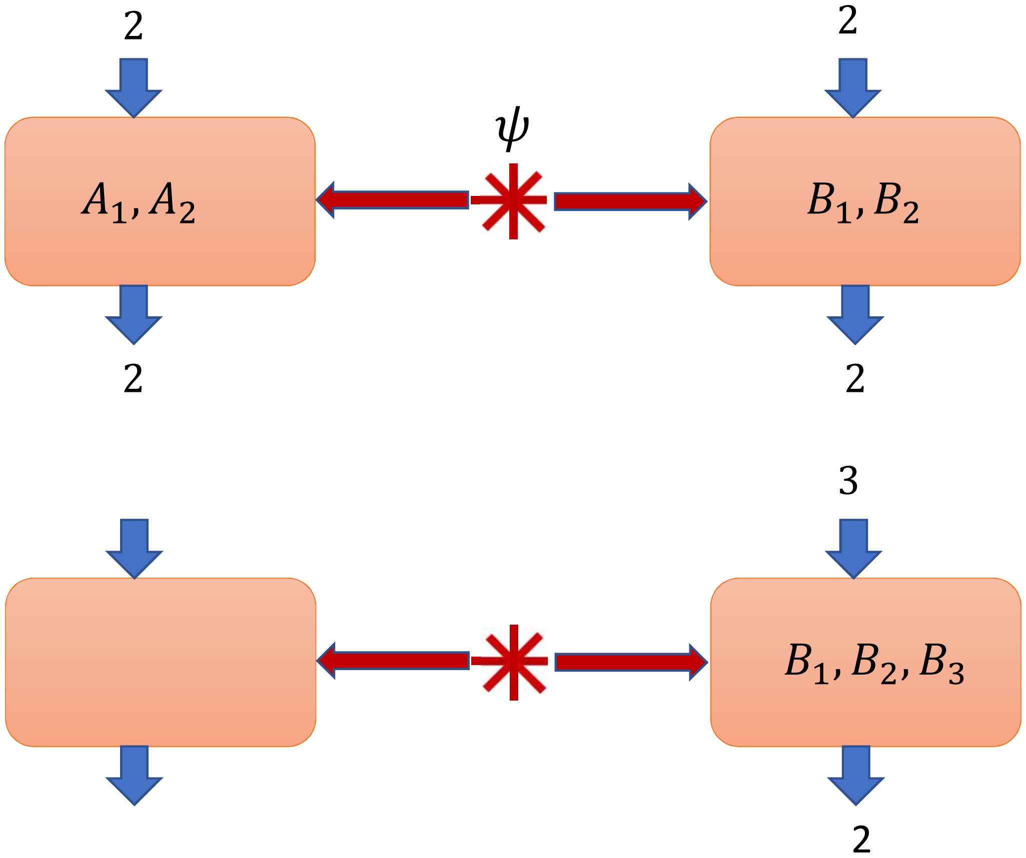
$$\sigma_{a|x} = \mathrm{Tr}_A(A_{a|x} \otimes \mathbf{1}\psi)$$

$$\sigma_{a|x} = \sum_{\lambda} p(\lambda) p(a|x,\lambda) \rho_{\lambda}$$

When are the nonclassical features of quantum theory also sufficient for correlations that violate a Bell inequality?

Wiseman et. al., PRL (2006). *Werner PRA (1989).*

Does measurement incompatibility imply Bell nonlocality?



