CAUSAL NONSEPARABILITY AS AN OPERATIONAL RESOURCE: DEFINITIONS, CONVERSION, DISTILLATION

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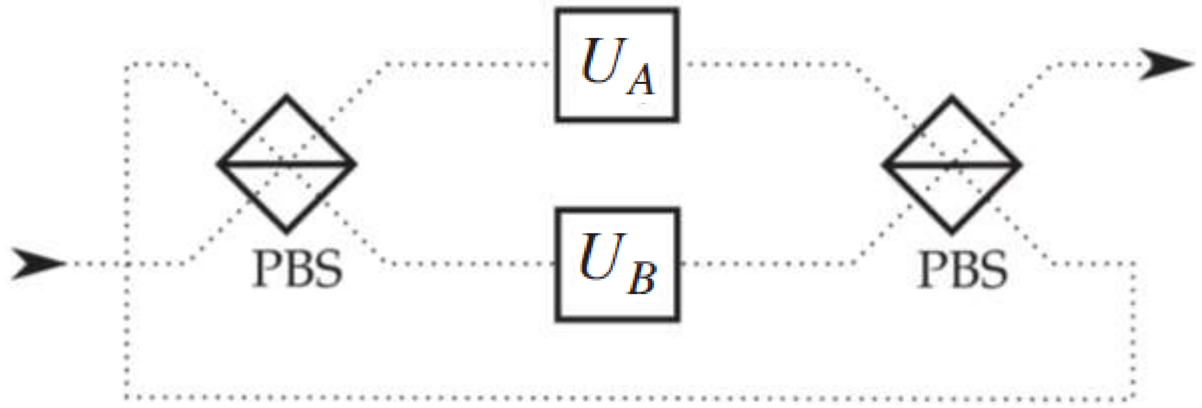
CAUSAL NONSEPARABILITY AN EXAMPLE



Araújo et al, PRL **113**, 250402 (2014)

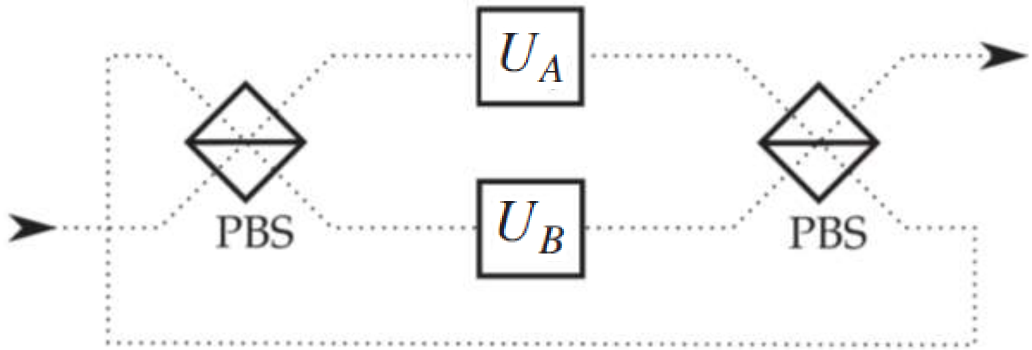
- $|H\rangle$: A→B, $|V\rangle$: B→A
- Quantum switch:

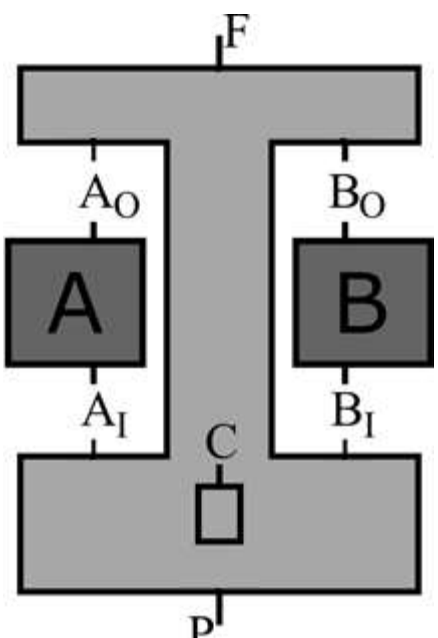
$$|H\rangle_C U_B U_A |\phi_0\rangle + |V\rangle_C U_A U_B |\phi_0\rangle$$

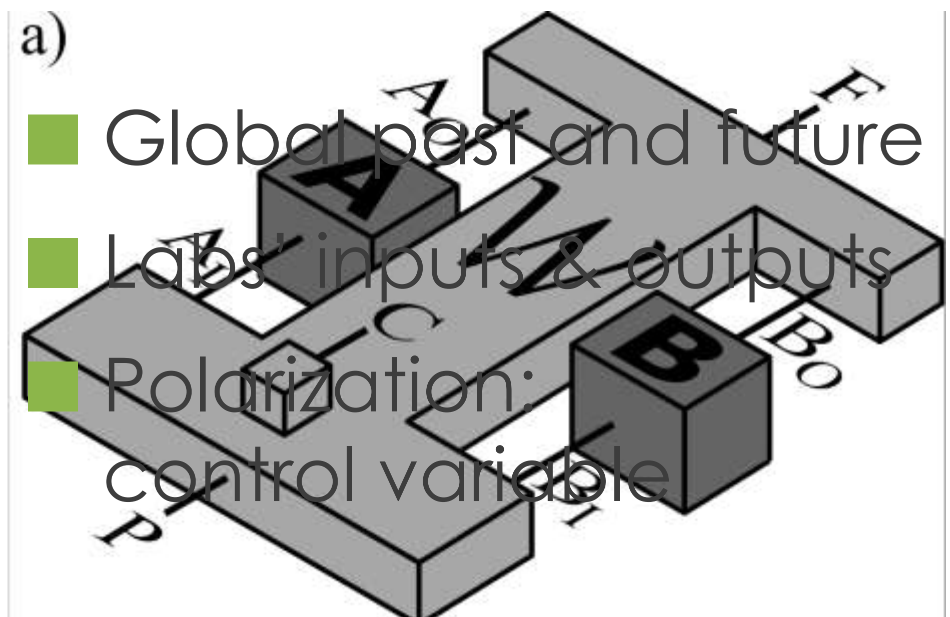
- $|\phi_0\rangle$: other dof (e.g. spatial)

