Quantum processes on time-delocalized systems

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Causality in the quantum world, Anacapri, 2019

Outline

0) Background

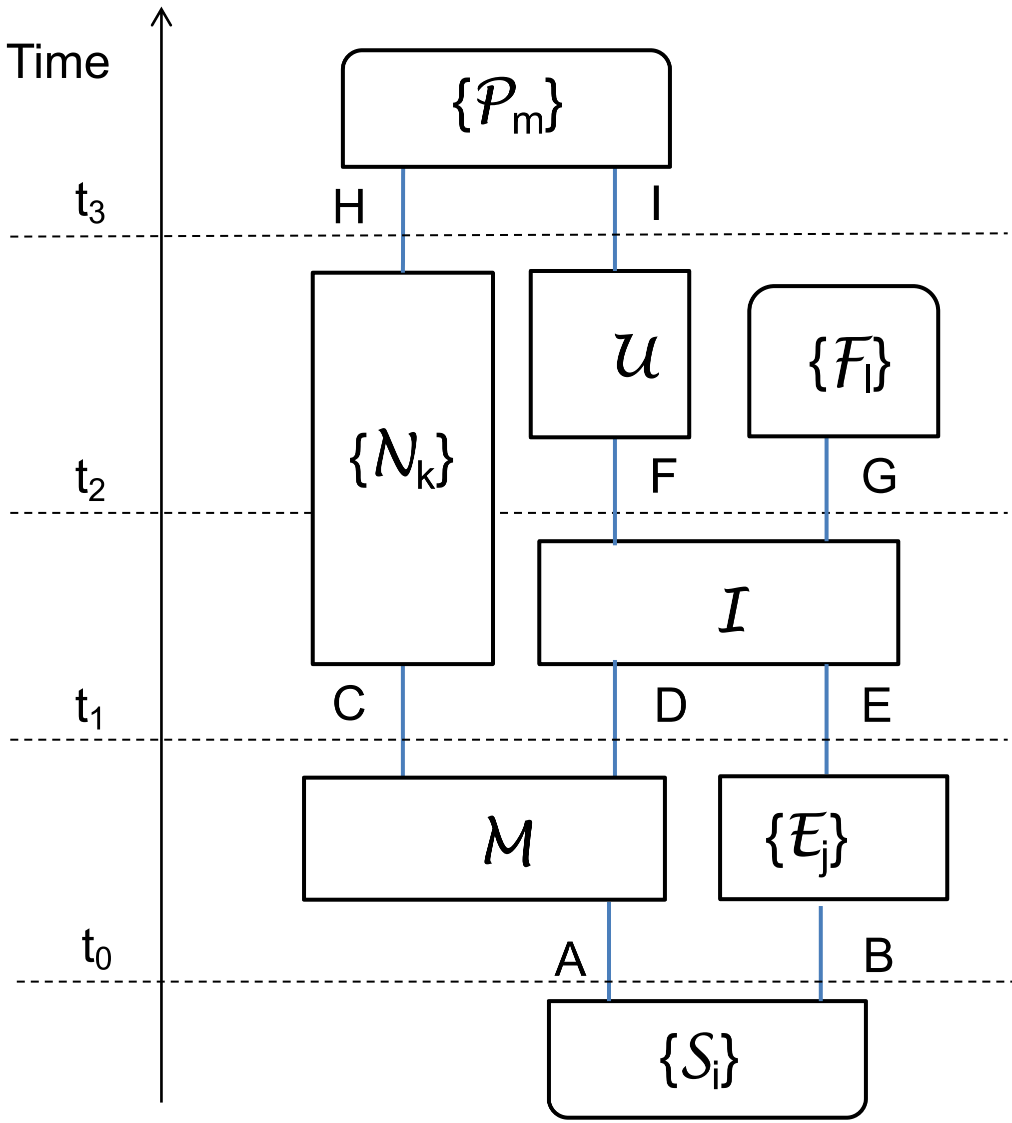
1) The quantum SWITCH on time delocalized systems

2) Realization of a class of isometric extensions of bipartite processes (based on arXiv:1801.07594)

3) **Recent** **(with R. Lorenz and J. Barret):** all unitarily extensible bipartite processes are causally separable

4) **New** **(with J. Wechs, Cyril Branciard, …):** realization of processes violating causal inequalities

Quantum systems



Joint probabilities

p(i, j, k, l, m)

Hardy, PIRSA:09060015; Chiribella, D'Ariano, Perinotti, PRA 81, 062348 (2010) [arXiv 2009]

Quantum systems

System A → Hilbert space $\mathcal{H}^A$

Quantum operation from A to B → quantum instrument (collection of CP maps):

$$\{\mathcal{M}_i^{A\to B}\}_{i\in O} \,, \quad \text{where} \quad \sum_{i\in O} \mathcal{M}_i^{A\to B} = \overline{\mathcal{M}}^{A\to B} \quad \text{is CPTP.}$$

From a global perspective, a system is generally a ***subsystem*** (tensor factor of a subspace) of a larger Hilbert space (P. Zanardi, PRL (2001); Zanardi, Lidar, Lloyd, PRL (2004)):

$$(\mathcal{H}^A \otimes \mathcal{H}^{\overline{A}}) \oplus \mathcal{H}^K \cong \mathcal{H}^{large} \quad \text{(usually at a given time)}$$

The most general faithful encoding of quantum information

Viola, Knill, Laflamme, JPA (2001); E. Knill, PRA (2006); Kribs and Spekkens PRA (2006)

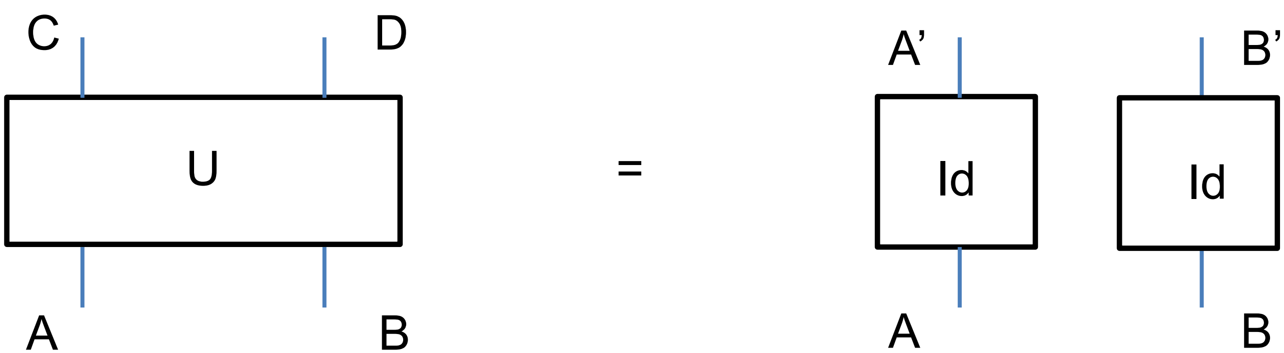
Quantum systems

A subsystem is defined by the algebra of operators acting (locally) on that systems:

$$\mathcal{A}^A := (\mathcal{A}^A \otimes \mathbb{1}^{\bar{A}}) \oplus 0^K$$

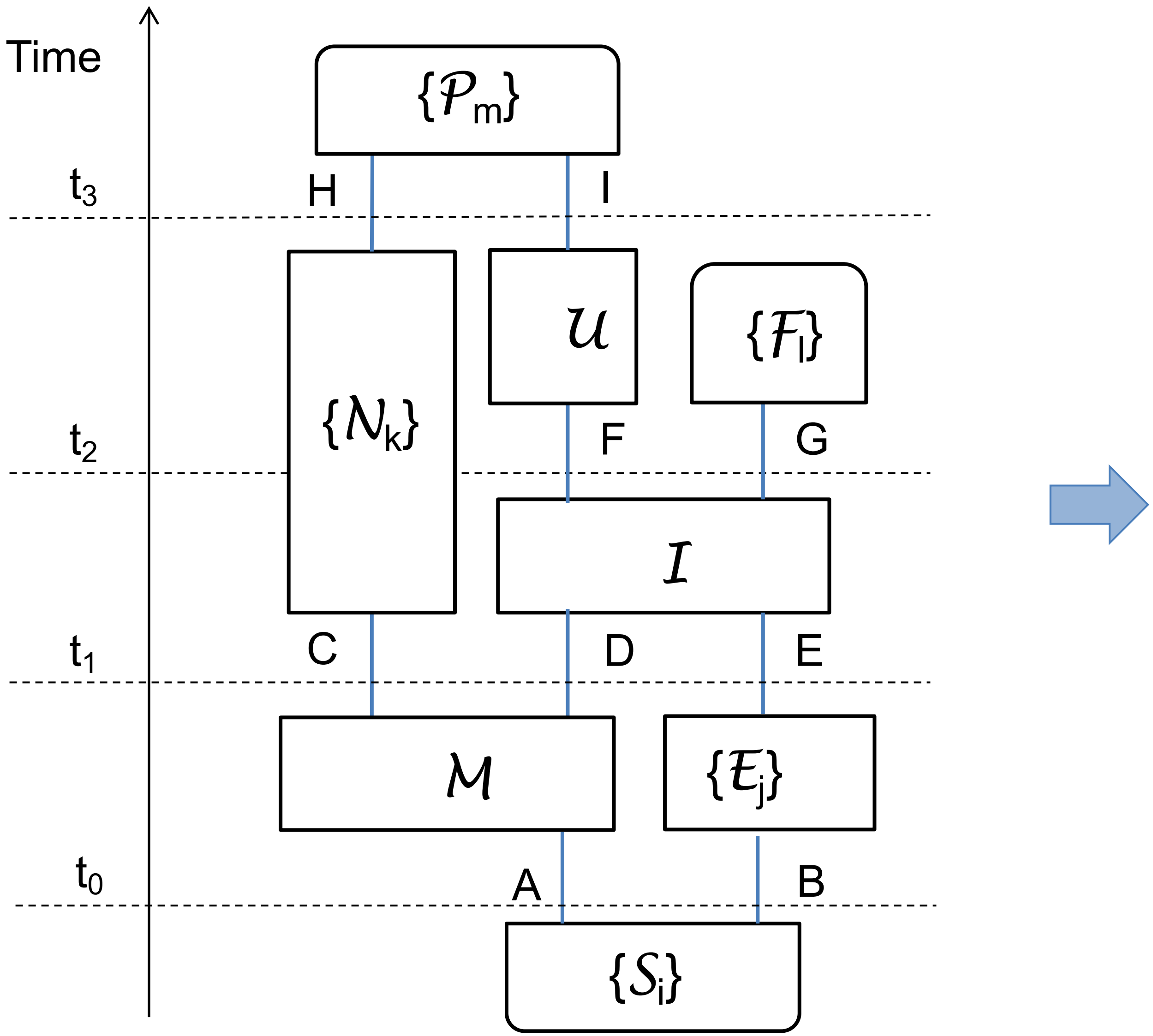
Example:

$$\mathcal{H}^C \otimes \mathcal{H}^D \cong \mathcal{H}^{A'} \otimes \mathcal{H}^{B'}$$



$$O^{A'} \equiv O^{A'} \otimes \mathbb{1}^{B'} := U(O^A \otimes \mathbb{1}^B)U^\dagger$$

