Quantifying Bell nonclassicality across arbitrary resource types

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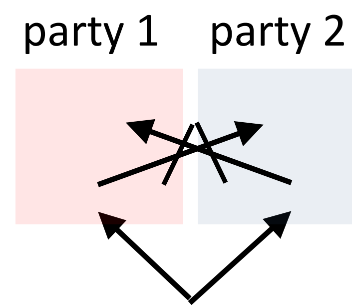
arXiv:1909.04065

Common-cause scenario



-e.g. Bell scenarios/experiments at space-like separation

Quantum theory allows for interesting no-signaling resources:

-entangled quantum states
-nonlocal boxes
-steering assemblages
-distributed measurements
-teleportages
-measurement-device-independent assemblages
-channel-steering assemblages
-Bob-with-input assemblages

Useful for/Studied via Distributed Games:

-nonlocal games
-semiquantum games
-teleportation experiments
-entanglement-witnesses
-measurement-device-independent experiments

for citations, see arXiv:1909.04065

Seminal results:

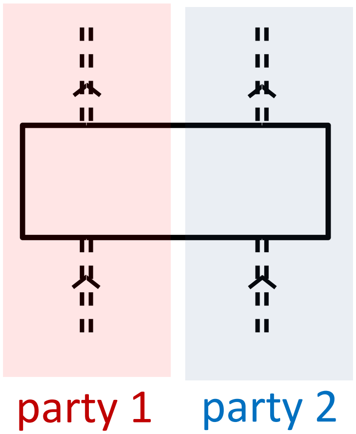
- Not every entangled state is useful for a nonlocal game
- Every entangled state is useful for some semiquantum game
- Every entangled state is useful for teleportation
- Semiquantum/teleportation games can witness entanglement of any state
- Entanglement can be witnessed in a measurement-device-independent way

Type-independent resource theory of local operations and shared randomness

Types of Resources and Scenarios

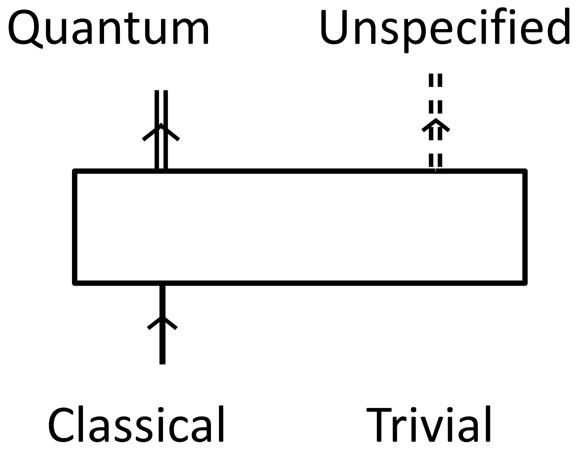
Resources:

no-signaling quantum channels
distributed among various parties
(focus on bipartite for simplicity)

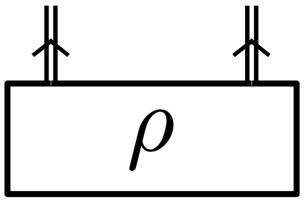


The **type** of a resource is determined by the nature of its input and output systems: quantum, classical, or trivial

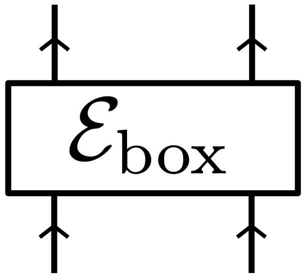
Graphical notation:



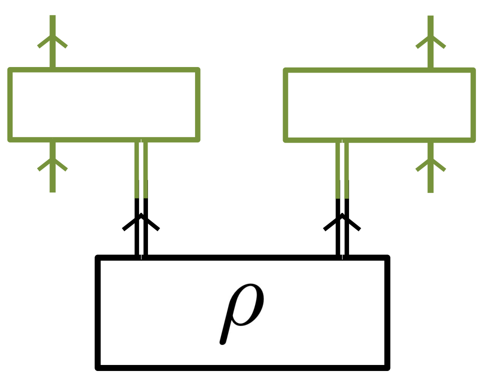
Resource type (examples)



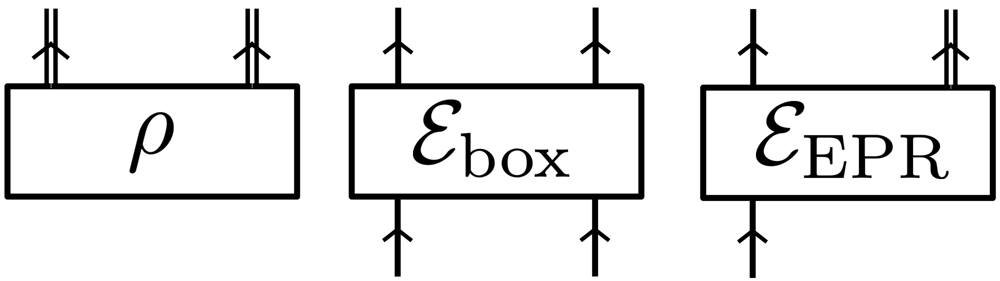
quantum
state



no-signaling
box



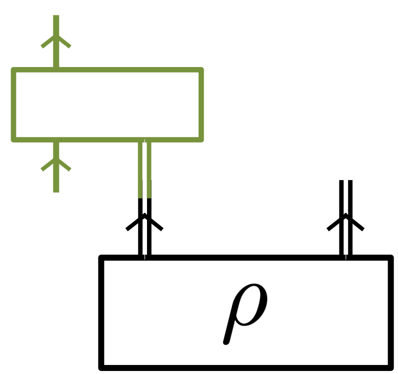
Resource type (examples)