SETUP: TWO INDEPENDENT LABS

- Inside labs A&B: well-defined temporal order



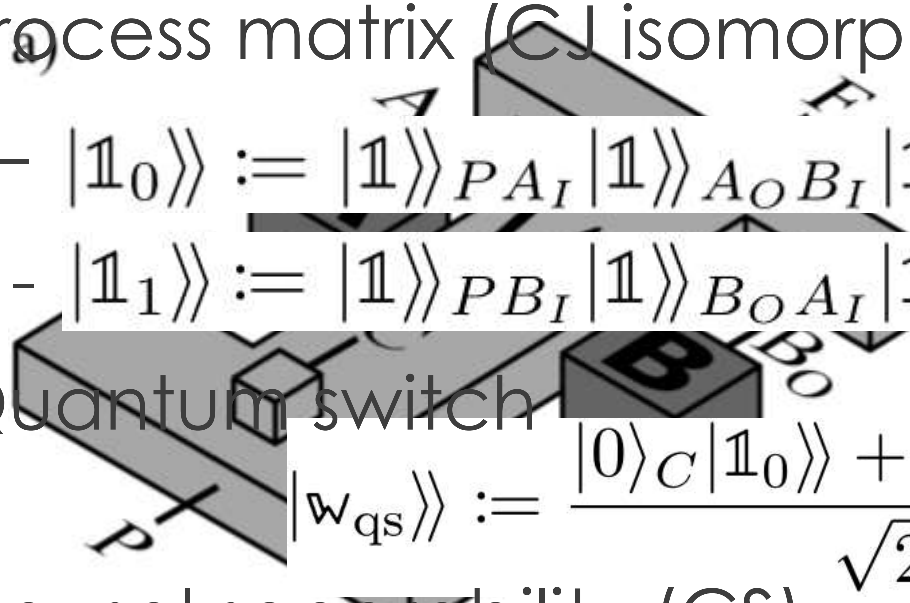




- Global past and future
- Labs' inputs & outputs
- Polarization: control variable

PROCESS IN SCENARIO WITH CONTROL

- Encapsulates entire state preparation, measurement, transmission **outside** labs (W)
- Process matrix (CJ isomorphism) $|\mathbb{1}\rangle\rangle := \sum_j |j\rangle \otimes |j\rangle$



—— $|\mathbb{1}_0\rangle\rangle := |\mathbb{1}\rangle\rangle_{PA_I} |\mathbb{1}\rangle\rangle_{A_O B_I} |\mathbb{1}\rangle\rangle_{B_O F}$

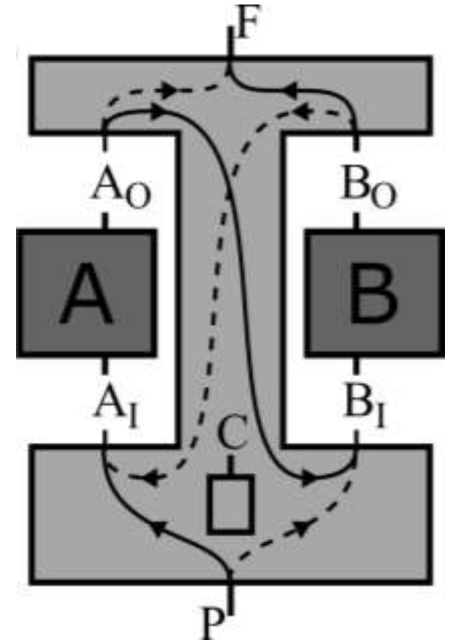
\- - - $|\mathbb{1}_1\rangle\rangle := |\mathbb{1}\rangle\rangle_{PB_I} |\mathbb{1}\rangle\rangle_{B_O A_I} |\mathbb{1}\rangle\rangle_{A_O F}$

- Quantum switch

$$|w_{qs}\rangle\rangle := \frac{|0\rangle_C |\mathbb{1}_0\rangle\rangle + |1\rangle_C |\mathbb{1}_1\rangle\rangle}{\sqrt{2}}$$

- Causal separability (CS)

$$W_{cs} = p\, W_{A\to B} + (1-p)\, W_{B\to A}$$



PROCESS IN SCENARIO WITH CONTROL

- Not restricted to processes where control takes active role

$$W_{OCB} = 1_C \otimes 1_P \otimes \frac{1}{4}\left(\mathbb{1} + \frac{\mathbb{1}_{A_I} Z_{A_O} Z_{B_I} \mathbb{1}_{B_O} + Z_{A_I} \mathbb{1}_{A_O} X_{B_I} Z_{B_O}}{\sqrt{2}}\right) \otimes 1_F$$

[Oreshkov, Costa, Brukner, Nat Comms **3**, 1092 (2012)]