Two incompatible measurements imply Bell nonlocality

Three incompatible measurements <u>do not</u> imply Bell nonlocality

E. Bene et. al., NJP (2018). *F. Hirsch et. al., PRA (2018).* *M. Wolf et. al., PRL (2009).*

<u>Necessary but not sufficent</u>

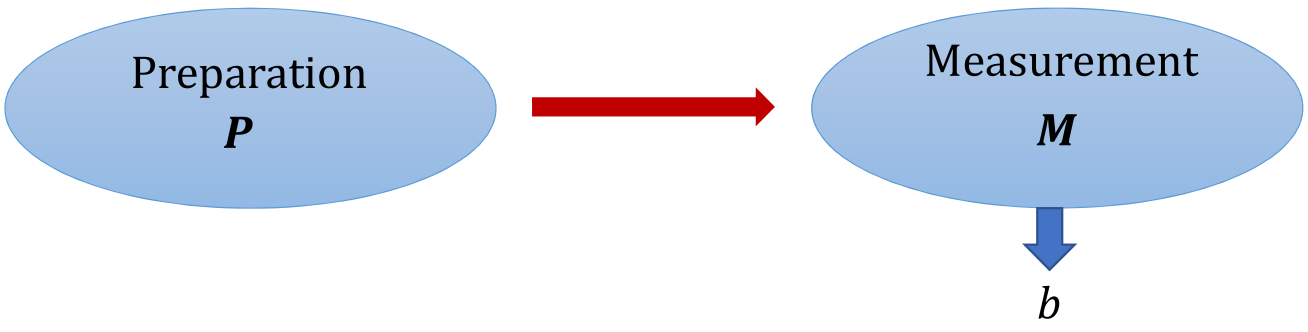
Bell nonlocality $\not\Longleftarrow$ Entanglement

Bell nonlocality $\not\Longleftarrow$ Measurement incompatibility

Bell nonlocality $\not\Longleftarrow$ Steerability

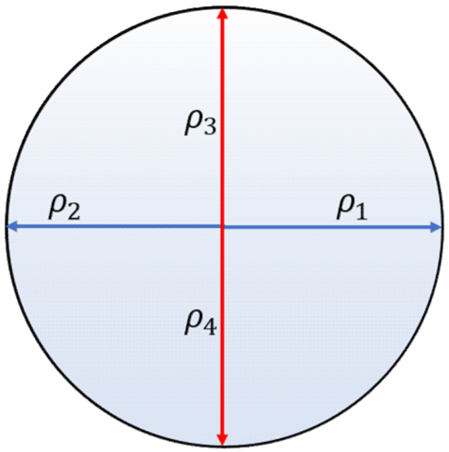
Can quantum features also be <u>sufficient</u> for some other form of quantum correlations?

Ontological models and contextuality

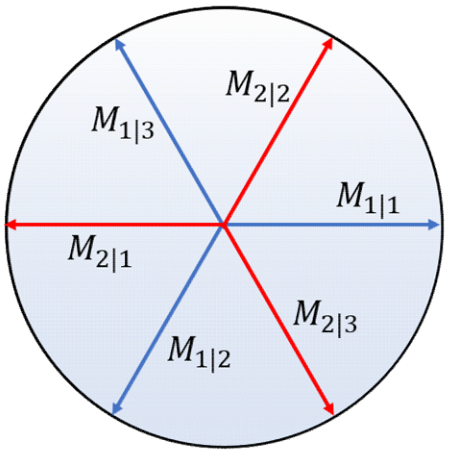


Ontological model

$$p(b|\boldsymbol{P},\boldsymbol{M}) = \sum_{\lambda} p(\lambda|\boldsymbol{P})p(b|\boldsymbol{M},\lambda)$$