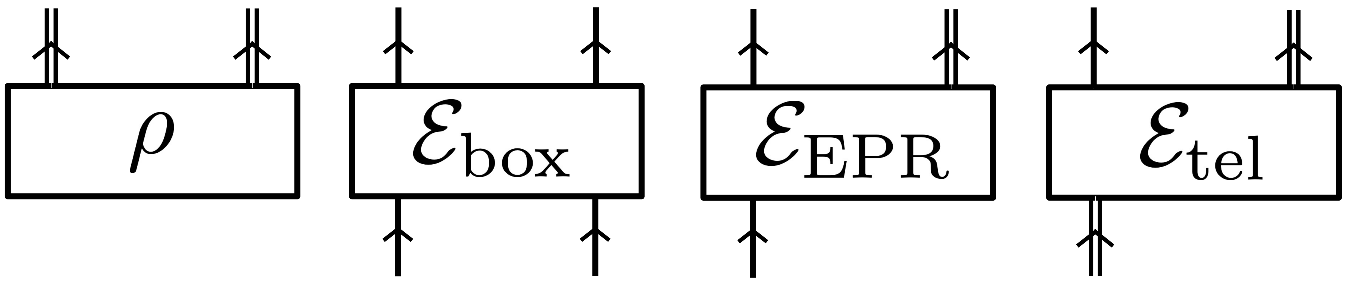
quantum state

no-signaling box

steering assemblage

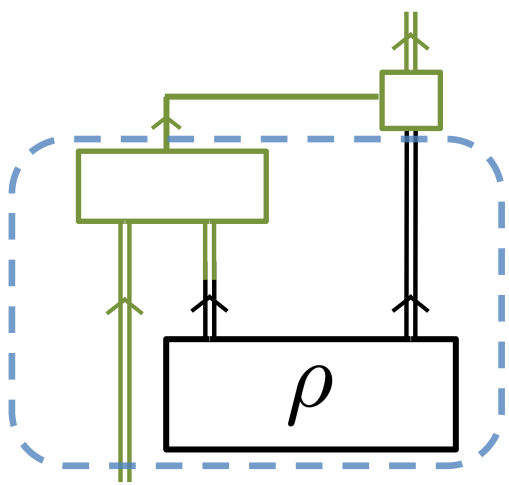


Resource type (examples)



quantum state | no-signaling box | steering assemblage | quantum teleportage

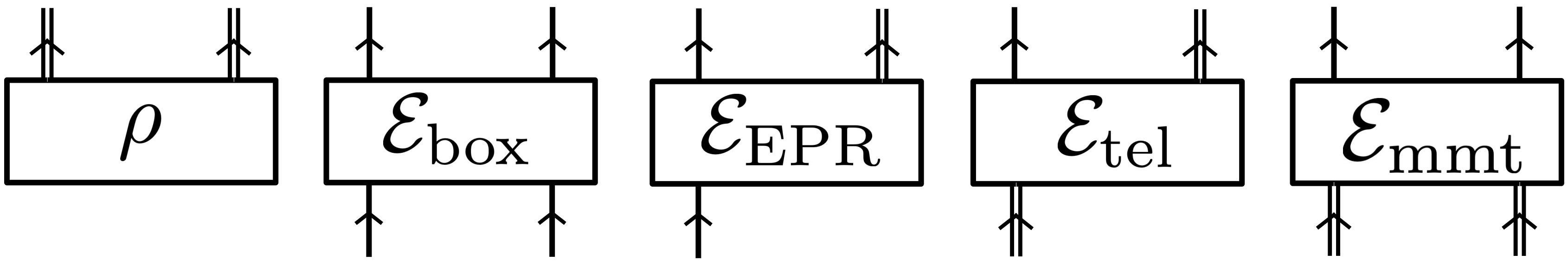
teleportation protocol:



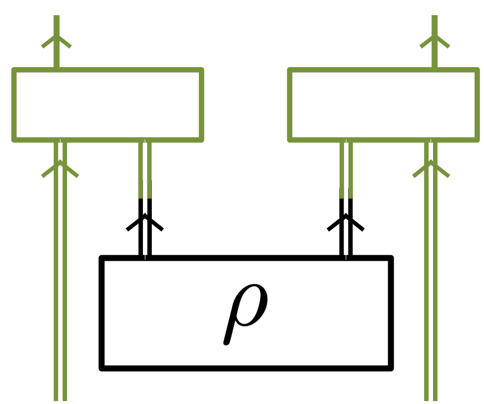
contains the relevant information for teleportation*