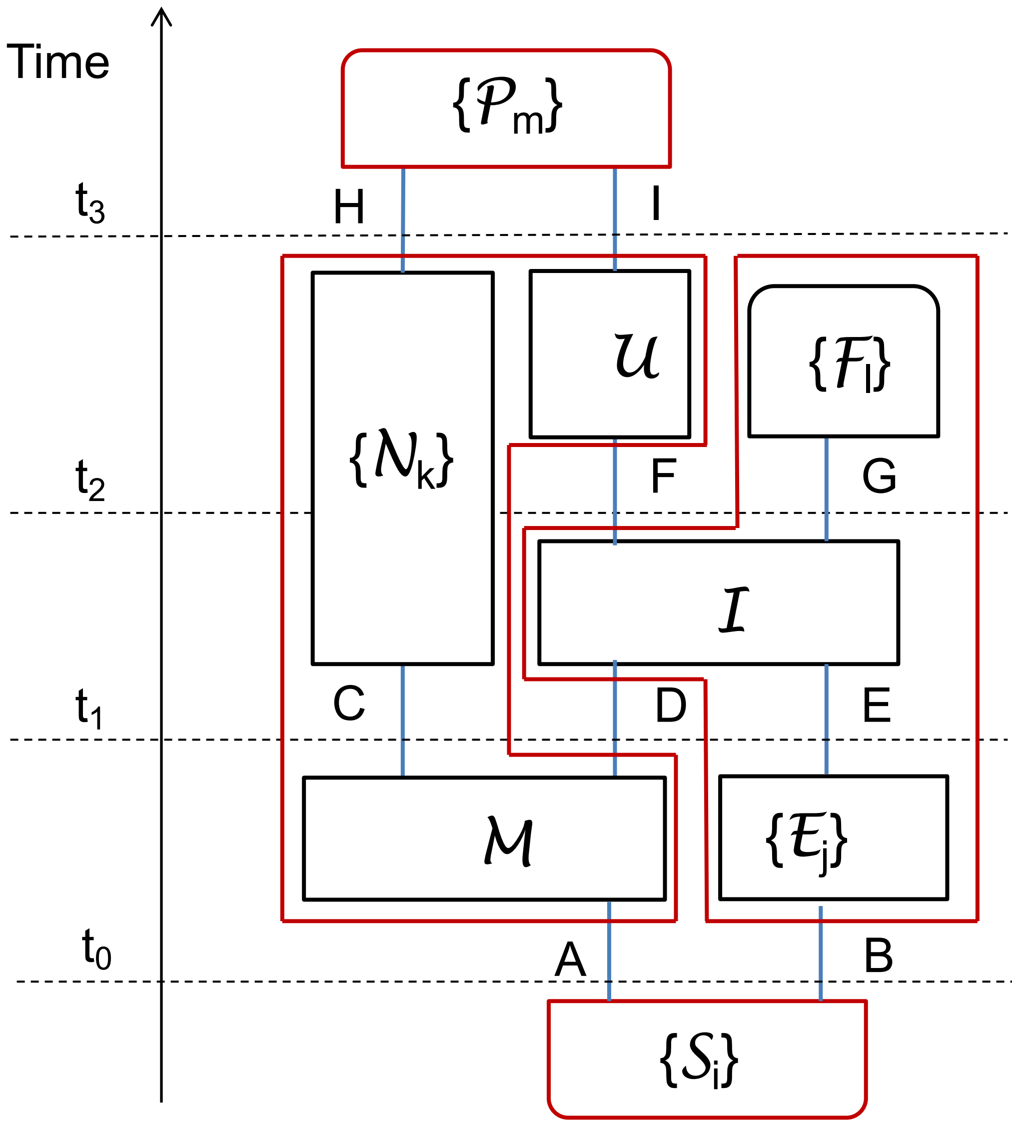
Quantum systems



Joint probabilities

p(i, j, k, l, m)

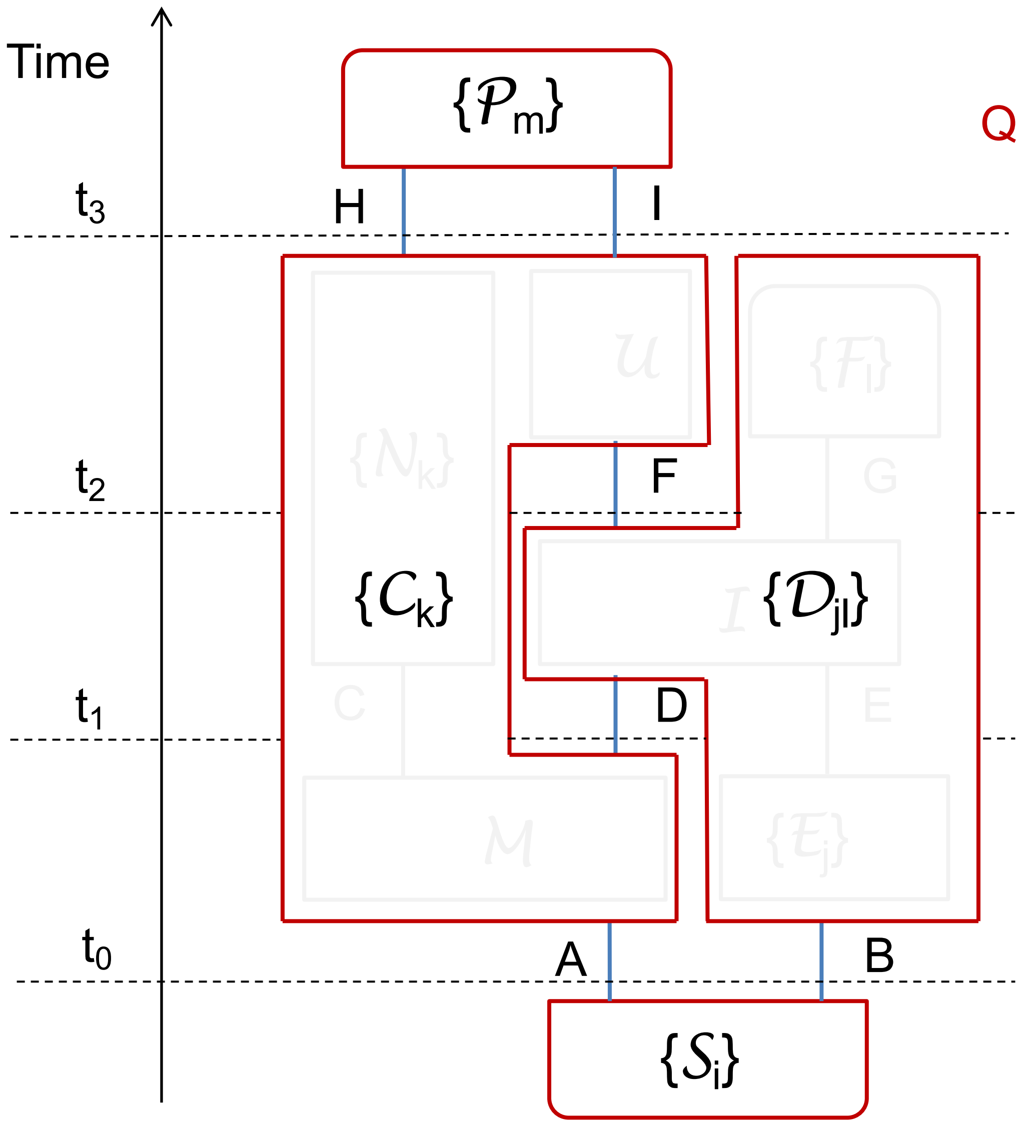
Quantum systems



Joint probabilities

$p(i, j, k, l, m)$

Quantum systems



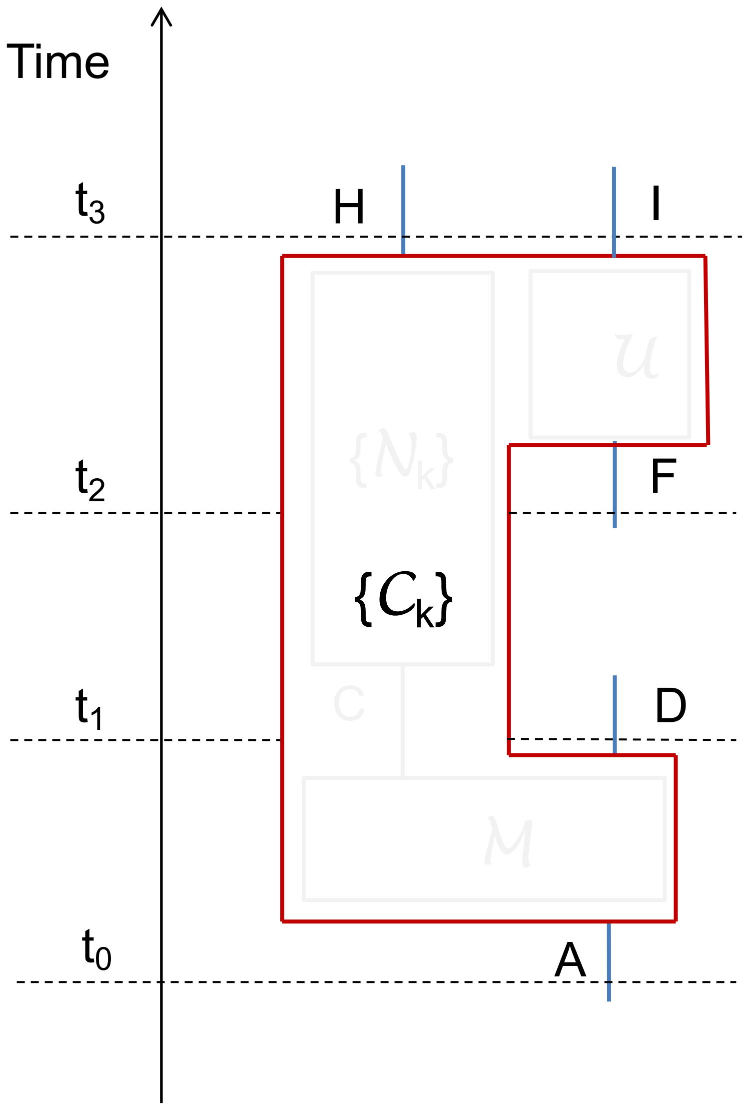
Quantum combs:

Joint probabilities

p(i, j, k, l, m)

Chiribella, D'Ariano, Perinotti, PRA (2009)

Quantum systems

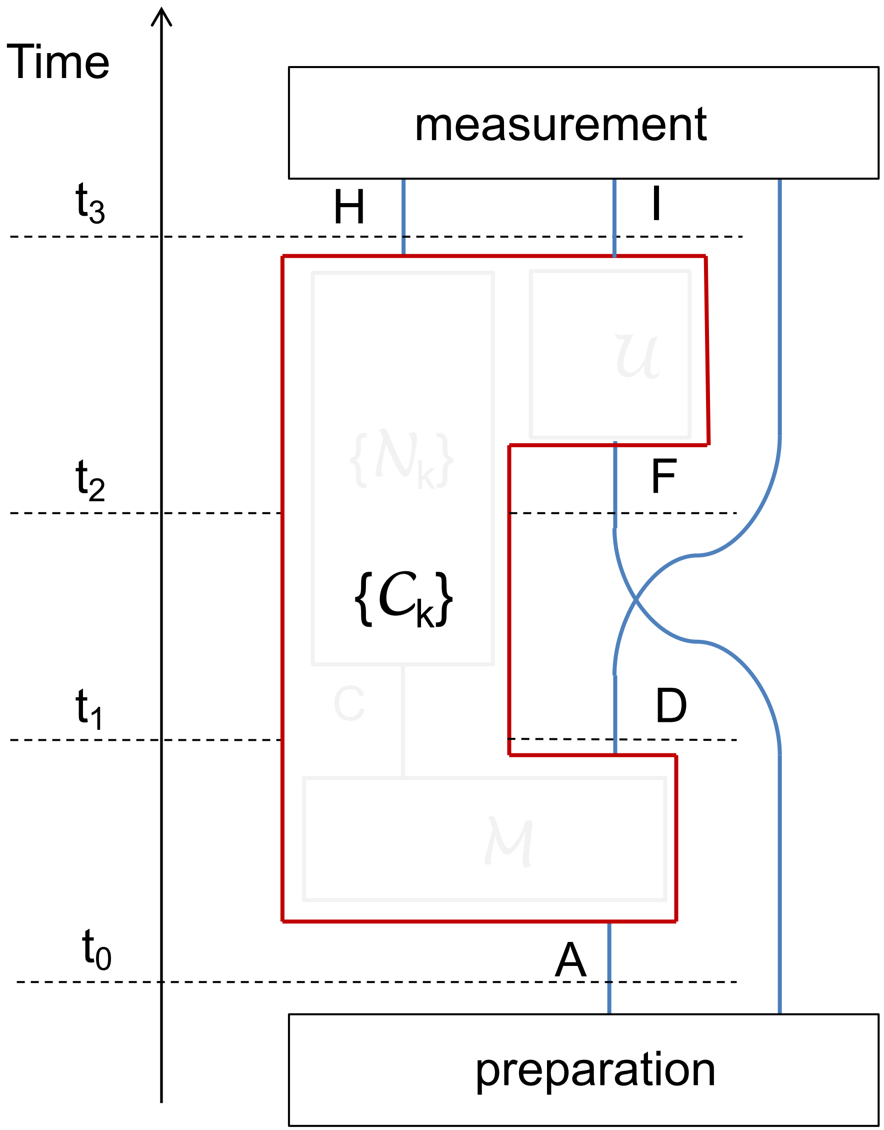


Quantum combs:

← *An operation from AF to DHI*

Chiribella, D'Ariano, Perinotti, PRA (2009)