Two contexts for the same preparation

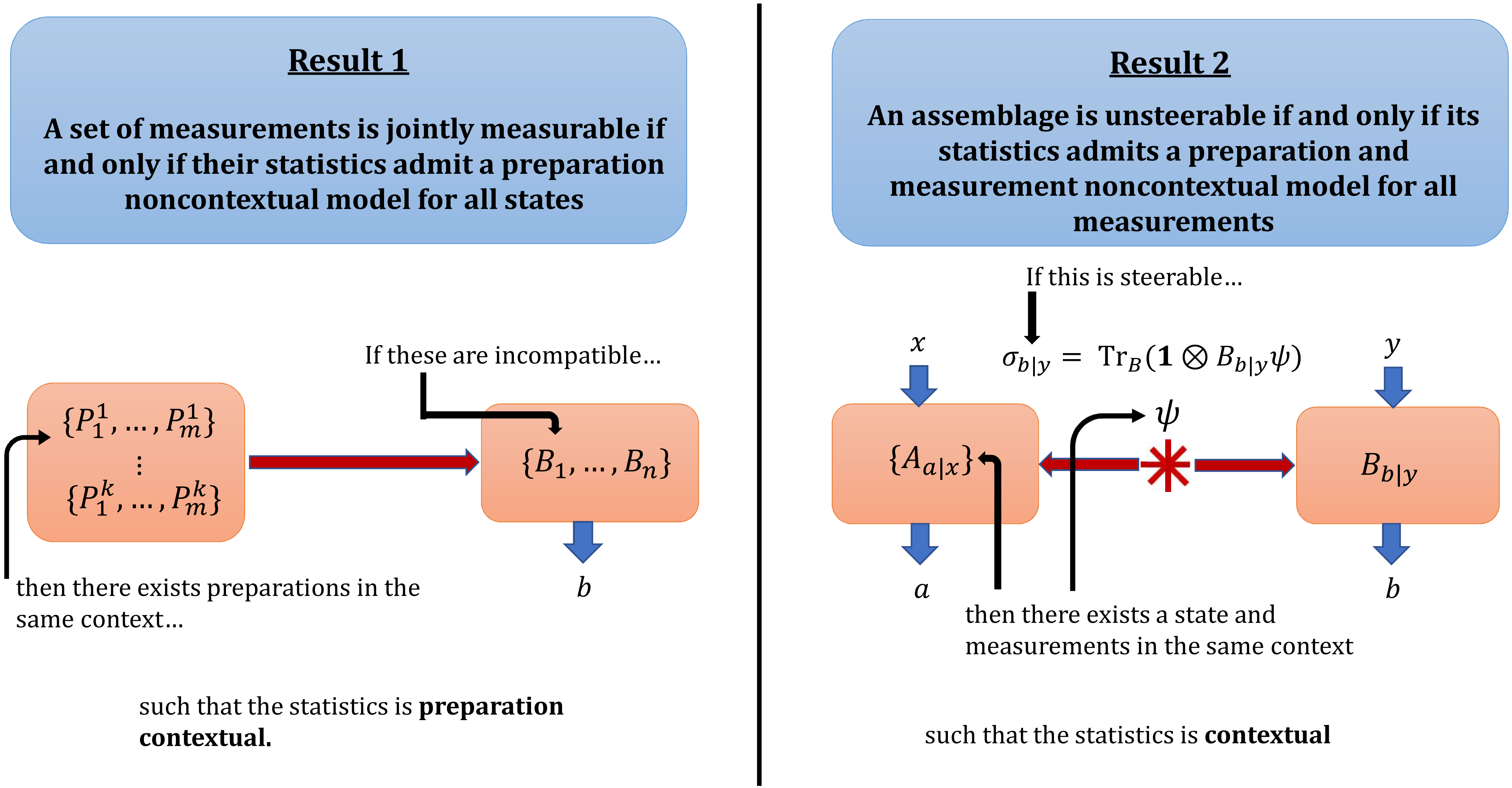


Two contexts for the same measurement

Preparation noncontextual models are independent of the context of *P*

Measurement noncontextual models are independent of the context of *M*

R. Spekkens, PRA (2005)


Result 1
A set of measurements is jointly measurable if and only if their statistics admit a preparation noncontextual model for all states
If these are incompatible...
$\{P_1^1, \dots, P_m^1\}$
$\vdots$
$\{P_1^k, \dots, P_m^k\}$
$\{B_1, \dots, B_n\}$
b
then there exists preparations in the same context...
such that the statistics is **preparation contextual.**
Result 2
An assemblage is unsteerable if and only if its statistics admits a preparation and measurement noncontextual model for all measurements
If this is steerable...
$\sigma_{b|y} = \mathrm{Tr}_B(\mathbf{1} \otimes B_{b|y}\psi)$
x
y
ψ
$\{A_{a|x}\}$
$B_{b|y}$
a
b
then there exists a state and measurements in the same context
such that the statistics is **contextual**

Sketch of a very simple proof

Assume that the POVM $A_{a|x}$ always produces preparation noncontextual statistics, for any state.