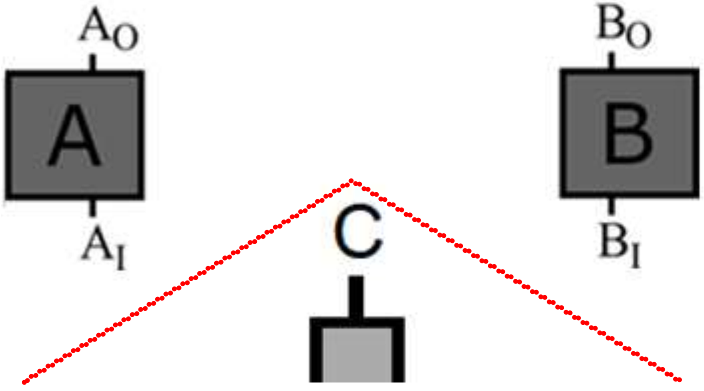
- Different from quantum switch

$$W_{\text{qs}} = \left(\frac{|0\rangle_C |\mathbb{1}_0\rangle\rangle + |1\rangle_C |\mathbb{1}_1\rangle\rangle}{\sqrt{2}}\right)\left(\frac{\langle 0|_C \langle\langle \mathbb{1}_0| + \langle 1|_C \langle\langle \mathbb{1}_1|}{\sqrt{2}}\right)$$

- **How to specify these different forms of causal nonseparability?**

DEFINITION: QUANTUM CONTROL OF CAUSAL ORDERS

- Specific resource present in quantum switch
- Goes beyond causal separability (CS)
- Depends also on AB|C separability (S)



$$\mathrm{Tr}_{PF}[W] = W|_{ABC} = \sum_j p_j \rho_C^{(j)} \otimes W_{AB}^{(j)}$$

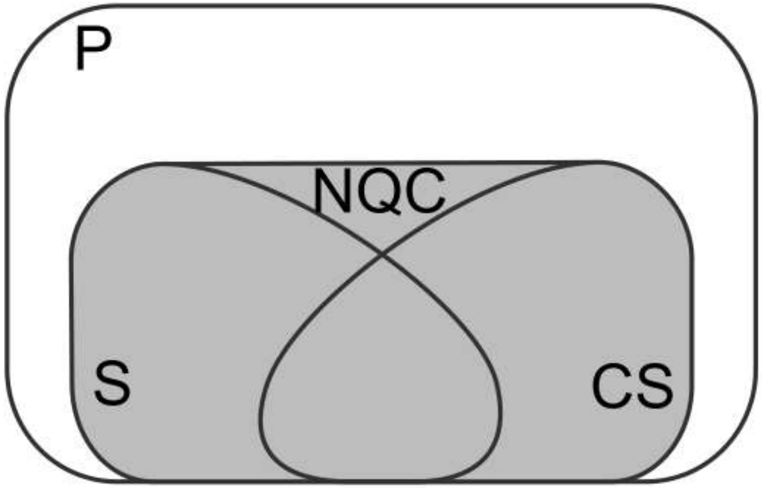
DEFINITION: QUANTUM CONTROL OF CAUSAL ORDERS

- CS

$$W_{\mathrm{cs}} = p\,W_{A\to B} + (1-p)\,W_{B\to A}$$

- S

$$\mathrm{Tr}_{PF}[W] = W|_{ABC} = \sum_j p_j \rho_C^{(j)} \otimes W_{AB}^{(j)}$$



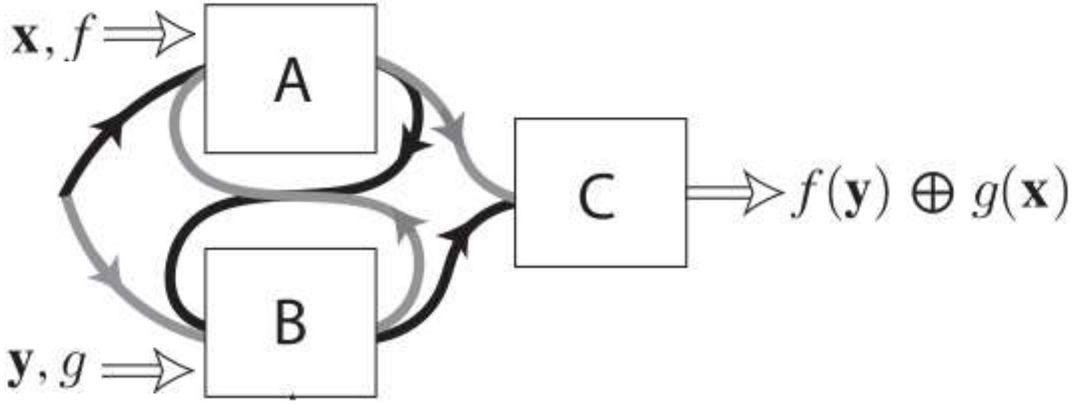
- Definition: processes have **Quantum Control of Causal Orders** (QCCO) when outside convex hull of CS and S
- Why convex hull?

AS A RESOURCE?

CAUSAL NONSEPARABILITY AS A RESOURCE FOR QUANTUM TASKS

- Chiribella's and Costa's talks
- Araujo et al, PRL **113**, 250402 (2014)
  (Anti-)commutation of unitaries
- [Experiment with more than 2 unitaries?]
- Guerin et al, PRL **117**, 100502 (2016)
  Advantage in one-way communication

Leandro Aolita's talk on Thursday @ 11:30



RESOURCE THEORIES

- Resource theories hinge on two sets

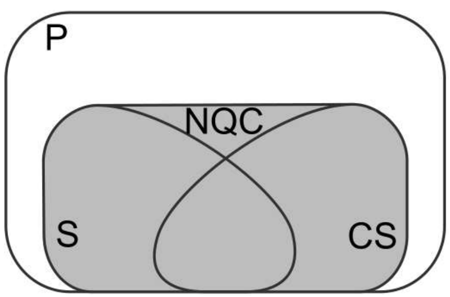
Free states
Free operations

- What are the relevant sets of free operations?