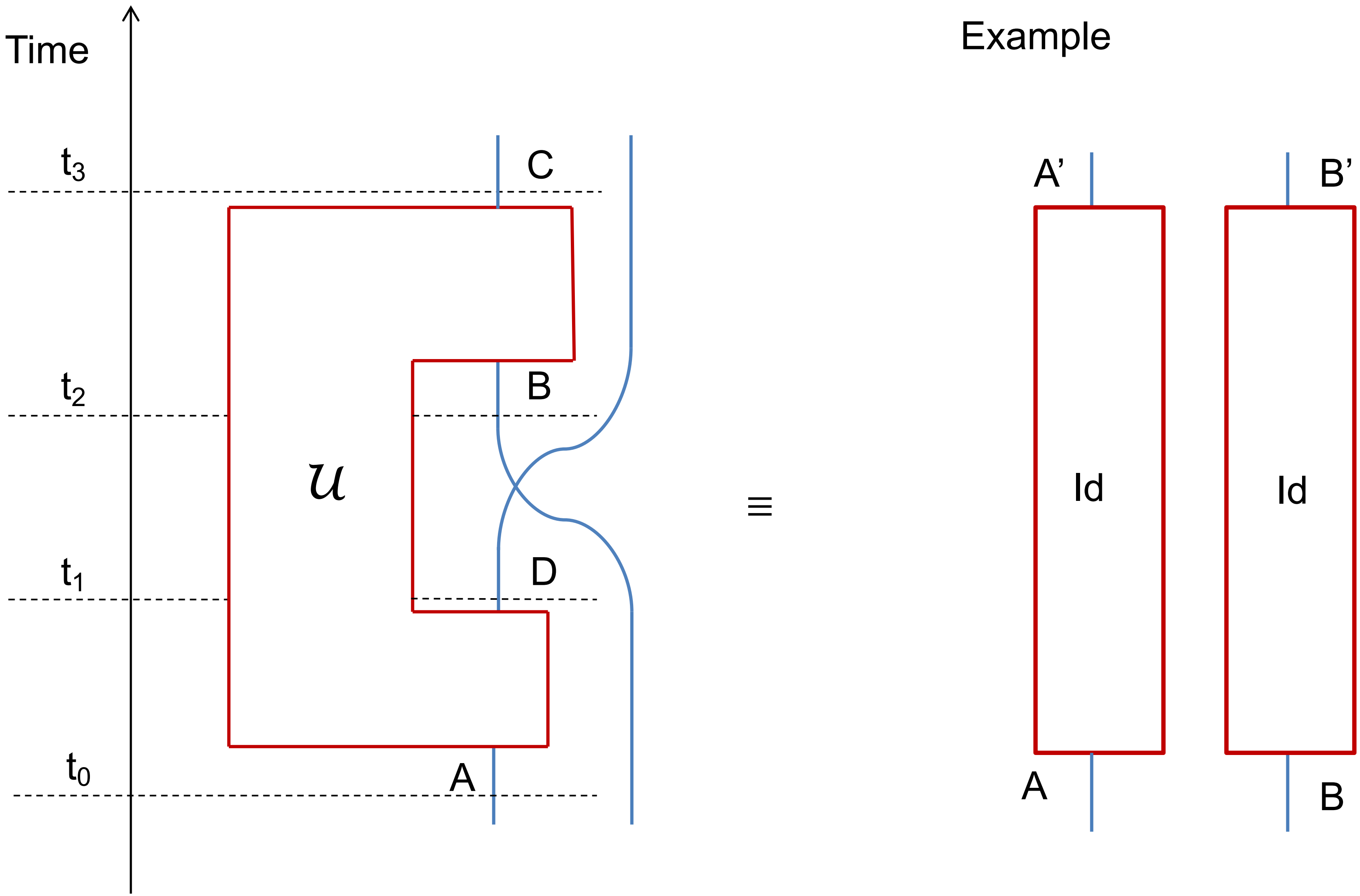
Quantum systems



Can do tomography on it.

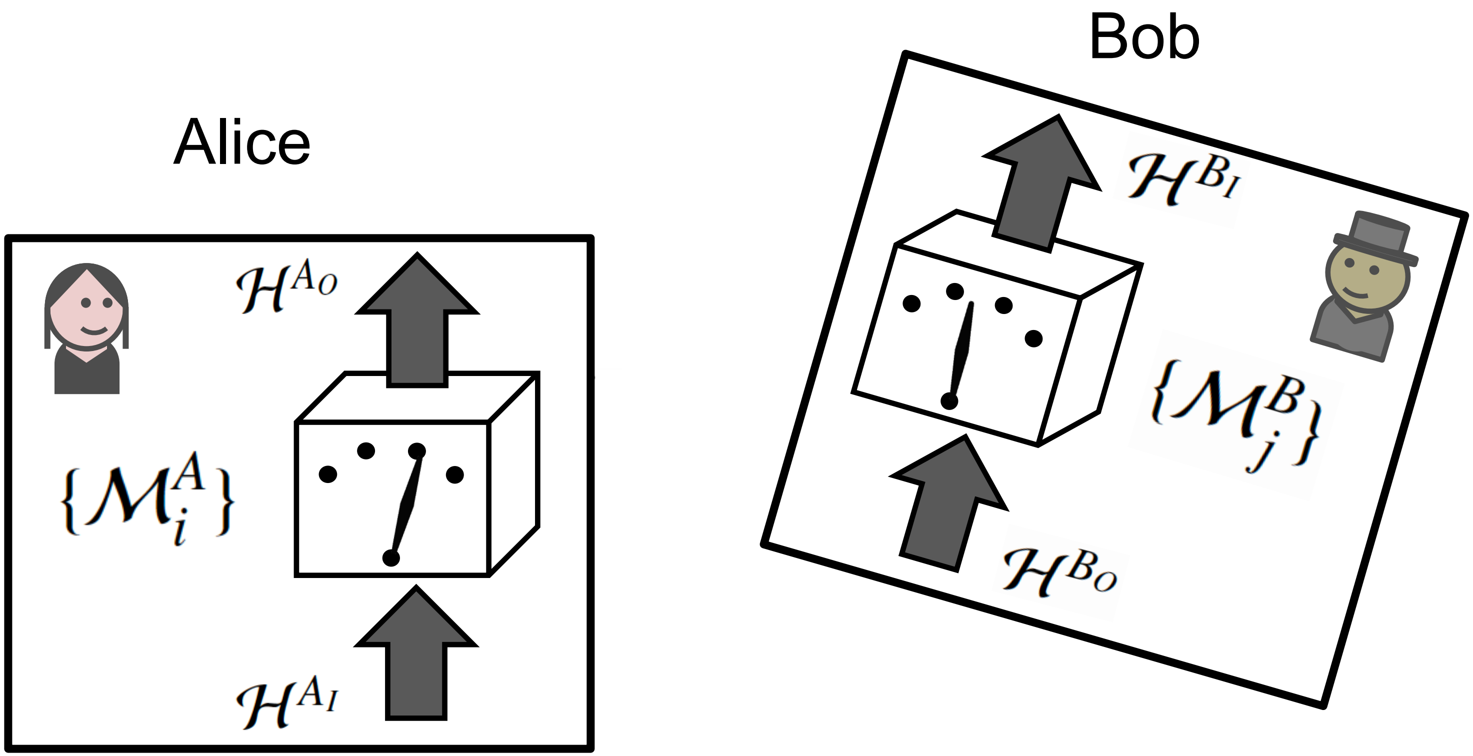
Time-delocalized systems: subsystems of Hilbert spaces that are tensor products of systems at different times

Time-delocalized systems: subsystems of Hilbert spaces that are tensor products of systems at different times



$$\mathcal{H}^{C} \otimes \mathcal{H}^{D} \cong \mathcal{H}^{A'} \otimes \mathcal{H}^{B'}$$

The quantum process matrix framework



No assumption of global causal order

O. O., F. Costa, and Č. Brukner, Nat. Commun. 3, 1092 (2012).

The quantum process matrix framework

Joint probabilities

$$p(\mathcal{M}_i^A, \mathcal{M}_j^B, \cdots) = \mathrm{Tr}\left[W^{A_{\mathrm{in}}A_{\mathrm{out}}B_{\mathrm{in}}B_{\mathrm{out}}\cdots}\left(M_i^{A_{\mathrm{in}}A_{\mathrm{out}}} \otimes M_j^{B_{\mathrm{in}}B_{\mathrm{out}}} \otimes \cdots\right)\right]$$

Process matrix

CJ operators

The quantum process matrix framework

Joint probabilities

$$p(\mathcal{M}_i^A, \mathcal{M}_j^B, \cdots) = \mathrm{Tr}\left[W^{A_{\rm in}A_{\rm out}B_{\rm in}B_{\rm out}\cdots}\left(M_i^{A_{\rm in}A_{\rm out}} \otimes M_j^{B_{\rm in}B_{\rm out}} \otimes \cdots\right)\right]$$

Process matrix

CJ operators

1. Non-negative probabilities: $W^{A_{\rm in}A_{\rm out}B_{\rm in}B_{\rm out}\cdots} \geq 0$

2. Probabilities sum up to 1:

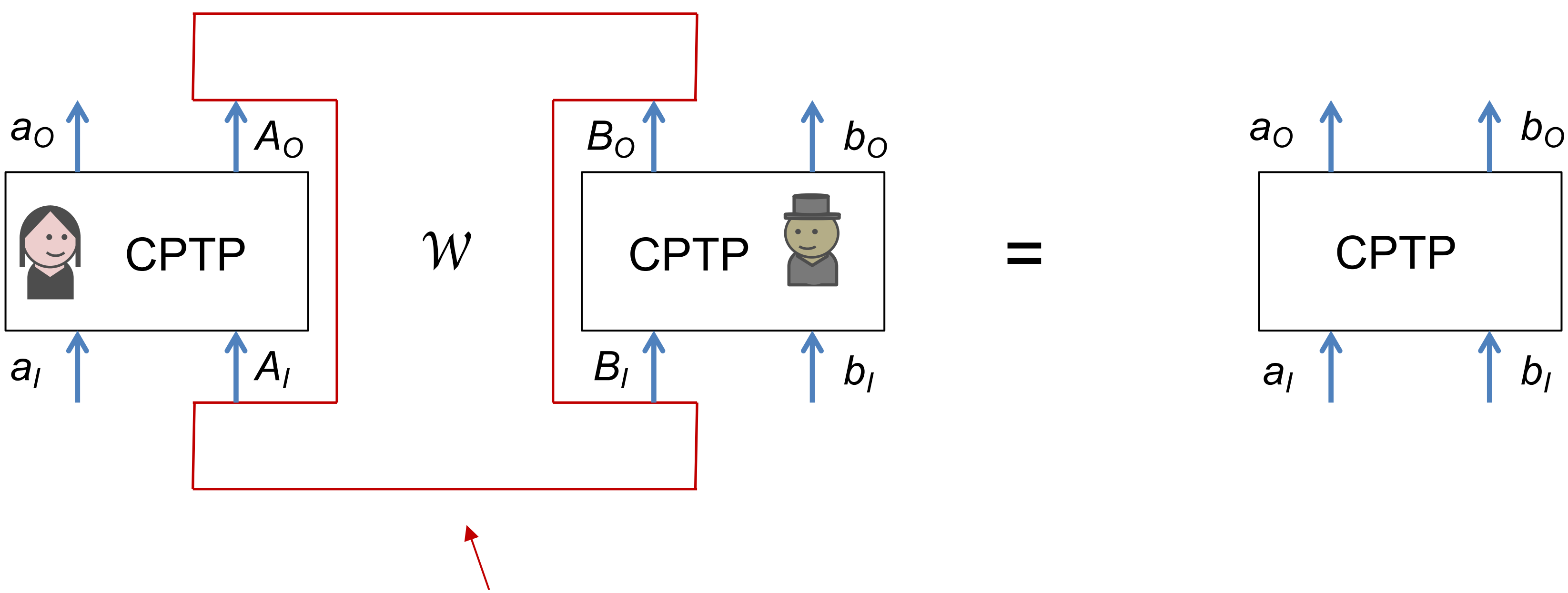
$$\mathrm{Tr}\left[W^{A_{\rm in}A_{\rm out}B_{\rm in}B_{\rm out}\cdots}\left(M^{A_{\rm in}A_{\rm out}} \otimes N^{B_{\rm in}B_{\rm out}} \otimes \cdots\right)\right] = 1$$

on all CPTP $M^{A_{\rm in}A_{\rm out}}$, $N^{B_{\rm in}B_{\rm out}}$, ...