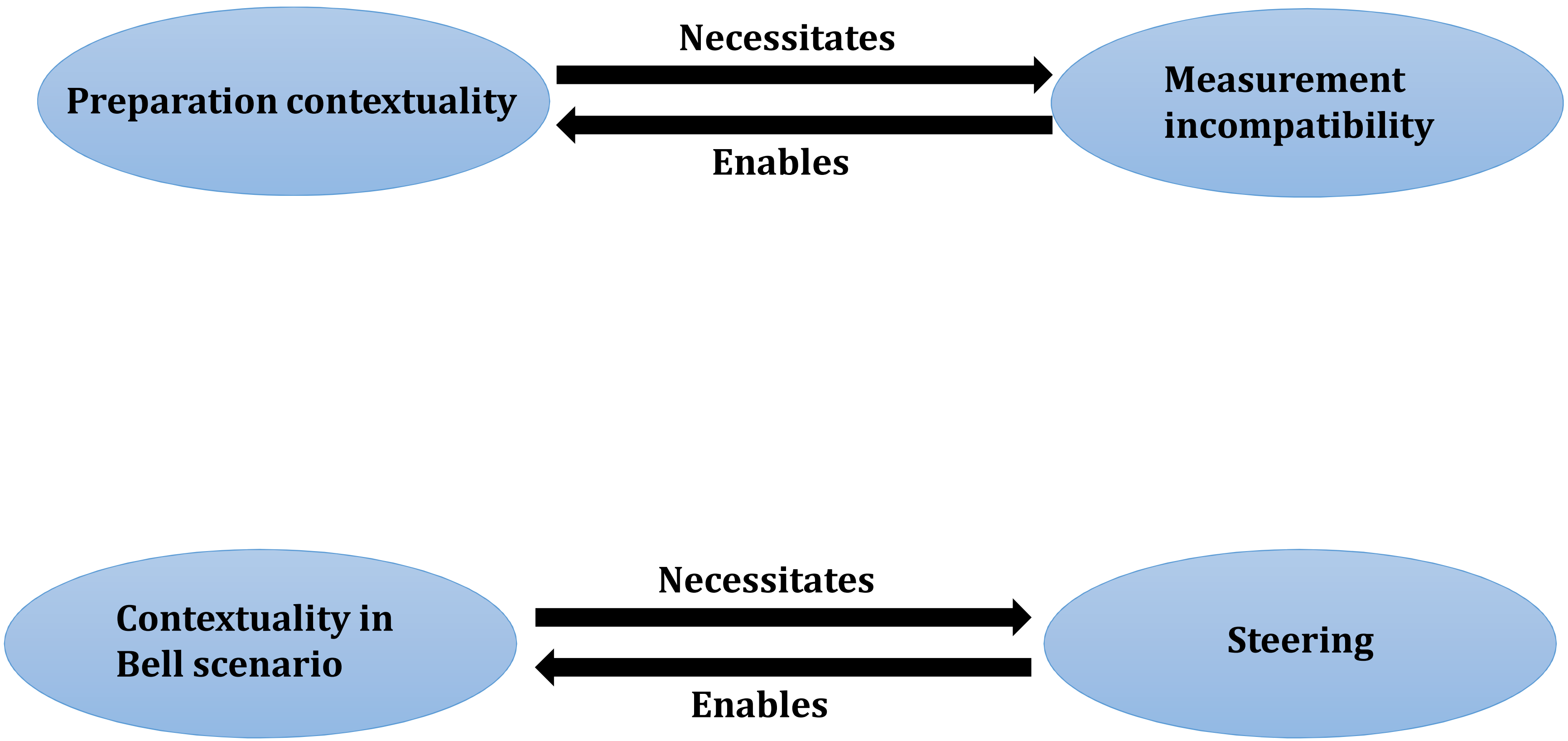
$$\forall \mathbf{P} \in \mathcal{P}_\rho : p(a|x,\mathbf{P}) = \sum_\lambda p(\lambda|\rho) p(a|x,\lambda)$$