*Cavalcanti, Skrzypczyk, Supić
arXiv:1607.03249

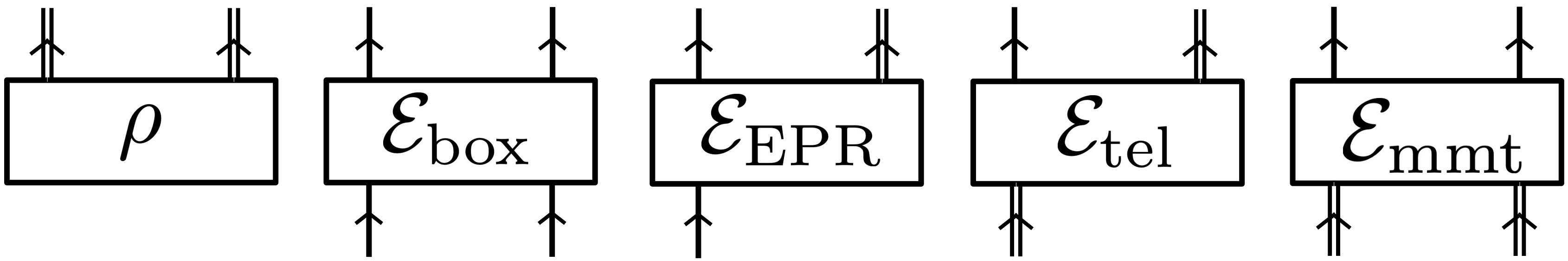
Resource type (examples)



quantum state | no-signaling box | steering assemblage | quantum teleportage | distributed measurement



Resource type (examples)



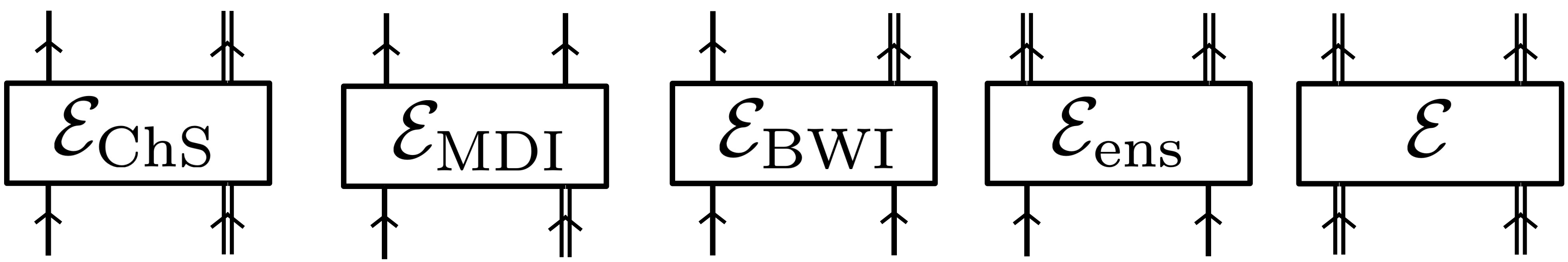
quantum state

no-signaling box

steering assemblage

quantum teleportage

distributed measurement



channel assemblage

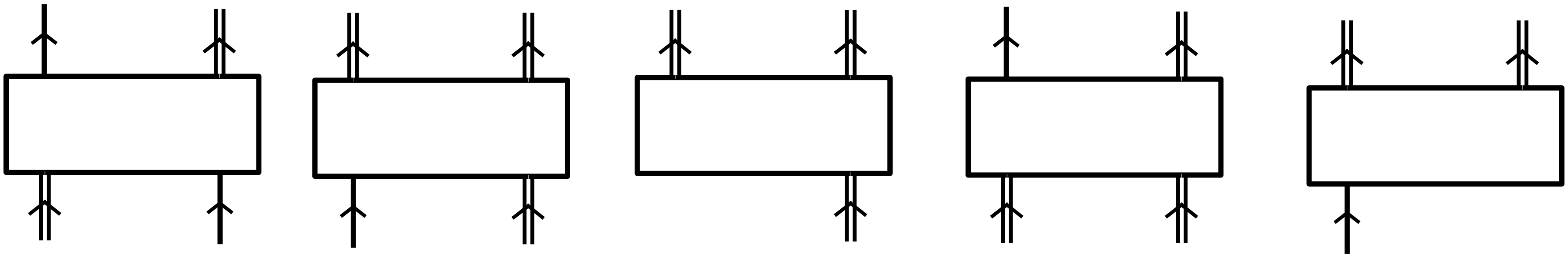
measurement-device-independent steering

Bob-with-input assemblage

ensemble-preparing channel

bipartite channel

Five new nontrivial bipartite scenarios/resource types:



Open question: foundational or practical significance?

-five new notions of “nonlocality”

(Type-independent) Resource Theory

The KEY step in any resource theoretic research is identifying the right set of free operations.

we want to quantify nonclassicality of states and of boxes…
-for states, entanglement has usually been characterized by LOCC
-for boxes, nonclassicality is best characterized by LOSR

(E. Wolfe, D. Schmid, A.B. Sainz, R. Kunjwal, R.W. Spekkens, arXiv 1903.06311)

<u>What are the physical restrictions in the scenario under study?</u>
-no cause-effect relations (no communication)
-no local restrictions
-common causes are allowed

So, we allow local quantum operations and classical common causes.
Then, anything nonfree requires a *nonclassical* common cause

local operations and shared randomness (LOSR)

Previous work on LOSR:

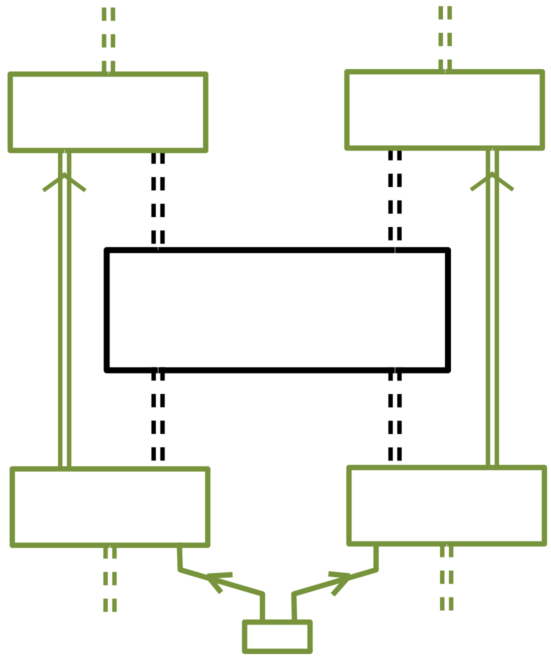
states: Buscemi (2012)

assemblages (and states): Cavalcanti, Hall, and Wiseman (2013)

boxes: de Vicente (2014)
Gallego and Aolita, (2017)
Wolfe, Schmid, Sainz, Kunjwal, Spekkens, (2019)

Here, we allow local operations to change resource types.

Free (type-changing) LOSR transformations



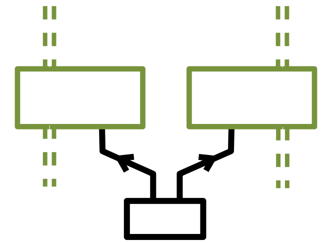
G. Chiribella, G. M. D'Ariano, and P. Perinotti, "Quantum Circuit Architecture," Phys. Rev. Lett. **101**, 060401 (2008).

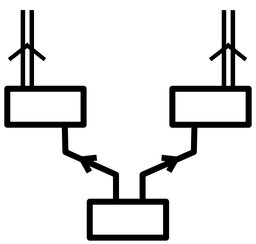
Free LOSR resources:

those simulable by

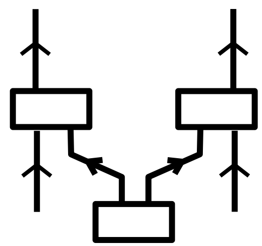
-local operations

-shared randomness

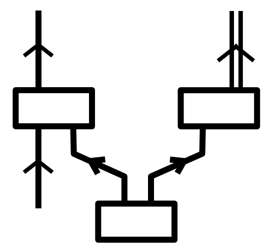




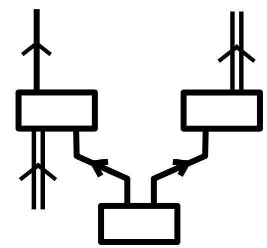
separable
state



local
box



unsteerable
assemblage



classical
teleportage

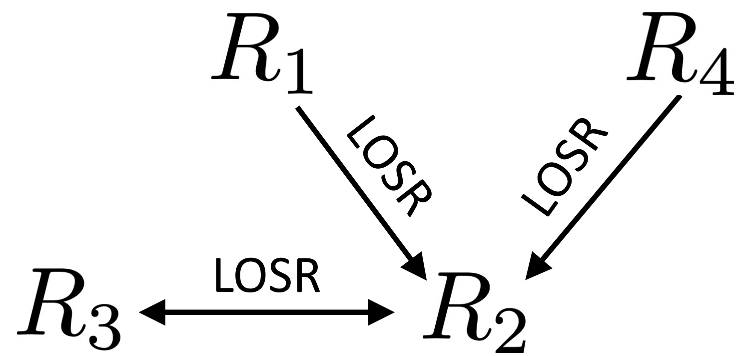
etc

In every case, the `useless’ set is the LOSR free set!

Open question: characterize geometry of free set in each scenario

Quantifying the nonclassicality of resources

R is **at least as nonclassical as** R'
if R can be freely converted to R'