The quantum process matrix framework

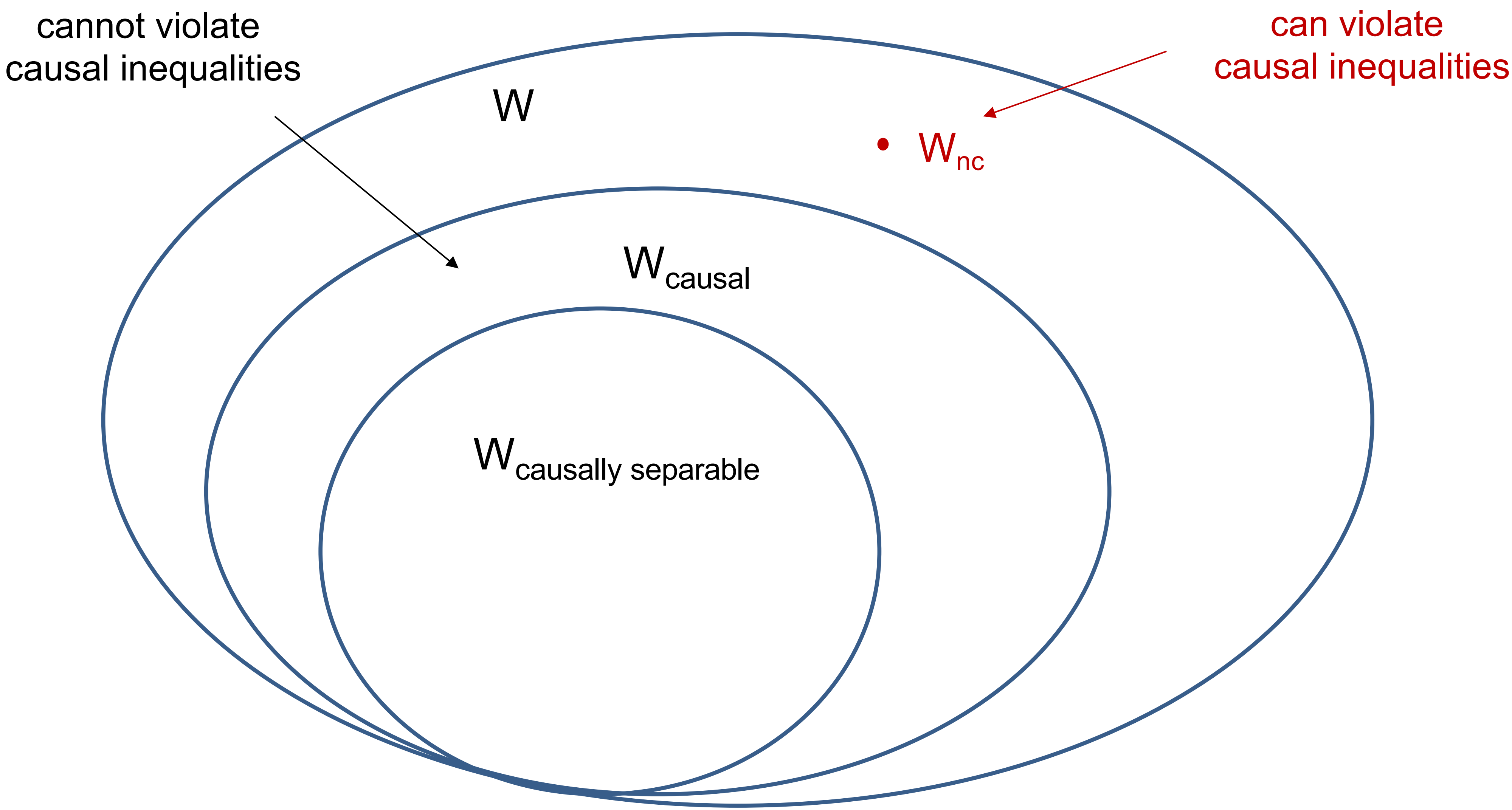
An equivalent formulation as a second-order transformation:

[Quantum supermaps, Chiribella, D'Ariano, and Perinotti, EPL 83, 30004 (2008)]



a channel from $A_O B_O$ to $A_I B_I$

The process matrix framework

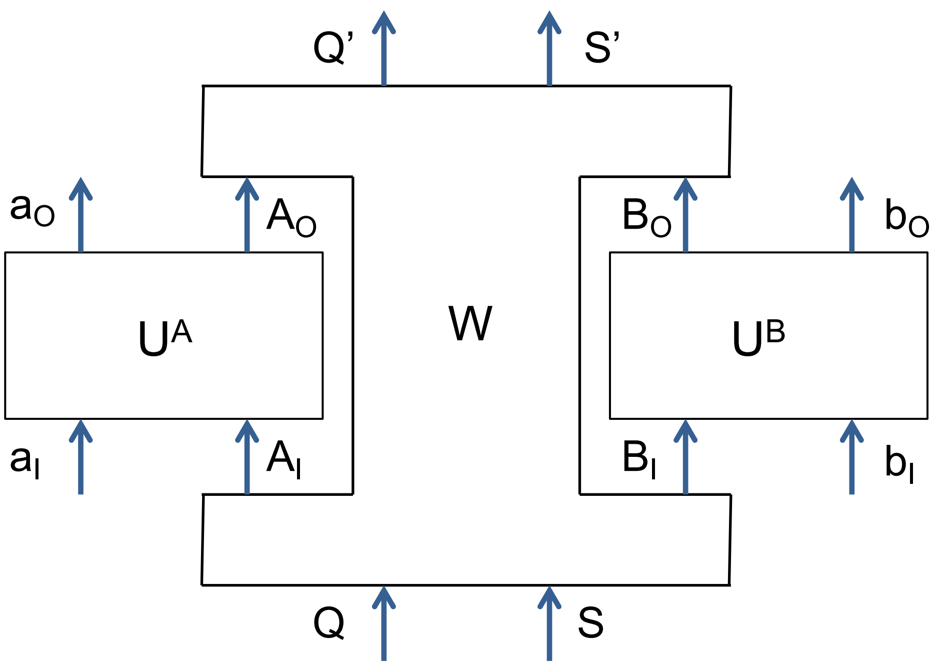


Do such processes have a physical realization?

Physical realization: there can be an experiment in which every element of the mathematical description of the process (Hilbert spaces, operations on them, …) has a physical counterpart that can be considered a realization of that element.

The quantum SWITCH

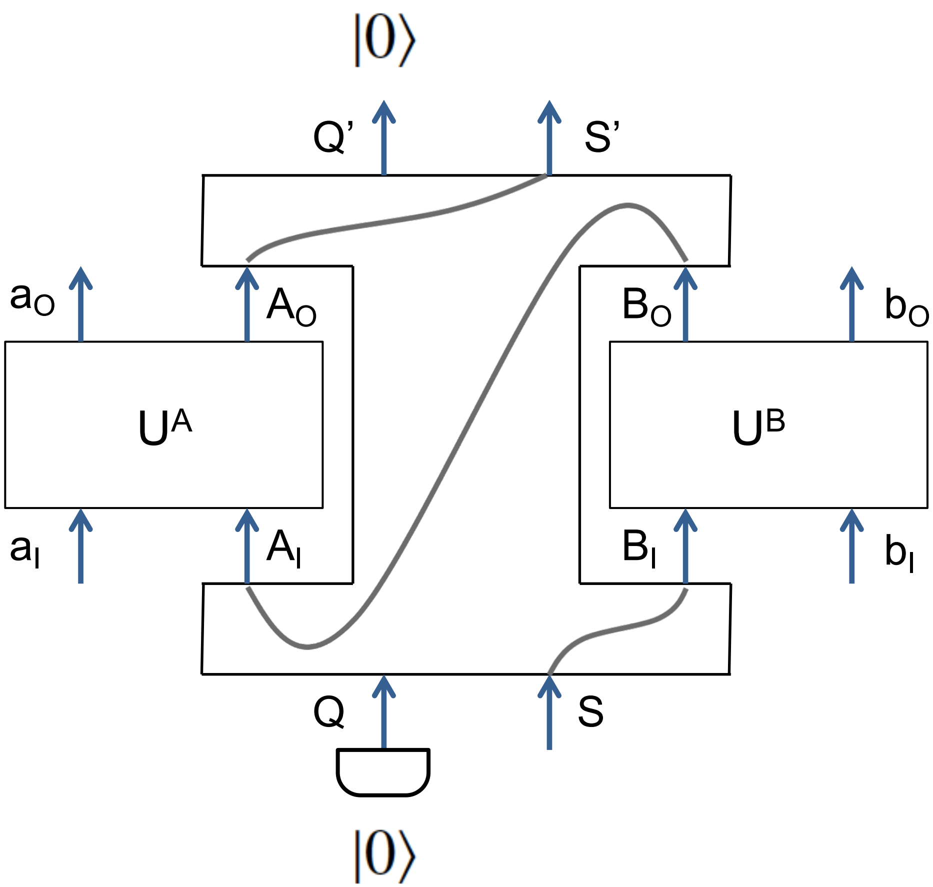
A supermap:



Chiribella, D'Ariano, Perinotti, and Valiron, PRA 88, 022318 (2013), arXiv:0912.0195 (2009)

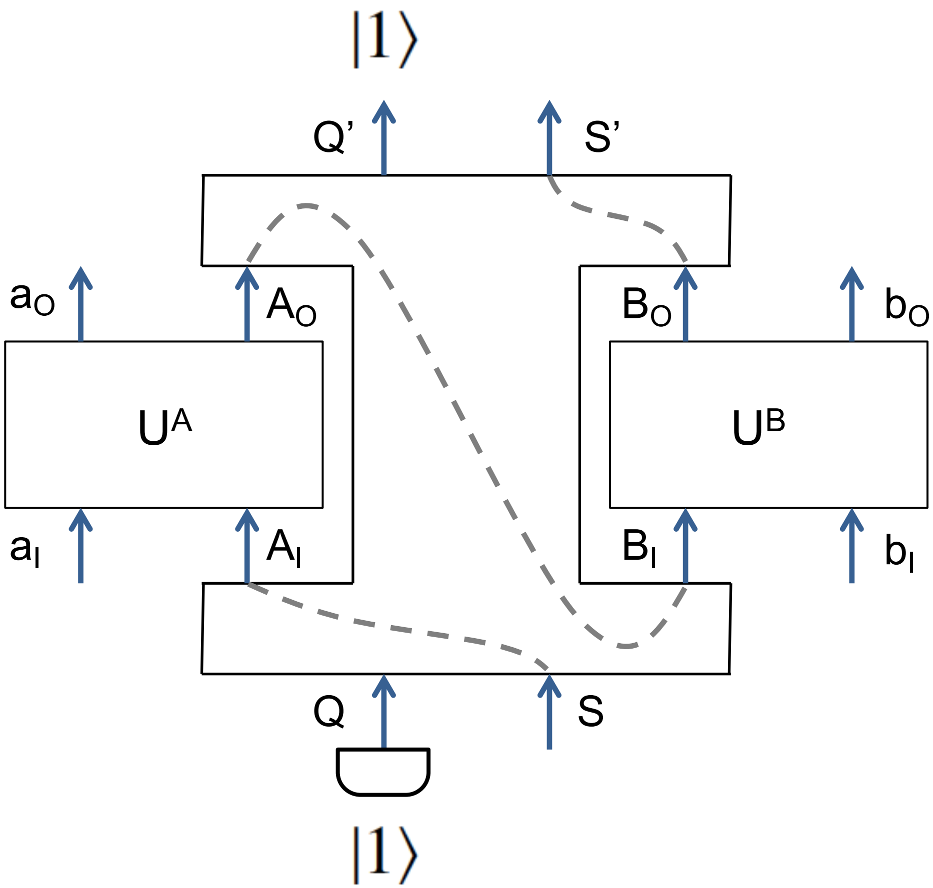
The quantum SWITCH

A supermap:



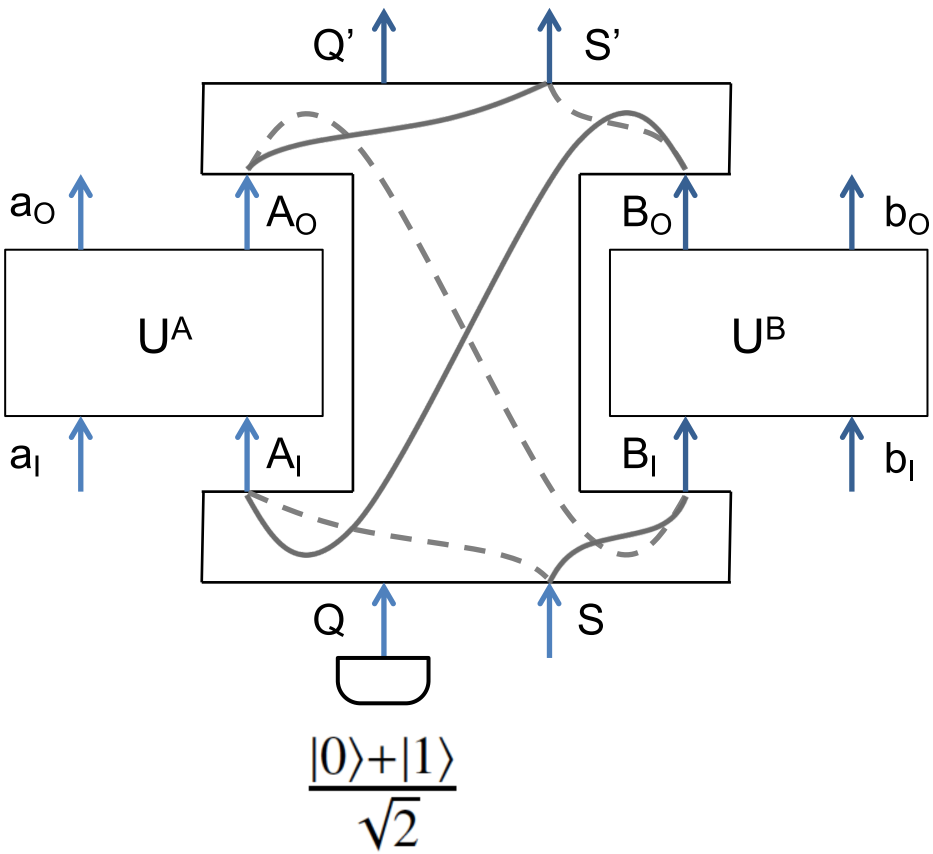
The quantum SWITCH

A supermap:



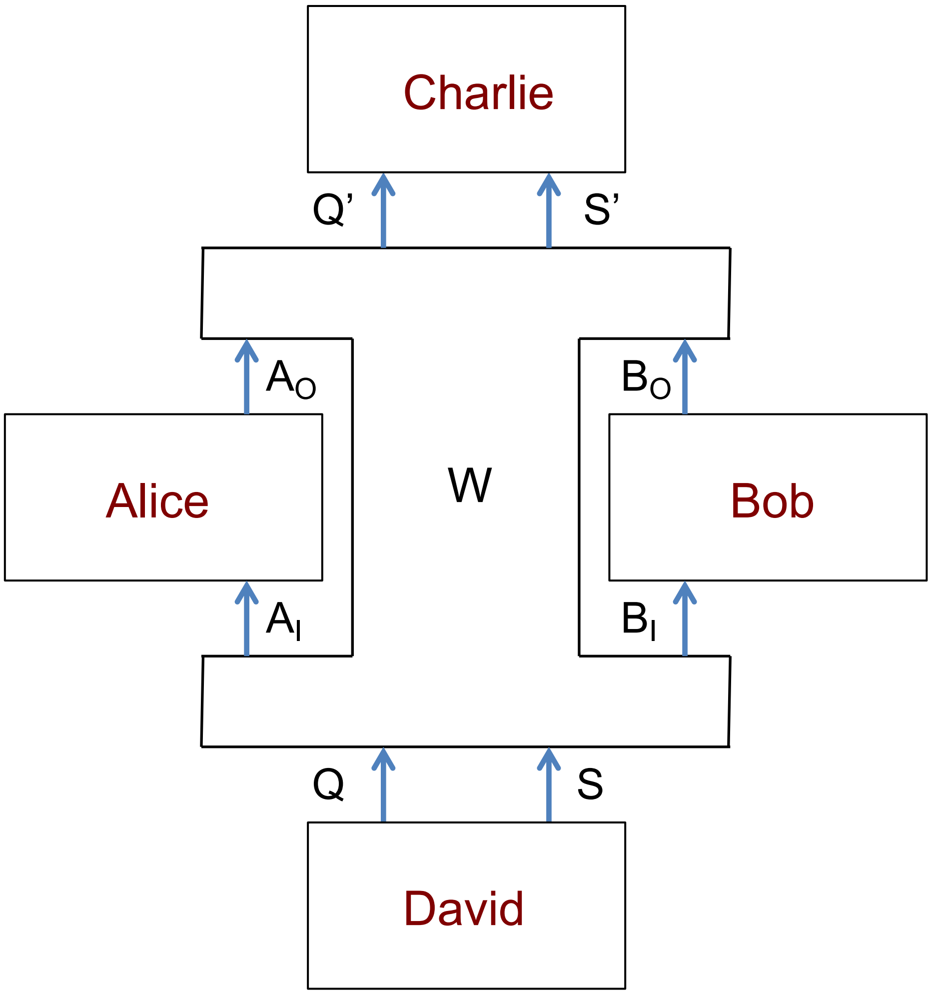
The quantum SWITCH

A supermap:



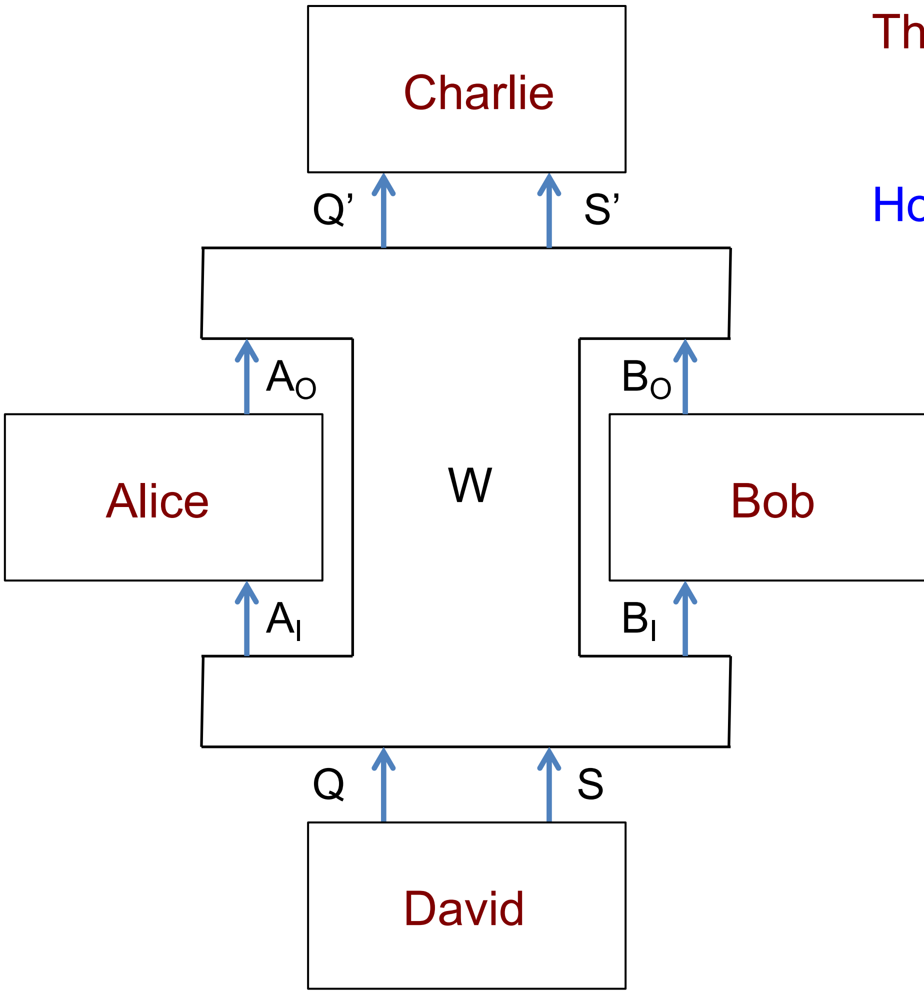
The quantum SWITCH

A process matrix:



The quantum SWITCH

A process matrix:



The process matrix is **not causally separable.**

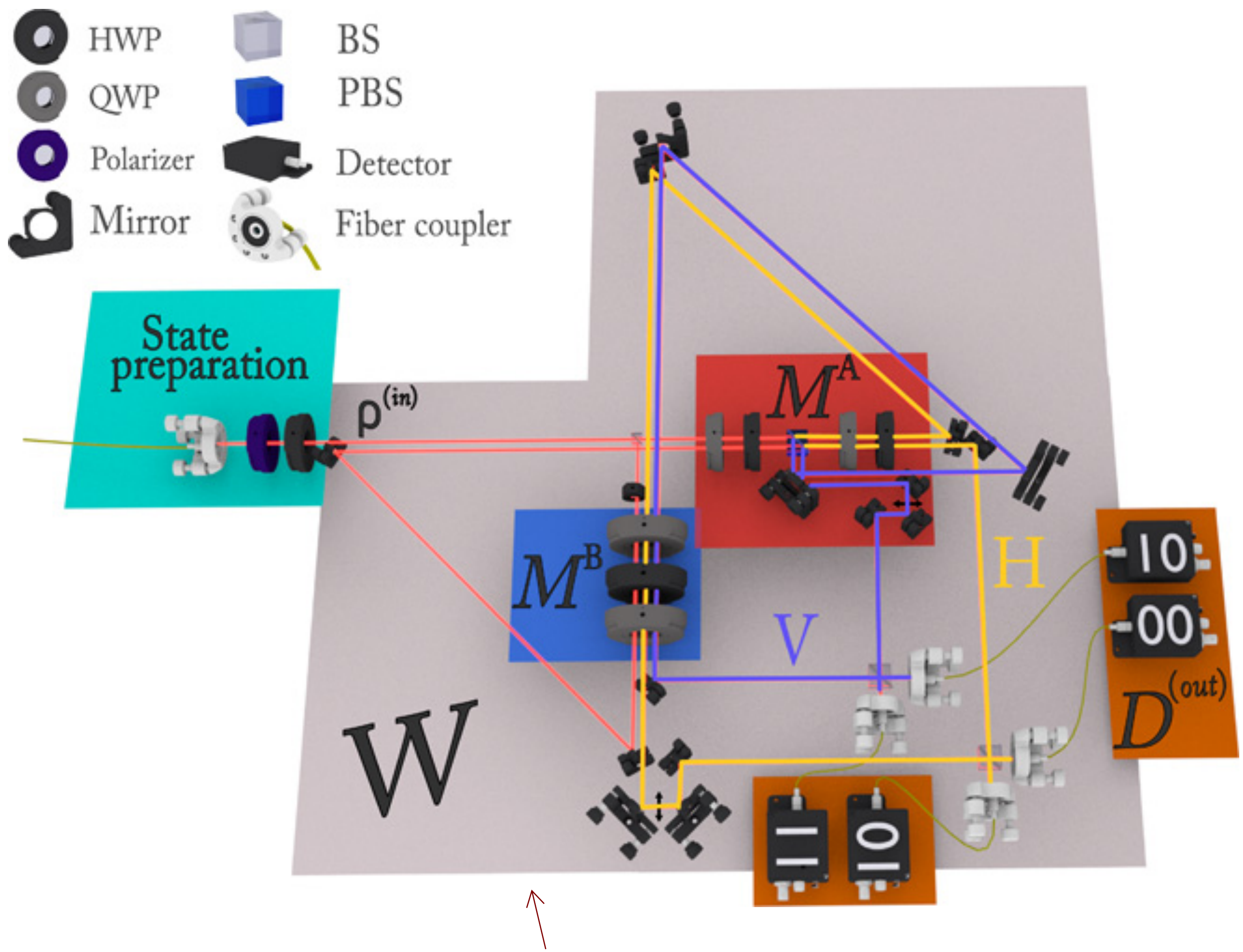
However, it cannot violate causal inequalities.