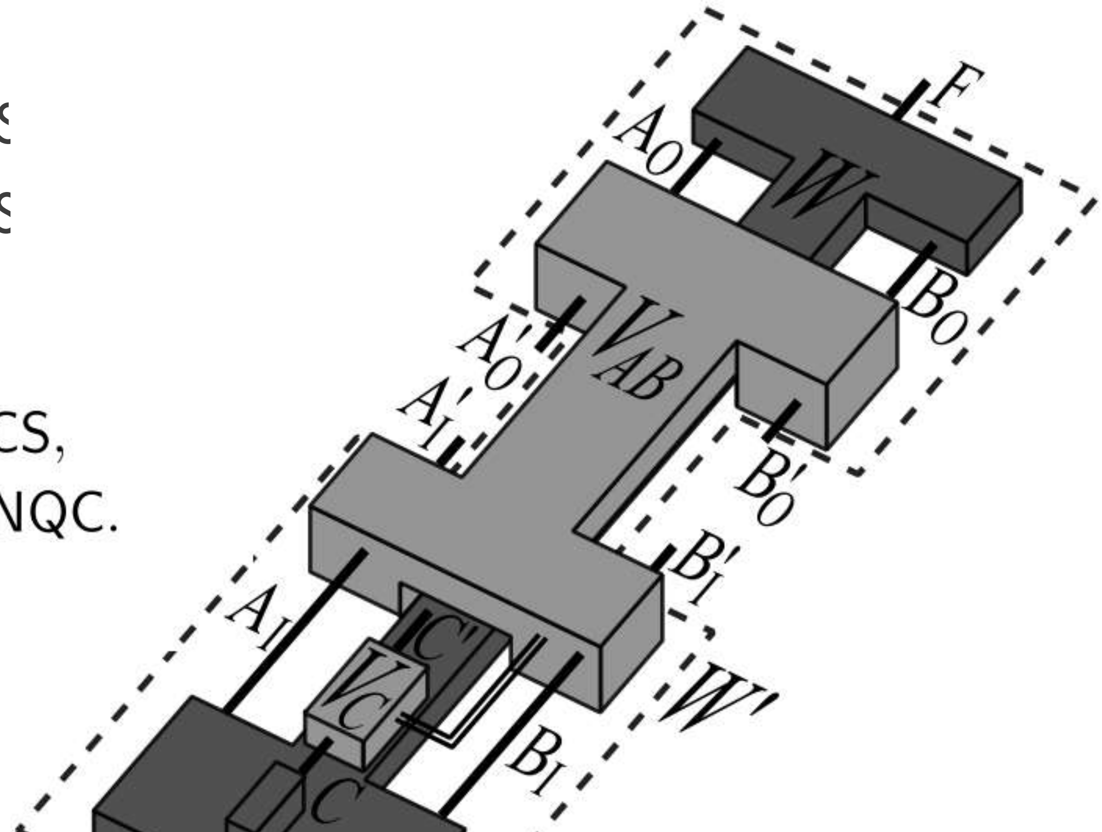
FREE OPERATIONS

- Relatively simple s of free operations
- Preserve both CS

$$\mathcal{V}(W)\begin{cases}\in \text{CS} & \text{if } W \in \text{CS},\\ \in \text{NQC} & \text{if } W \in \text{NQC}.\end{cases}$$