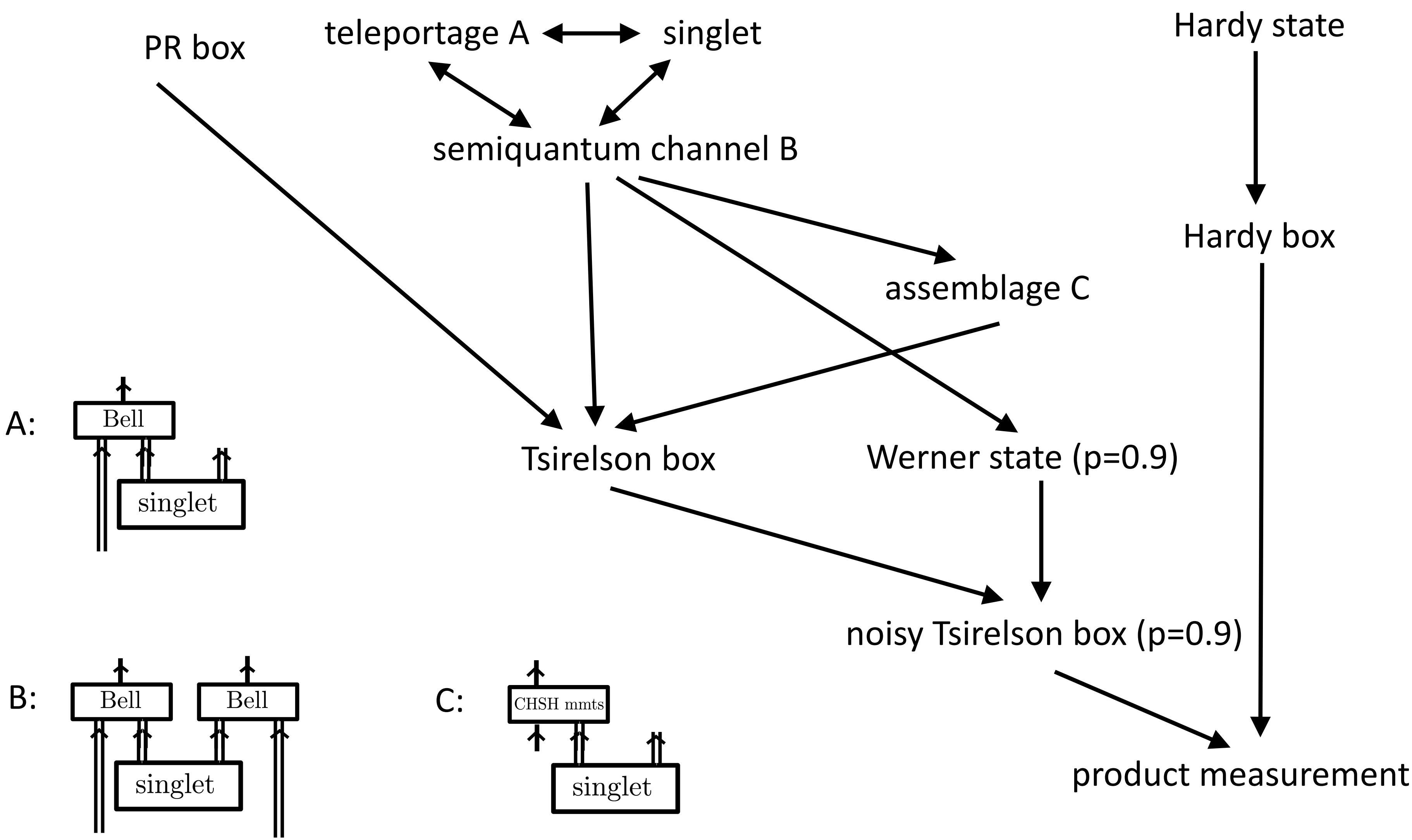


R and R' are **equally nonclassical**
if they are freely interconvertible
("same equivalence class")

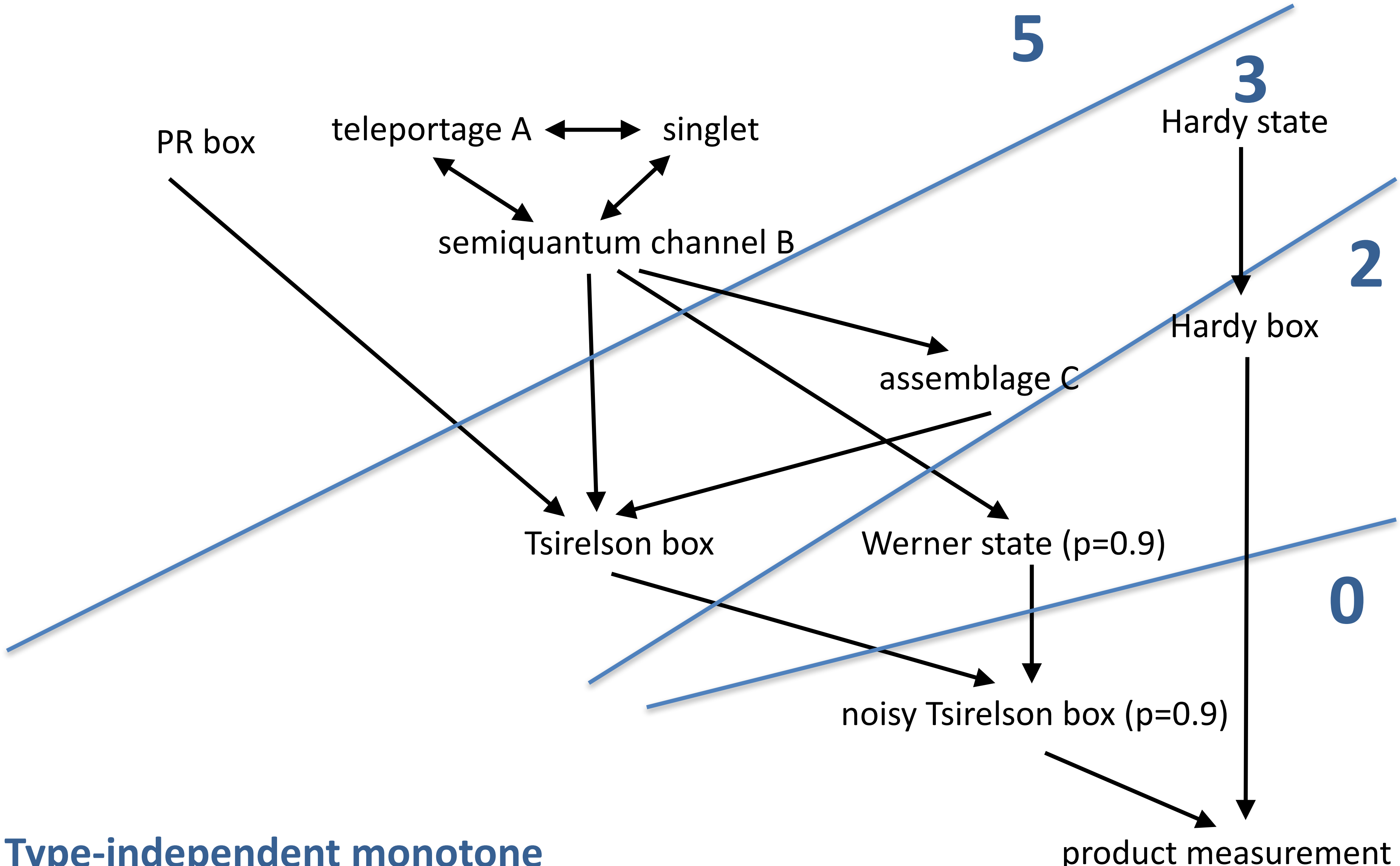
R and R' are **incomparable** if neither
can be freely converted to the other

The value (nonclassicality) of a resource is fully determined by which conversions are possible and which are not.
(monotones/witnesses are just a means of getting partial information about the conversions)

Can compare resources of different types!