W= |W><W|

Oreshkov and Giarmatzi, NJP 18, 093020 (2016)

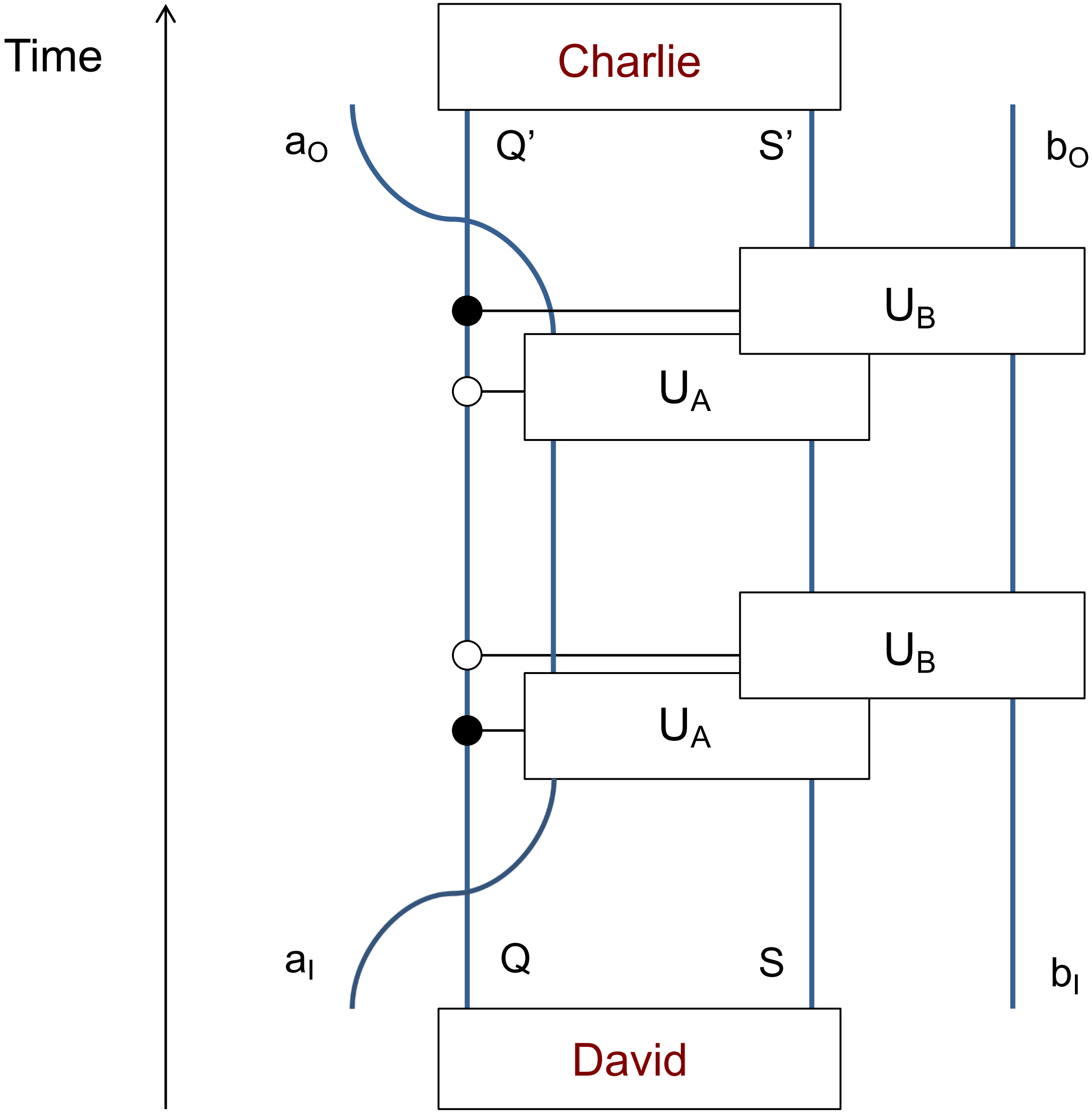
Araujo et al., NJP 17, 102001 (2015)

Experimental realizations of the quantum SWITCH



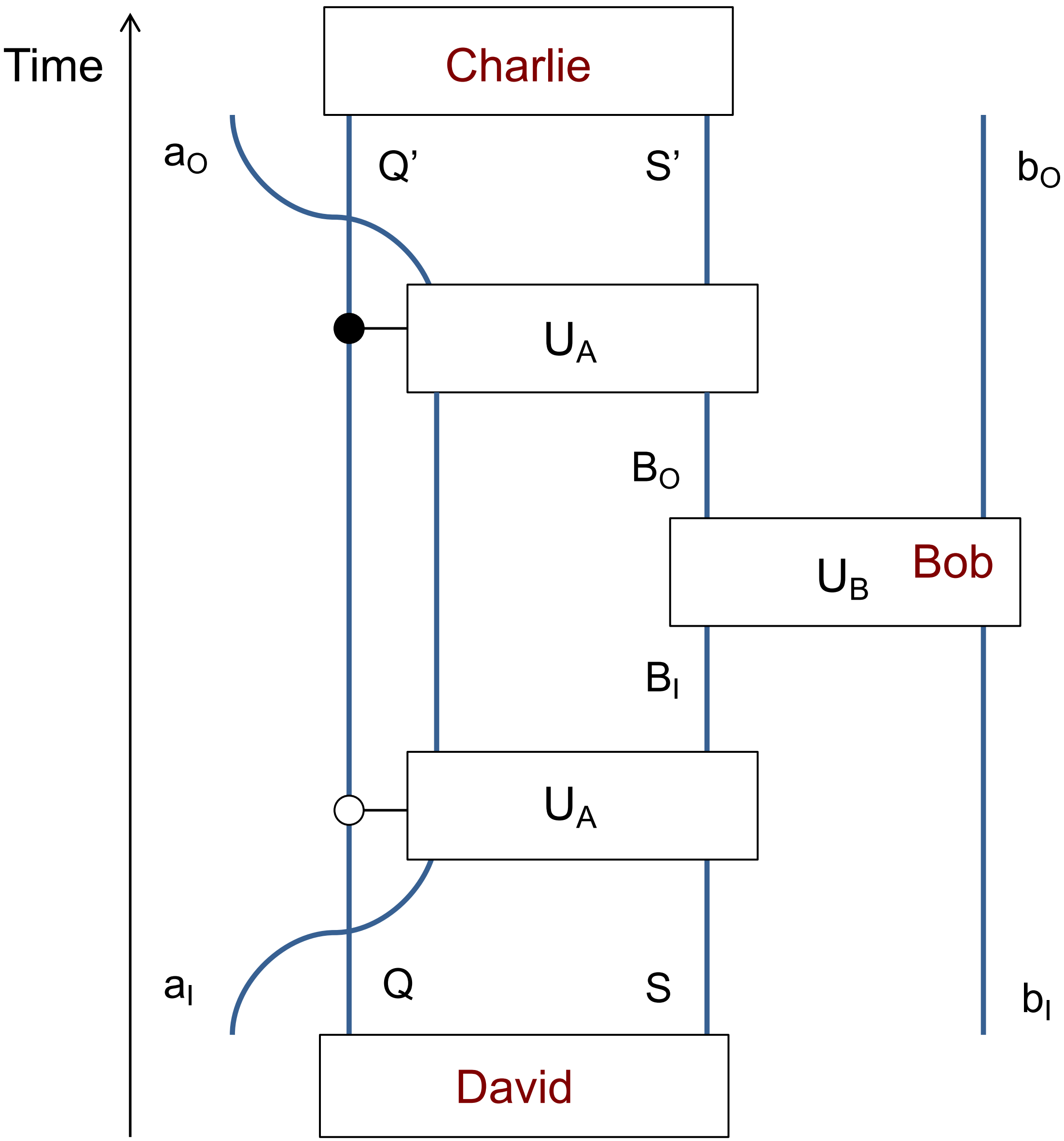
Procopio et al., Nat. Commun. (2015)   Rubio et al., Sci. Adv. (2017)   Goswami et al., PRL (2018)   ...

Where are the operations of Alice and Bob?



Temporal description

Where are the operations of Alice and Bob?

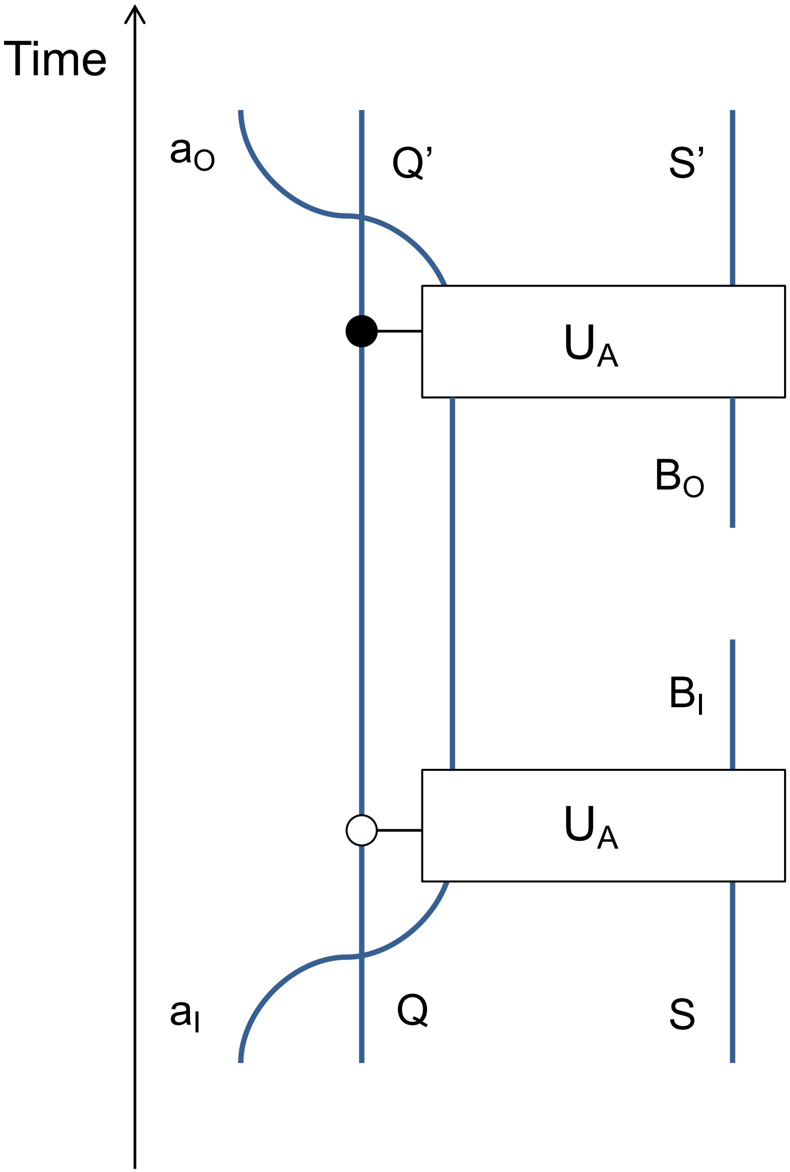


Temporal description (simple version)

Bob at a fixed time

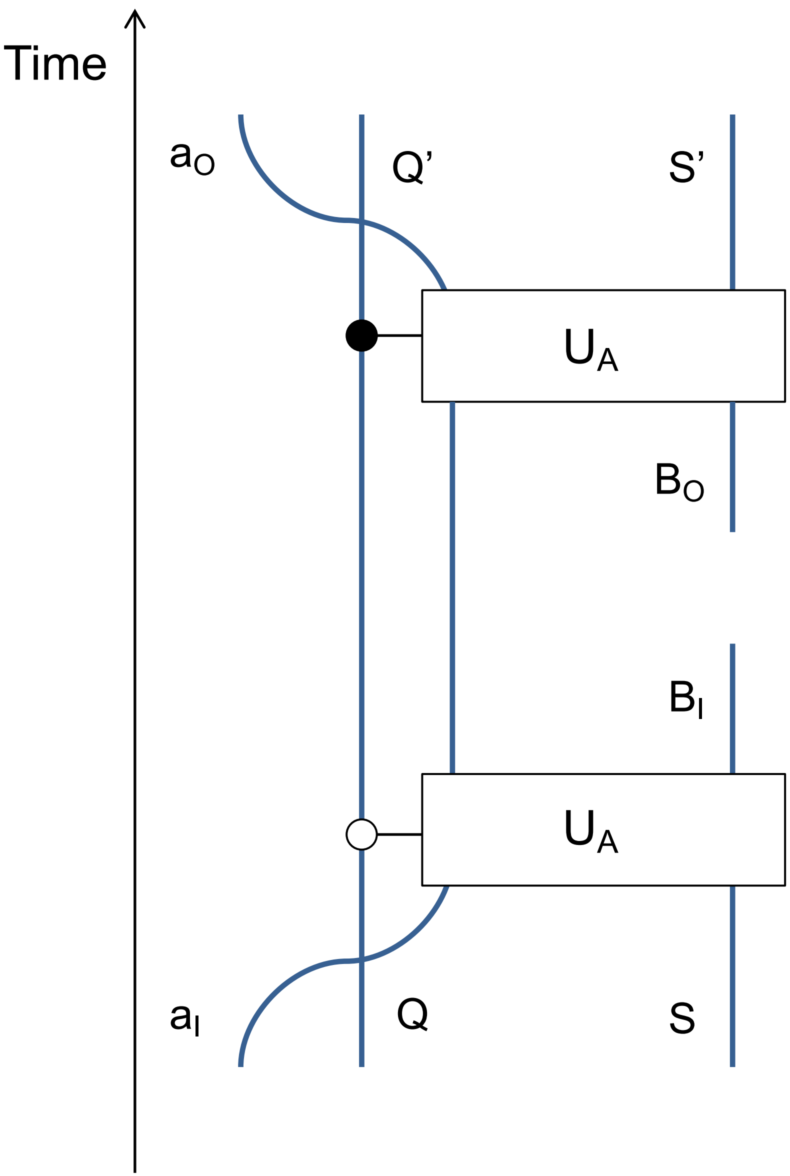
O. O., arXiv:1801.07594

Identifying Alice’s operation



O. O., arXiv:1801.07594

Identifying Alice's operation



Claim:

$$U^{a_I QS B_O \rightarrow a_O Q' S' B_I} = U_A^{a_I A_I \rightarrow a_O A_O} \otimes \mathbb{1}^{\overline{A_I} \rightarrow \overline{A_O}}$$

where A_I is a nontrivial *subsystem* of QSB_O, defined by the algebra of operators