Convexity-preserving map from quantum states to [0,1]

Riesz representation theorem: $p(\lambda|\rho) = \mathrm{Tr}(G_\lambda \rho)$ for some $0 \leq G_\lambda \leq \mathbf{1}$

$$\forall \mathbf{P} \in \mathcal{P}_\rho : p(a|x,\mathbf{P}) = \sum_\lambda p(a|x,\lambda)\, \mathrm{tr}\left[G_\lambda \rho\right]$$

Done!

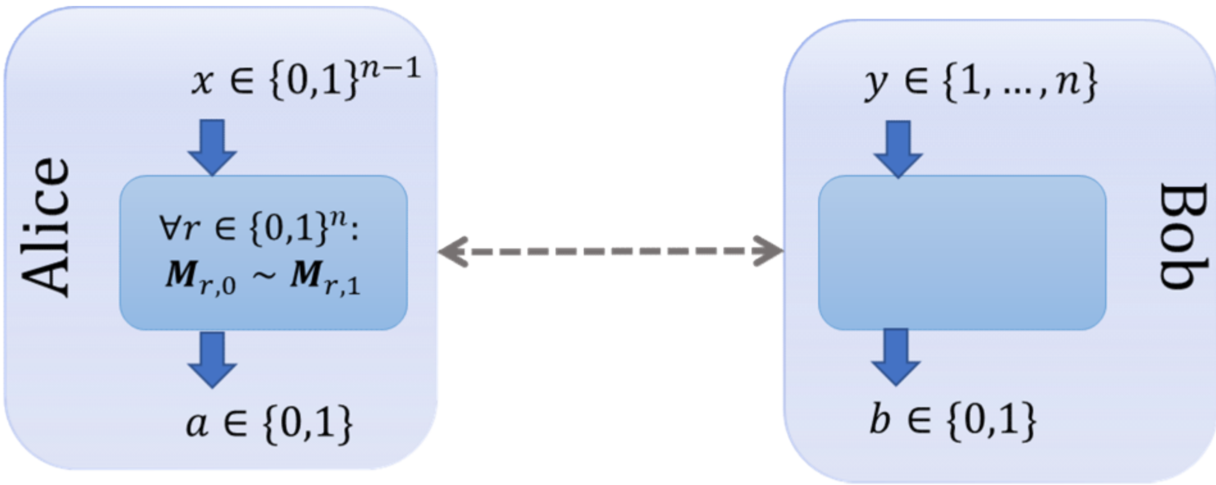

Preparation contextuality
Necessitates
Enables
Measurement incompatibility
Contextuality in Bell scenario
Necessitates
Enables
Steering

Applications

Can specific tests of contextuality ...

- **Optimally certify steerability of interesting states?**
- **Certify the incompatibility of an interesting class of mesurements?**

$$\sum_{x,y} p\big(a+b=(a,x+a\,)_y \big| x,y\big) \leq 2^{n-2}(n+1)$$



Class of states

$$\psi = v|\psi^-\rangle\langle\psi^-| + \frac{1-v}{4}\mathbf{1}$$

$v_2 = 0.7071 \qquad v_3 = 0.5774 \qquad v_4 = 0.5547$

$v_5 = 0.5422 \qquad v_6 = 0.5270 \qquad v_7 = 0.5234.$

Class of measuremets

Dichotomic qubit measurements

Numerics: 10 000 random sets of incompatible measurements for n=2,...,7

J. Bavaresco, PRA (2017)

Conclusions

- Measurement incompatibility and steering are necessary and sufficient for contextuality
- Specific tests of contextuality can optimally certify interesting classes of states and measurements.
- Certification of incompatible measurements when no Bell inequality violation is possible.

Thank you for your attention!