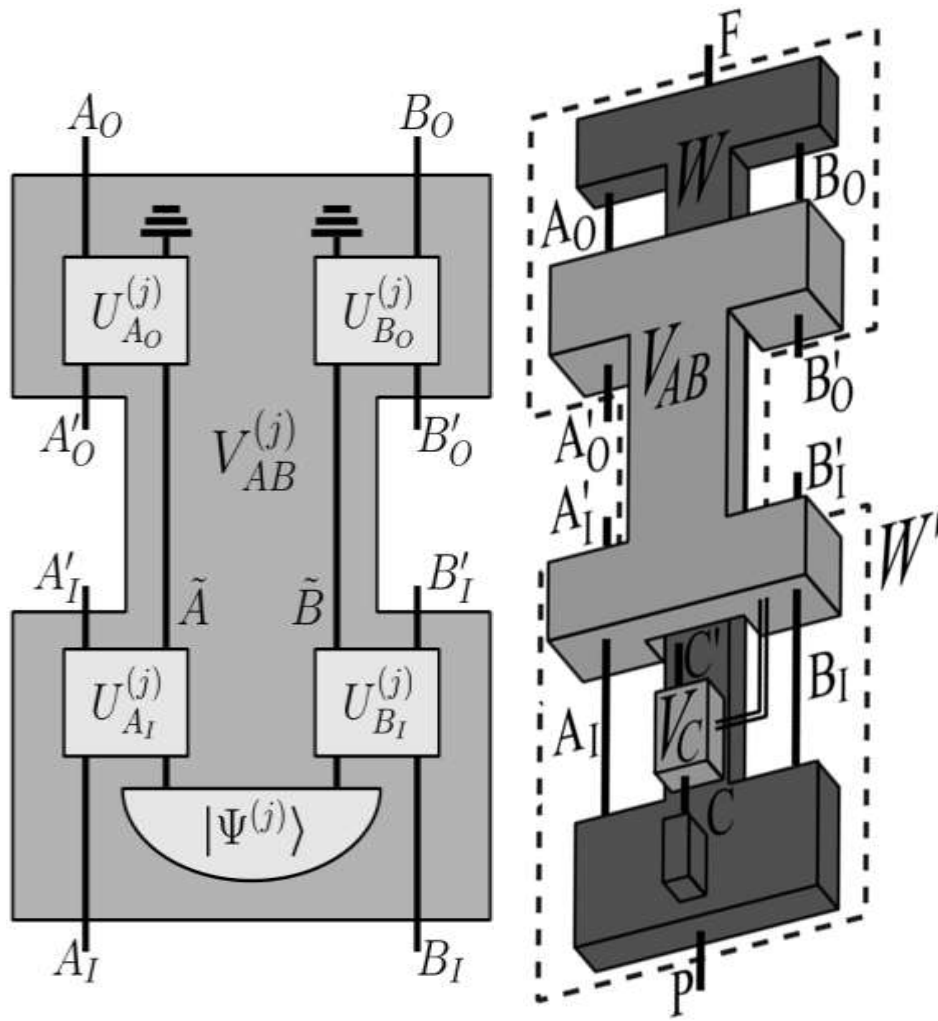
- Separable in AB |



$$V = \sum_j V_C^{(j)} \otimes V_{AB}^{(j)}$$

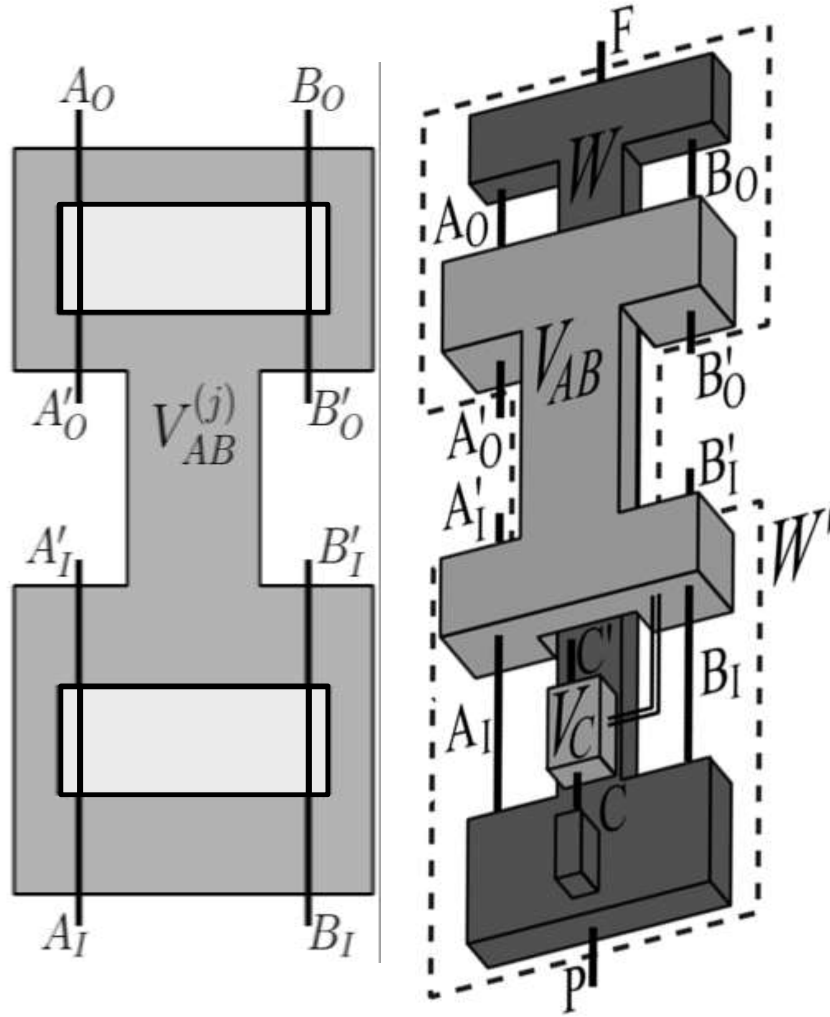
SETS OF FREE OPERATIONS: **LOAE**

- Operations without any A-B signaling*
- Local unitaries
- Shared entanglement allowed
- "Local Operations and Ancillary Entanglement"



SETS OF FREE OPERATIONS: PROB LAB SWAPS

- Signaling A-B allowed
- Either full lab swap
- Or identity channel
- Probabilistic combination (incoherently controlled)
- PLS (“probabilistic lab swaps”)

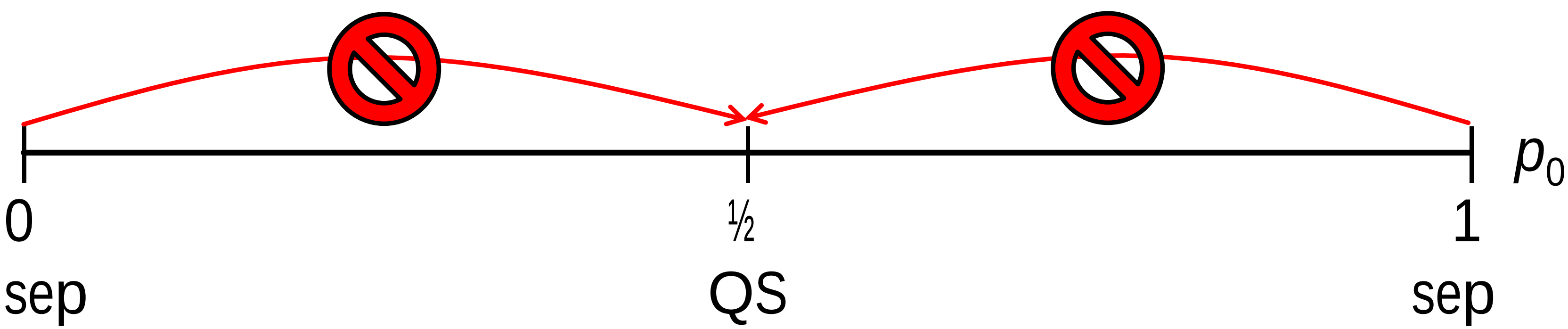


NOW, WHAT CAN WE DO WITH THESE OPERATIONS?