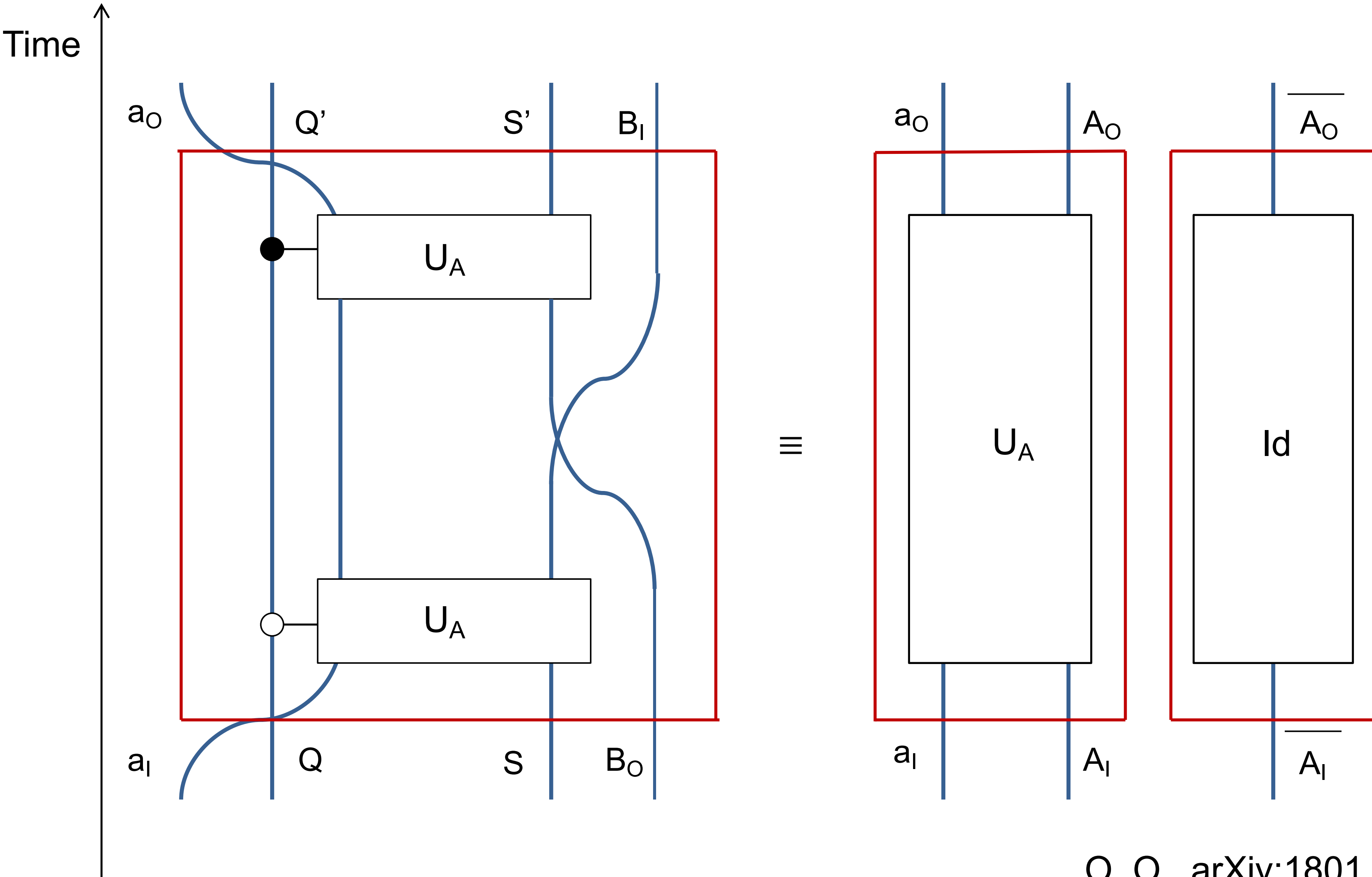
$$O^{A_I} \equiv |0\rangle\langle 0|^Q \otimes O^S \otimes \mathbb{1}^{B_O} + |1\rangle\langle 1|^Q \otimes \mathbb{1}^S \otimes O^{B_O},$$

and A_O is a nontrivial *subsystem* of $Q'S'B_I$, defined by the algebra of operators

$$O^{A_O} \equiv |0\rangle\langle 0|^{Q'} \otimes \mathbb{1}^{S'} \otimes O^{B_I} + |1\rangle\langle 1|^{Q'} \otimes O^{S'} \otimes \mathbb{1}^{B_I}.$$

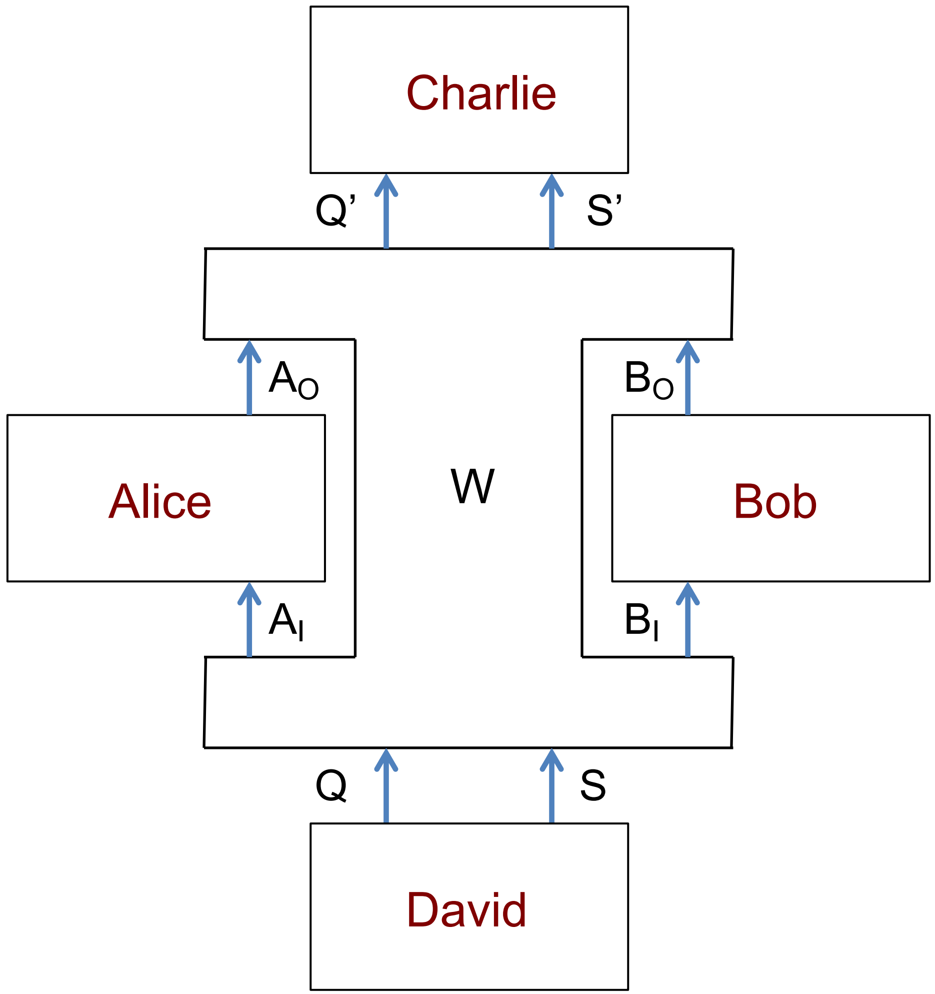
O. O., arXiv:1801.07594

Identifying Alice's operation

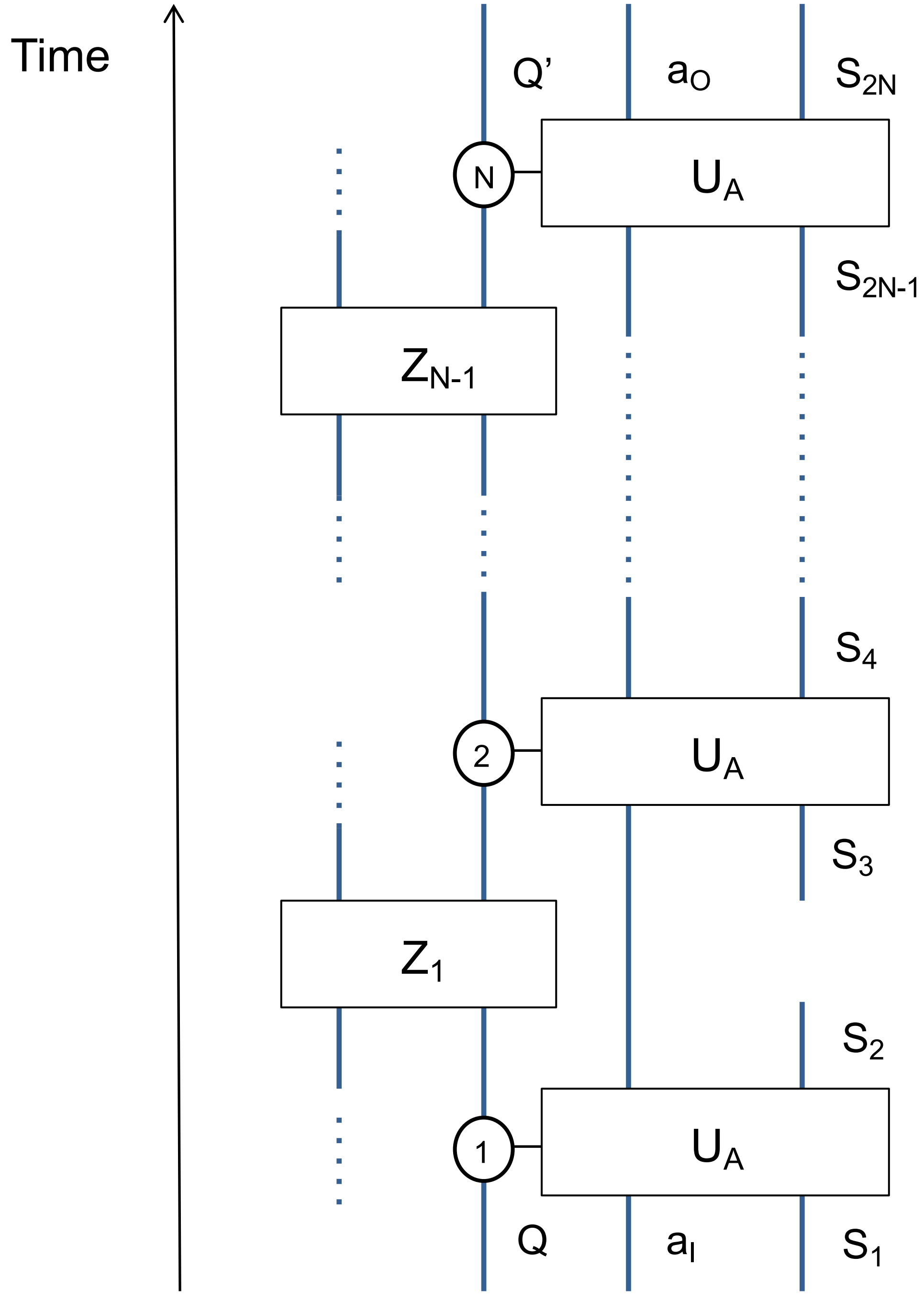


O. O., arXiv:1801.07594

With respect to A_I and A_O, the experiment has the structure of a circuit with a cycle.