Can compare resources of different types!



Type-independent monotone

-assigns a value to every resource of every type; non-increasing under LOSR

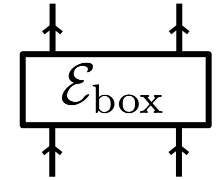
Some Results

Can every entangled state be transformed into…

a nonclassical box?

NO! Werner states admit of local HV models.

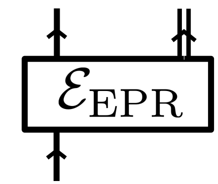
Barrett, Phys. Rev. A, 65, 042302 (2002)



an unsteerable assemblage?

NO! "Inequivalence of entanglement and steering"

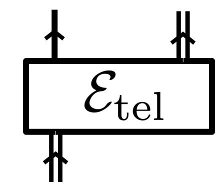
Bowles et. al., Phys. Rev. Lett. 112, 200402 (2014)



a nonclassical teleportage?

YES! All entangled states are useful for teleportation

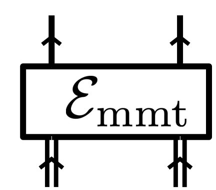
Cavalcanti et. al., Phys. Rev. Lett. 119, 110501 (2017)



a nonclassical distributed measurement?

YES! "All entangled quantum states are nonlocal"

Buscemi, Phys. Rev. Lett. 108, 200401 (2012)



(using free LOSR transformations)

Perfectly

resources of any given type

Encoding nonclassicality of ~~quantum states~~ into resources of another type

formalities in arXiv:1909.04065

Can every resource of type T' be freely converted into *some* resource of type T in the same LOSR equivalence class?

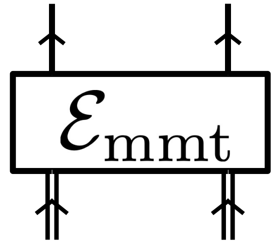
If yes, we say type T ***encodes*** type T'

$T' \backslash T$	$I \to C$	$I \to Q$	$C \to C$	$C \to Q$	$Q \to C$	$Q \to Q$
$I \to C$	✓ embed	✓ embed	✓ embed	✓ embed	✓ embed	✓ embed
$I \to Q$	✗ trans.	✓ embed	✗ Werner states	✓ embed	✓ semi−quantum games	✓ embed
$C \to C$	✗ trans.	✗ LOSR cannot entangle	✓ embed	✓ embed	✓ embed	✓ embed
$C \to Q$	✗ trans.	✗ trans.	✗ trans.	✓ embed	✓ Thm 3	✓ embed
$Q \to C$	✗ trans.	✗ trans.	✗ trans.	?	✓ embed	✓ embed
$Q \to Q$	✗ trans.	✗ trans.	✗ trans.	?	✓ Thm 3	✓ embed

One can deduce all possible encodings from our analysis…

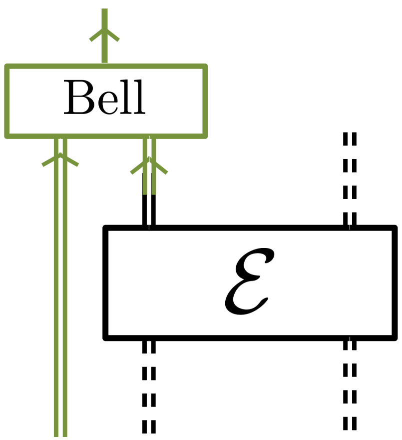
…with one important exception.

Theorem: The `distributed measurement' type encodes all other types.

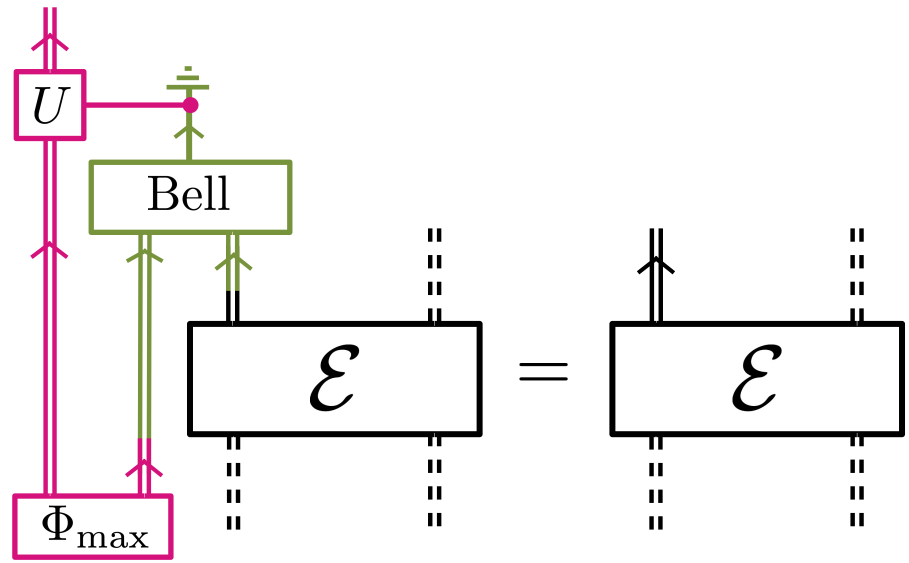


-*every* resource can be converted to one with only classical outputs without degrading its LOSR nonclassicality