APPLICATIONS OF THE FREE OPERATIONS
1) INTERCONVERSION

- Consider $|\mathbb{W}\rangle\rangle = \sqrt{p_0}|0\rangle_C|\mathbb{1}_0\rangle\rangle + \sqrt{p_1}|1\rangle_C|\mathbb{1}_1\rangle\rangle$

- If $p_0 = 0$ or $p_1 = 0$, causally separable;
  if $p_0 = ½ = p_1$, quantum switch



- Is it possible to transform one into the other?

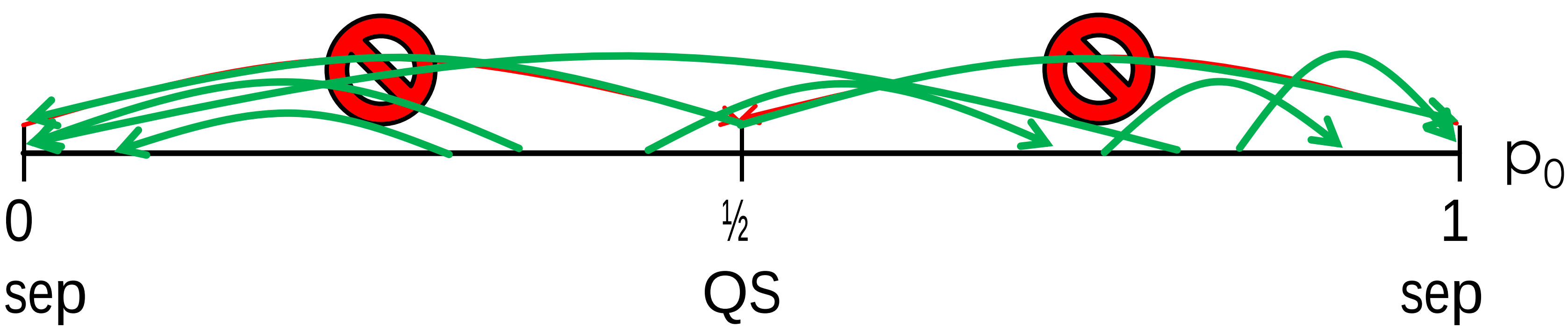
APPLICATIONS OF THE FREE OPERATIONS
1) INTERCONVERSION

$$|w\rangle\rangle = \sqrt{p_0}|0\rangle_C|\mathbb{1}_0\rangle\rangle + \sqrt{p_1}|1\rangle_C|\mathbb{1}_1\rangle\rangle$$

- Any process can be freely transformed into separable ones
- General: majorization-defined (“outwards”)

$$(p_0, p_1) = C_0\,(p_0', p_1') + C_1\,(p_1', p_0')$$

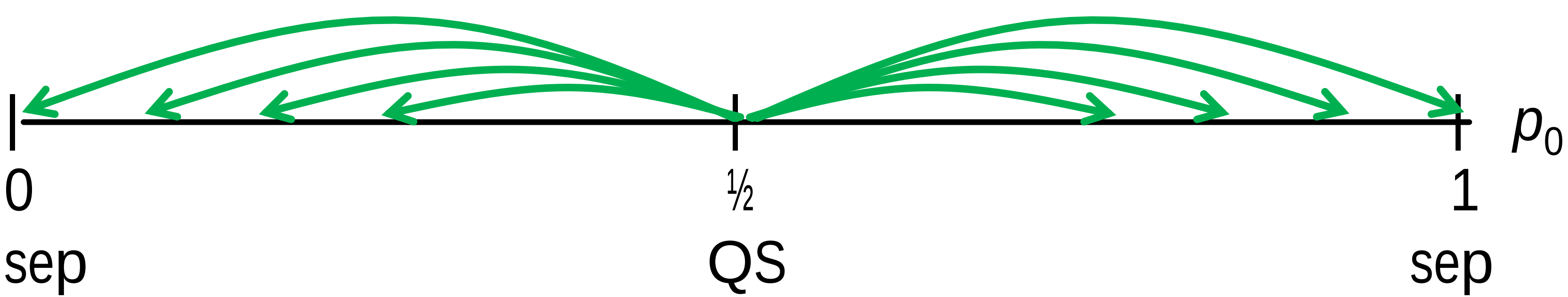
- **Hierarchy of Q Control of Causal Orders**



APPLICATIONS OF THE FREE OPERATIONS 1) INTERCONVERSION

$$|w\rangle\rangle = \sqrt{p_0}|0\rangle_C|\mathbb{1}_0\rangle\rangle + \sqrt{p_1}|1\rangle_C|\mathbb{1}_1\rangle\rangle$$

- Any process can be freely transformed into separable ones
- General: majorization-defined (“outwards”)
- **Hierarchy of Q Control of Causal Orders**
- QS on top (universal in this class)



APPLICATIONS OF THE FREE OPERATIONS 1) INTERCONVERSION

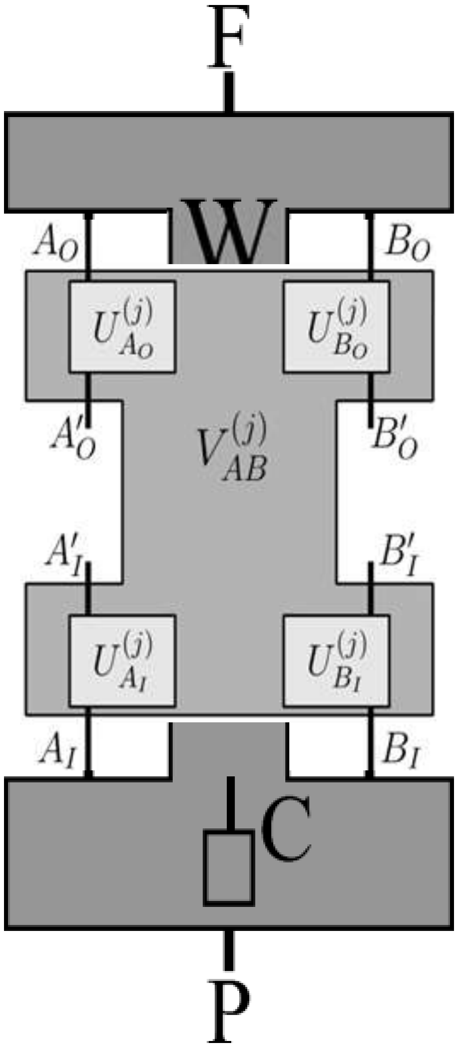
- Additional freedoms:
- Local unitaries at will before any lab input or after any lab output
- Control-qubit basis (also a unitary)
- General form ("generalized QS")

$$|\mathfrak{w}\rangle\rangle = \sqrt{p_0}\,|\Phi_0\rangle_C|\mathfrak{u}_0\rangle\rangle + \sqrt{p_1}\,|\Phi_1\rangle_C|\mathfrak{u}_1\rangle\rangle$$

$$|\mathfrak{u}_0\rangle\rangle := |\mathfrak{u}_{PA}\rangle\rangle_{PA_I}|\mathfrak{u}_{AB}\rangle\rangle_{A_O B_I}|\mathfrak{u}_{BF}\rangle\rangle_{B_O F}$$
$$|\mathfrak{u}_1\rangle\rangle := |\mathfrak{u}_{PB}\rangle\rangle_{PB_I}|\mathfrak{u}_{BA}\rangle\rangle_{B_O A_I}|\mathfrak{u}_{AF}\rangle\rangle_{A_O F}$$

with $\mathfrak{u}^\dagger_{PA}\mathfrak{u}_{BA}\mathfrak{u}^\dagger_{BF} = \mathbb{1} = \mathfrak{u}^\dagger_{PB}\mathfrak{u}_{AB}\mathfrak{u}^\dagger_{AF}$

- QS universal on this class



APPLICATIONS OF THE FREE OPERATIONS
1) INTERCONVERSION

- How?

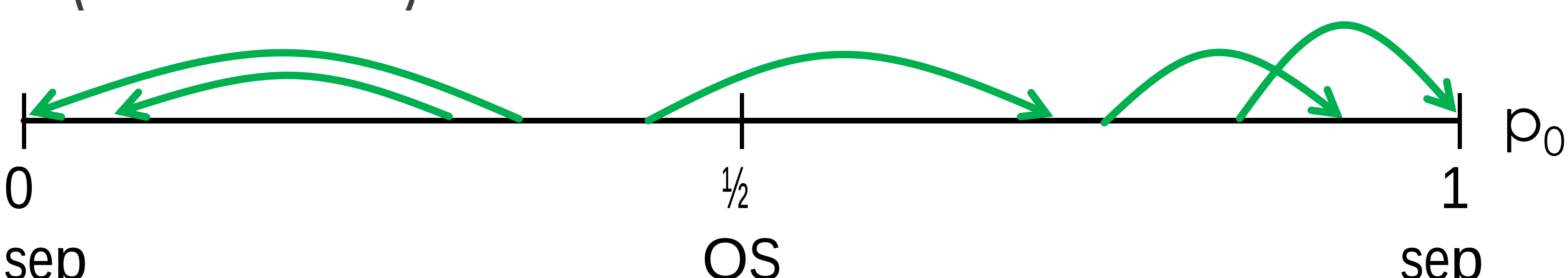
$$|\mathbb{W}\rangle\rangle = \sqrt{p_0}\,|\Phi_0\rangle_C|\mathbb{1}_0\rangle\rangle + \sqrt{p_1}\,|\Phi_1\rangle_C|\mathbb{1}_1\rangle\rangle$$

  Applying non-demolition measurements on control qubit:

$$\sqrt{C_0}\left(\sqrt{\frac{p_0'}{p_0}}|\Phi_0\rangle\langle\Phi_0|_C + \sqrt{\frac{p_1'}{p_1}}|\Phi_1\rangle\langle\Phi_1|_C\right)$$

$$\sqrt{C_1}\left(\sqrt{\frac{p_1'}{p_0}}|\Phi_0\rangle\langle\Phi_0|_C + \sqrt{\frac{p_0'}{p_1}}|\Phi_1\rangle\langle\Phi_1|_C\right)$$

- Plus lab swap if needed (heralded)



APPLICATIONS OF THE FREE OPERATIONS
2) DISTILLATION

- It is possible to distill Q control of causal orders!
- As reference, quantum switch

$$|w\rangle\rangle^{\otimes N} \xrightarrow{\text{free op}} |w_{qs}\rangle\rangle^{\otimes rN}$$

- Subtlety:

$$(W^{A\to B} + W^{B\to A})^{\otimes 2} = \quad \dots + W^{A\to B} \overset{???}{\otimes} W^{B\to A} + \dots$$

- Sidestepped: separate labs $\quad W^{A_1\to B_1} \otimes W^{B_2\to A_2}$
- Price: no joint operations (as in Costa's talk) [Explored in Guérin et al 1806.10374 (njp)]

APPLICATIONS OF THE FREE OPERATIONS
2) DISTILLATION

$$|\mathbb{W}\rangle\rangle^{\otimes N} \xrightarrow{\text{free op}} |\mathbb{W}_{\text{qs}}\rangle\rangle^{\otimes rN}$$