Proof:



same equivalence class as $\mathcal{E}$

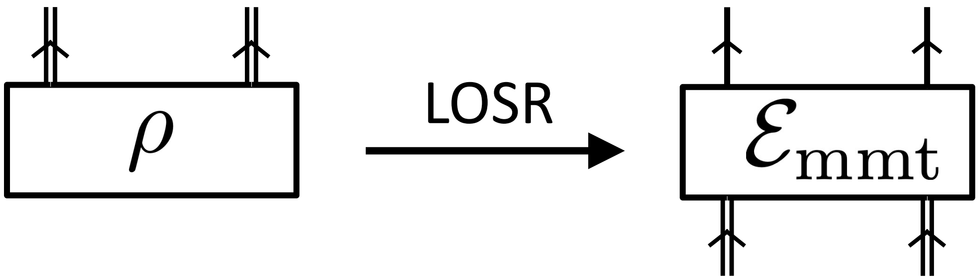


(same for all other quantum outputs)

T' \ T	$I \to C$	$I \to Q$	$C \to C$	$C \to Q$	$Q \to C$	$Q \to Q$
$I \to C$	✓ embed	✓ embed	✓ embed	✓ embed	✓ embed	✓ embed
$I \to Q$	✗ trans.	✓ embed	✗ Werner states	✓ embed	✓ semi–quantum games	✓ embed
$C \to C$	✗ trans.	✗ LOSR cannot entangle	✓ embed	✓ embed	✓ embed	✓ embed
$C \to Q$	✗ trans.	✗ trans.	✗ trans.	✓ embed	✓ Thm 3	✓ embed
$Q \to C$	✗ trans.	✗ trans.	✗ trans.	?	✓ embed	✓ embed
$Q \to Q$	✗ trans.	✗ trans.	✗ trans.	?	✓ Thm 3	✓ embed

Every encoding has practical consequences.

Example: Every entangled state can be freely transformed into a distributed measurement that is just as nonclassical.



without degrading its nonclassicality

Quantum systems require well-characterized quantum measurements to probe... ("device-dependent")

...but classical systems are easy to probe! ("device-independent")

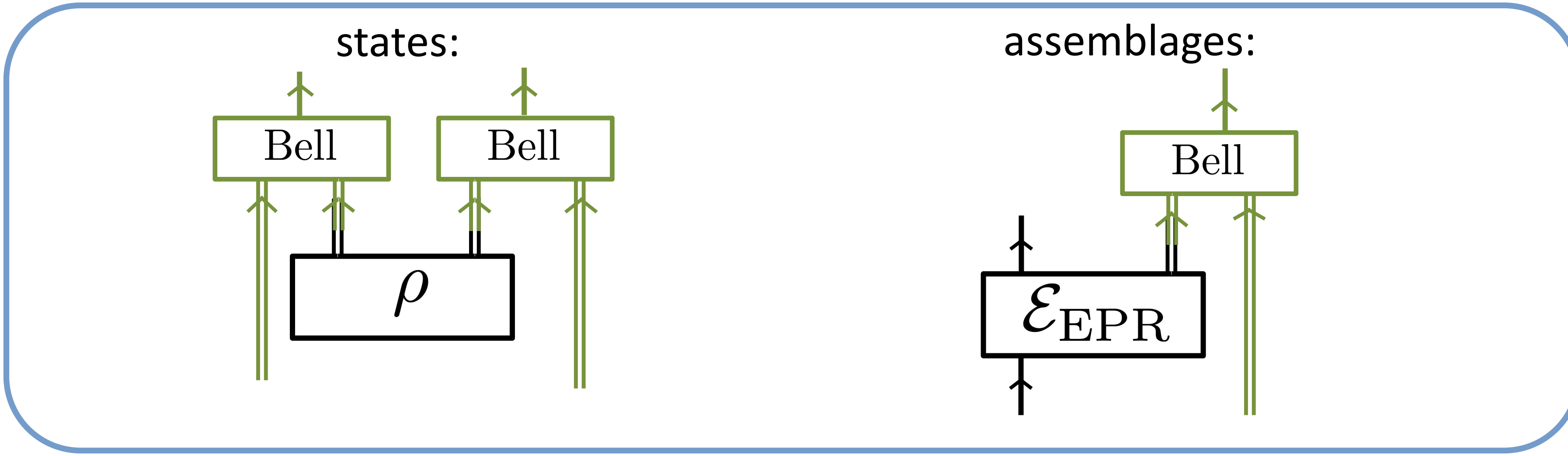
Hence:

All quantum states can have their nonclassicality characterized in a *measurement-device-independent* manner.