- Second* way:
  single-copy transitions (probabilistic)
- Local filtering on control qubit

$$\sqrt{x}|\Phi_0\rangle\langle\Phi_0|_C + \sqrt{y}|\Phi_1\rangle\langle\Phi_1|_C$$

$$\sqrt{1-x}|\Phi_0\rangle\langle\Phi_0|_C + \sqrt{1-y}|\Phi_1\rangle\langle\Phi_1|_C$$

$$x = \min\left\{\frac{p_1}{p_0}, 1\right\}$$

$$y = \min\left\{\frac{p_0}{p_1}, 1\right\}$$

APPLICATIONS OF THE FREE OPERATIONS
2) DISTILLATION

$$|\mathbb{W}\rangle\rangle^{\otimes N} \xrightarrow{\text{free op}} |\mathbb{W}_{\text{qs}}\rangle\rangle^{\otimes rN}$$

- Second* way:
  single-copy transitions (probabilistic)
- Local filtering on control qubit
- Distillation rate == success prob of each copy

$$r = 2 \min\{p_0, p_1\}$$

- Single-output production is **deterministic**
  on average with protected last copy

FINAL REMARKS AND OPEN QUESTIONS

- Definition of **Quantum Control of Causal Orders**
- Resource theory of both QCCO and CNS
- Free operations: LOAE, PLS
- Applications:
- Interconversion, hierarchy
- Distillation

FINAL REMARKS AND OPEN QUESTIONS

Open questions

- Resource theory for causal networks in general (LOAE, not PLS)?
- Are there different classes of QCCO? Or is the quantum switch really a universal maximum?
- What other classes of free operations are there? Are they physically interesting?

RANIERI V. NERY

LEANDRO AOLITA

THANK YOU FOR YOUR ATTENTION!

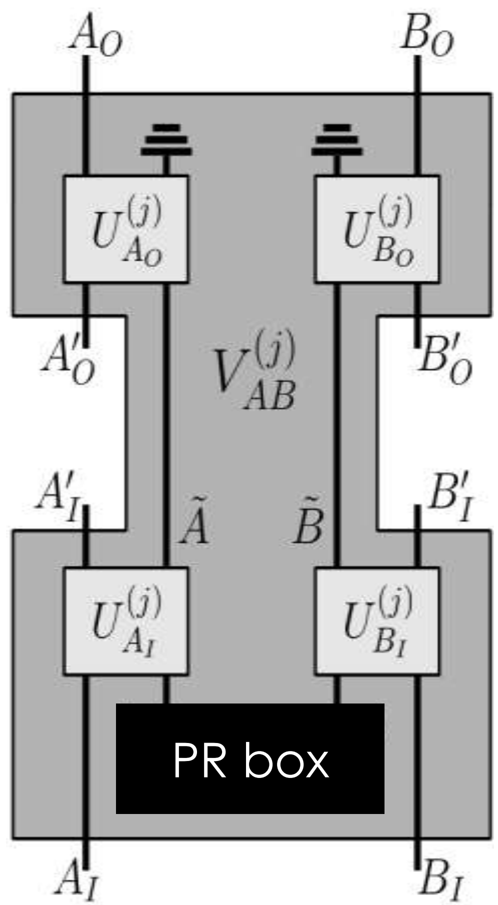
FOR MORE INFORMATION,
YOU CAN ASK ME QUESTIONS
AND ALSO CHECK **ARXIV:1903.06180**

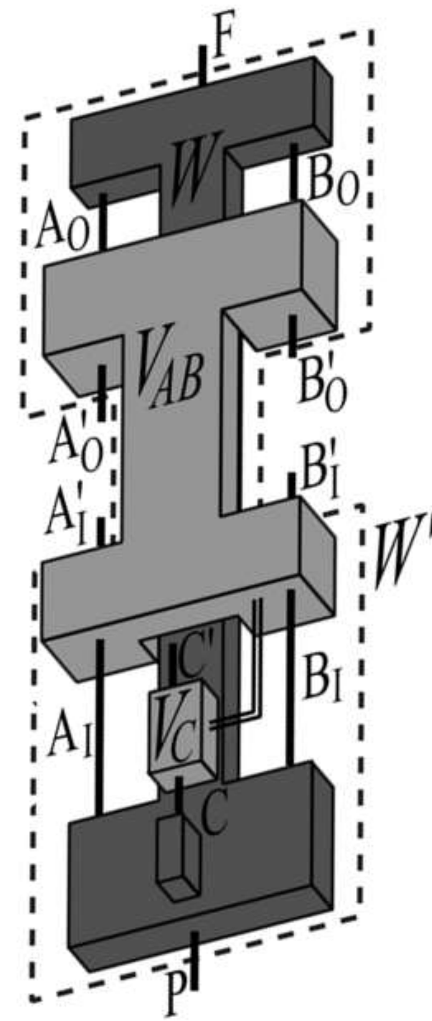
Leandro Aolita's talk on experimentally building an n-switch, Thursday @ 11:30

BACKUP SLIDES!

SETS OF FREE OPERATIONS: **NSO**

- Larger set defined strictly by lack of A-B signaling
- Superset of LOAE
- "Non-signaling operations"





APPLICATIONS OF THE FREE OPERATIONS
2) DISTILLATION

$$|\mathbb{W}\rangle\rangle^{\otimes N} \xrightarrow{\text{free op}} |\mathbb{W}_{\text{qs}}\rangle\rangle^{\otimes rN}$$

- First way:
  exploit joint operations on *control* qubits
- Measurements on
  "excitation-number subspaces"
- Low rate *r* because of restriction to joint operations on labs
- (feel free to ask me even more later)