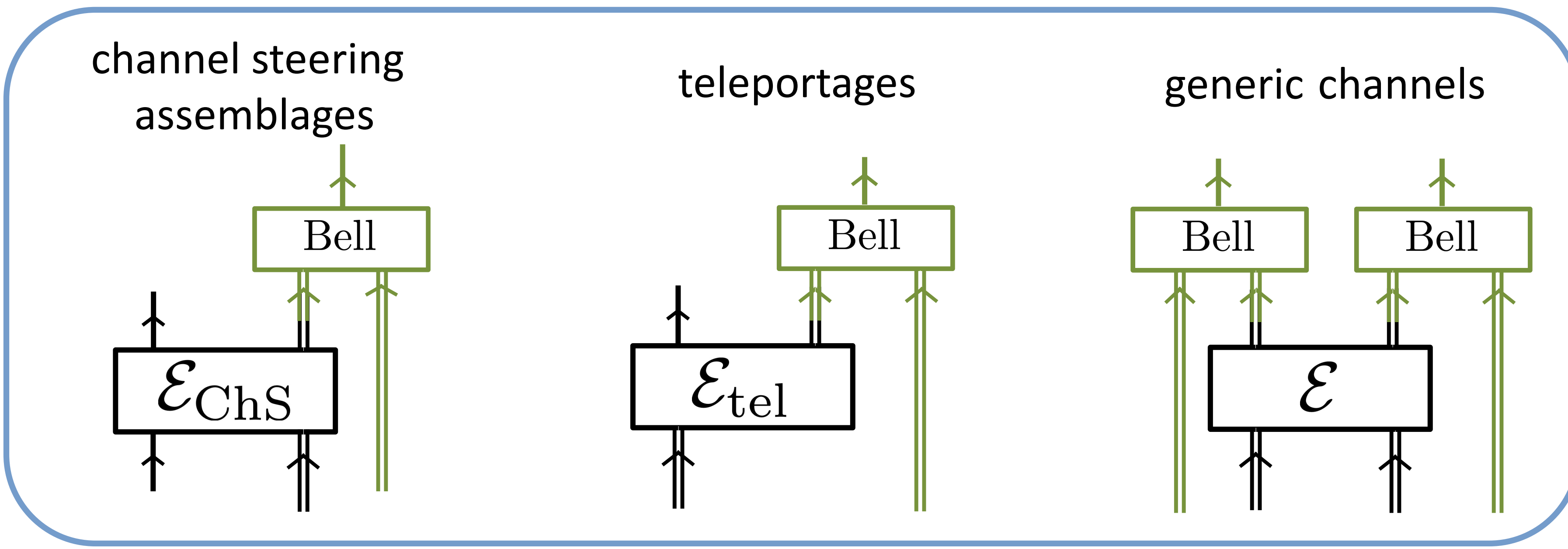
C. Branciard, D. Rosset, Y.-C. Liang, and N. Gisin, Physical Review Letters **110**, 060405 (2013).

E. G. Cavalcanti, M. J. W. Hall, and H. M. Wiseman, Phys. Rev. A **87**, 032306 (2013)

Measurement-device-independent characterization of:



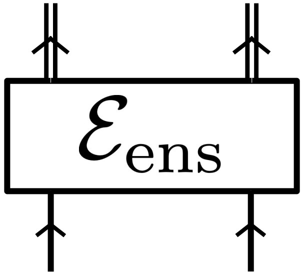
Our theorem extends this to all resources. e.g.:



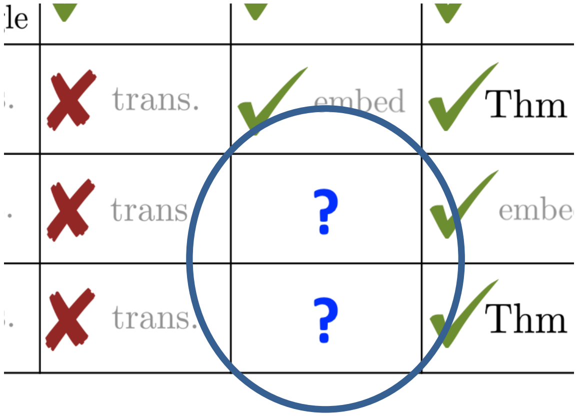
Preparation-device-independent tests of nonclassicality?

Can one convert *any* resource into another which has only *classical* inputs without degrading nonclassicality?

Do distributed ensemble-preparing channels encode all other types?



Open question:



Some encodings imply a new way of characterizing resources

(e.g. less demanding measurements/preparations)

Every encoding implies a new way of characterizing resources

(e.g. less demanding measurements/preparations)

(see Theorem 7 in arXiv:1909.04065)

Quantitative generalization of Cavalcanti, Skrzypczyk, Supić (arXiv:1607.03249):

-teleportation games perfectly characterize LOSR-entanglement of states

Open questions

- For all 15 nontrivial bipartite scenarios: geometry of free set, preorder over nonfree resources, monotones, witnesses, etc; more parties?
- More type-independent results relating different types (and games)
- Type-independent tools for characterizing nonclassicality in practice
  - E.g. computing values of monotones, finding explicit witnesses, etc (forthcoming)
- 5 novel types of `nonlocality' (and corresponding scenarios)
- Preparation-device-independent characterizations of nonclassicality?
- Inequivalent types of (maximal) nonclassicality
- Relation to self-testing
- quantifying nonclassicality of GPT or signaling resources
- quantifying the post-quantumness of resources (type-independent LOSE)
- relationships with separable operations and with LOCC operations

Lots of unanswered basic questions even in the Bell scenario!

Quantifying LOSR nonclassicality across arbitrary resource types

arXiv:1909.04065

Special thanks to Elie Wolfe and Rob Spekkens

Thank you!