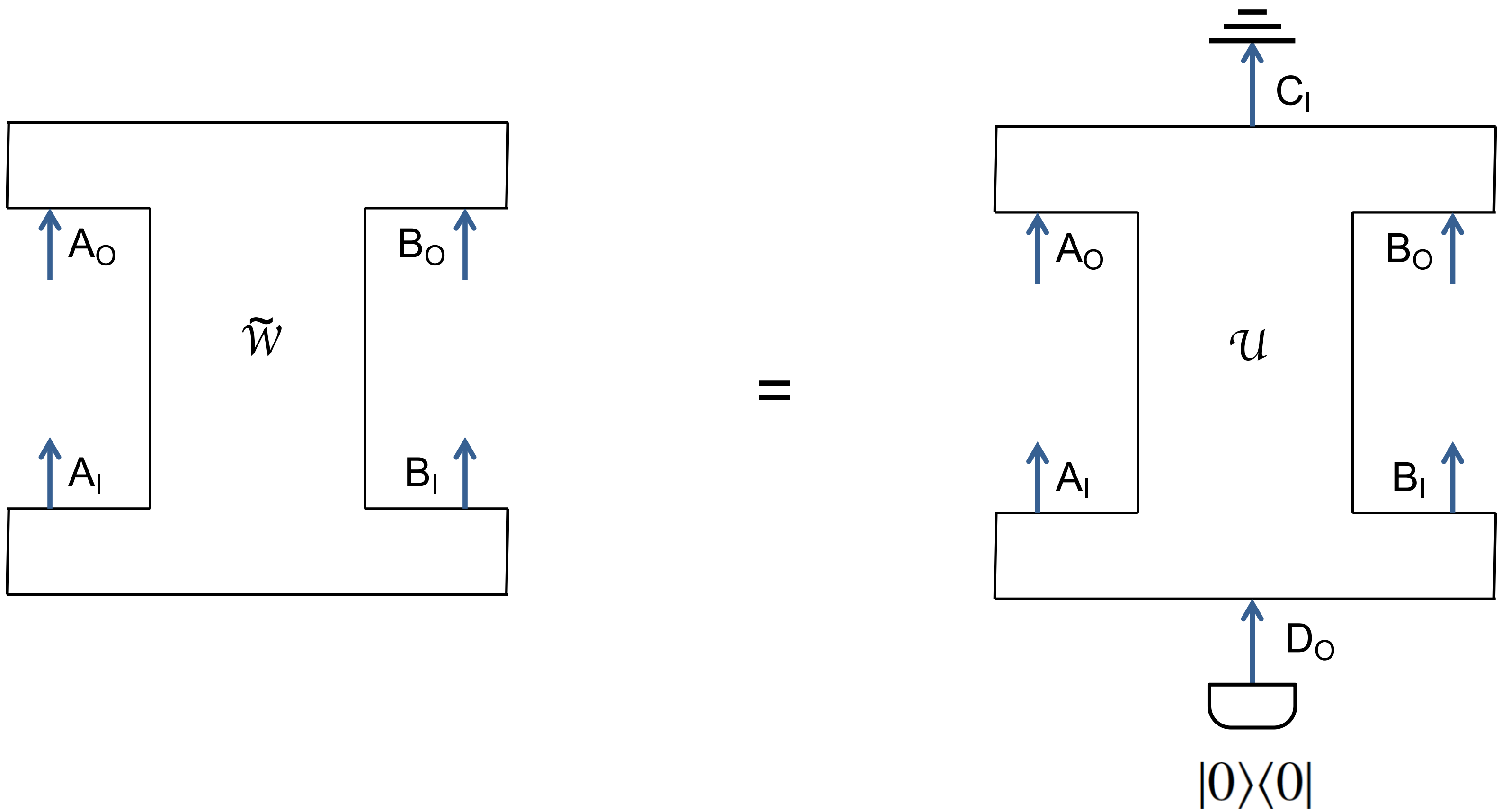
Are there more general processes that exist on time-delocalized systems?



Circuits with coherent control of the times of the operations

(includes the symmetric implementation of the SWITCH)

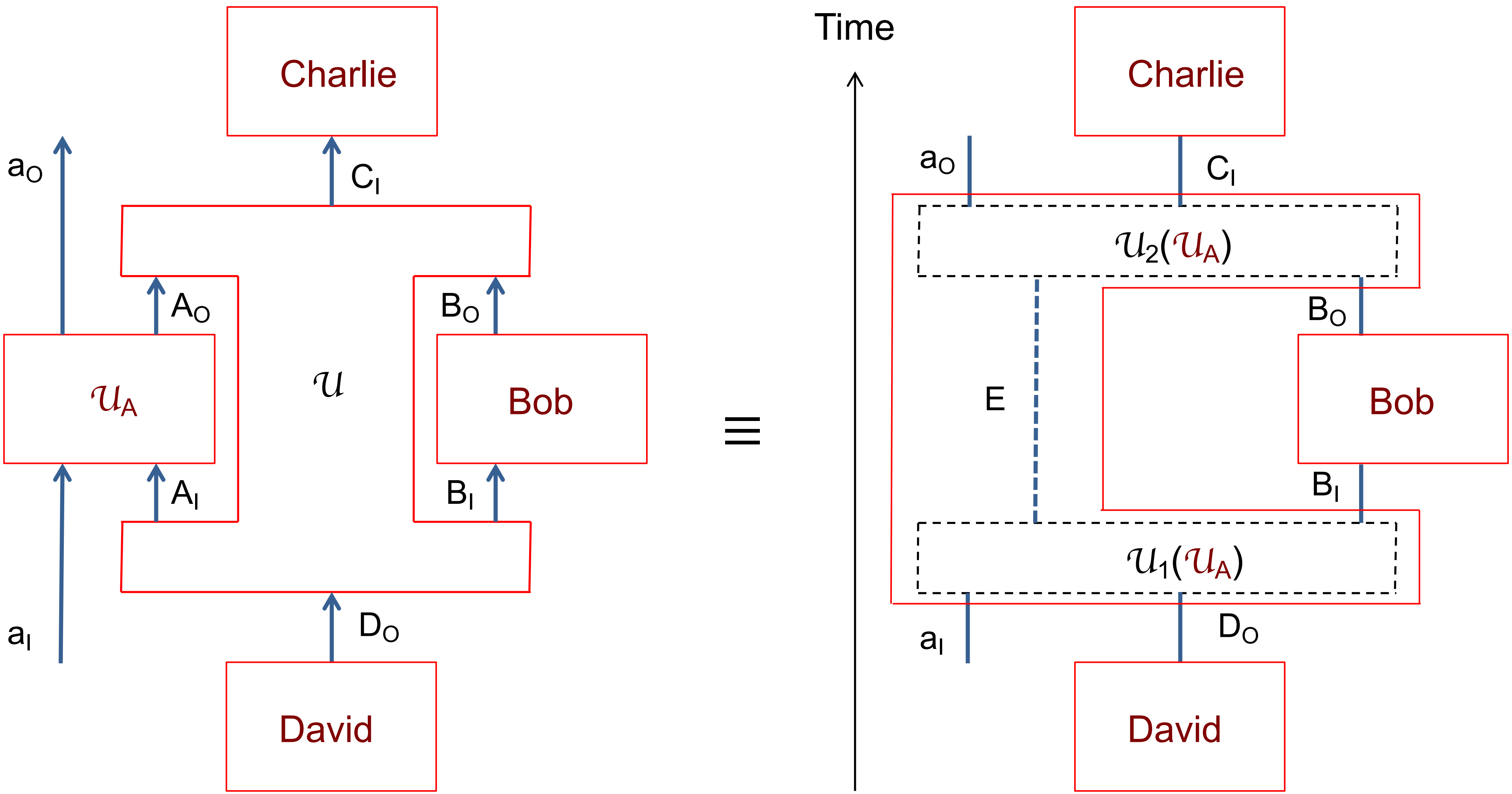
Unitarily extendible processes



Araujo, Feix, Navascues, Brukner, Quantum 1, 10 (2017)

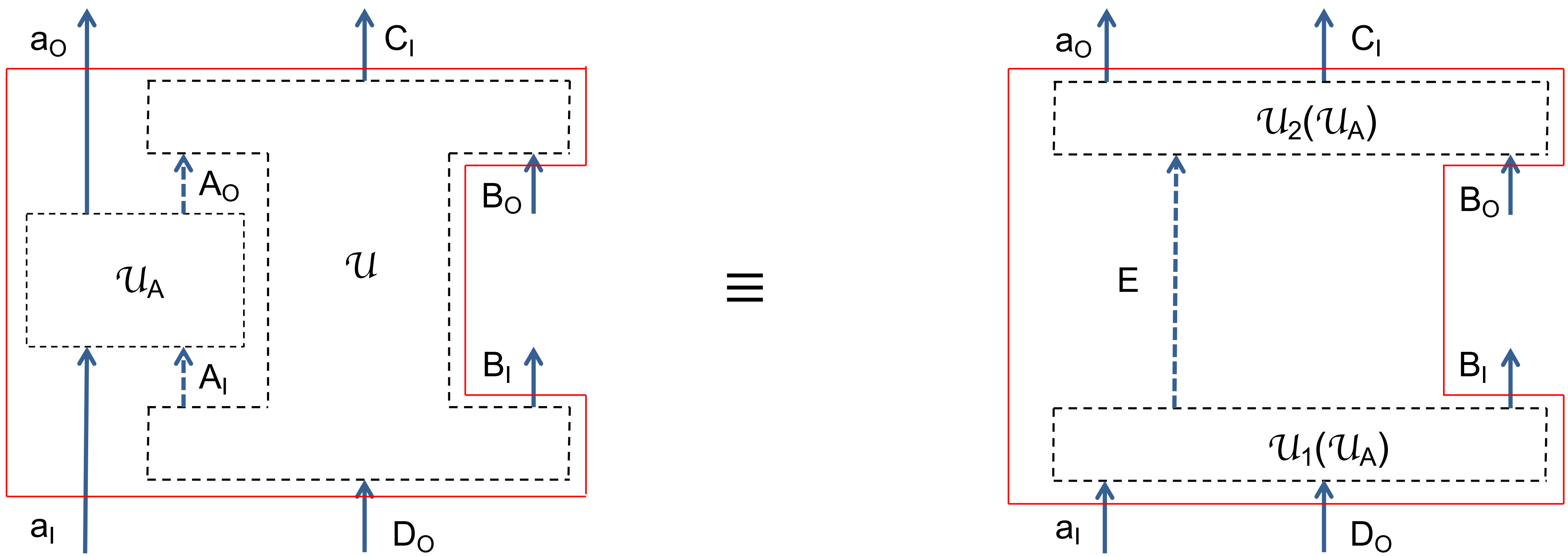
Claim:

All unitary extensions of bipartite processes have realizations on time-delocalized systems



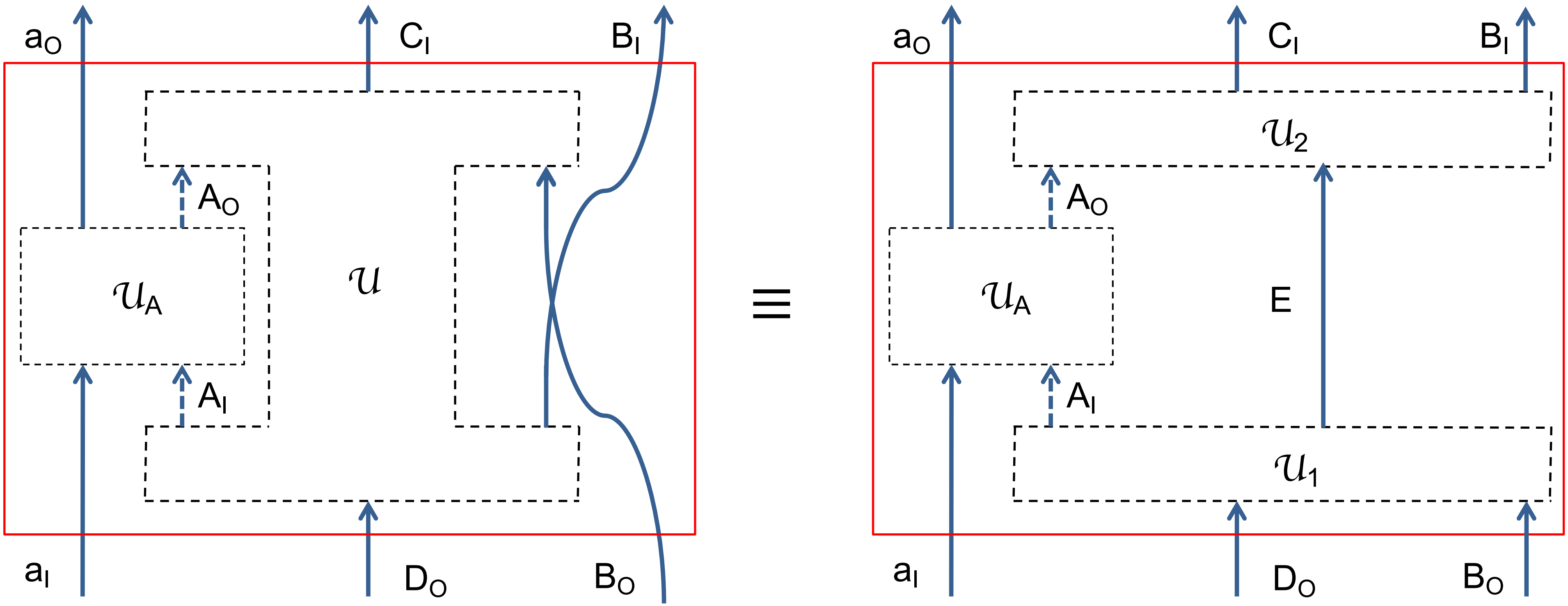
O. O., arXiv:1801.07594

Proof:



Chiribella, D'Ariano, and Perinotti, EPL 83, 30004 (2008)

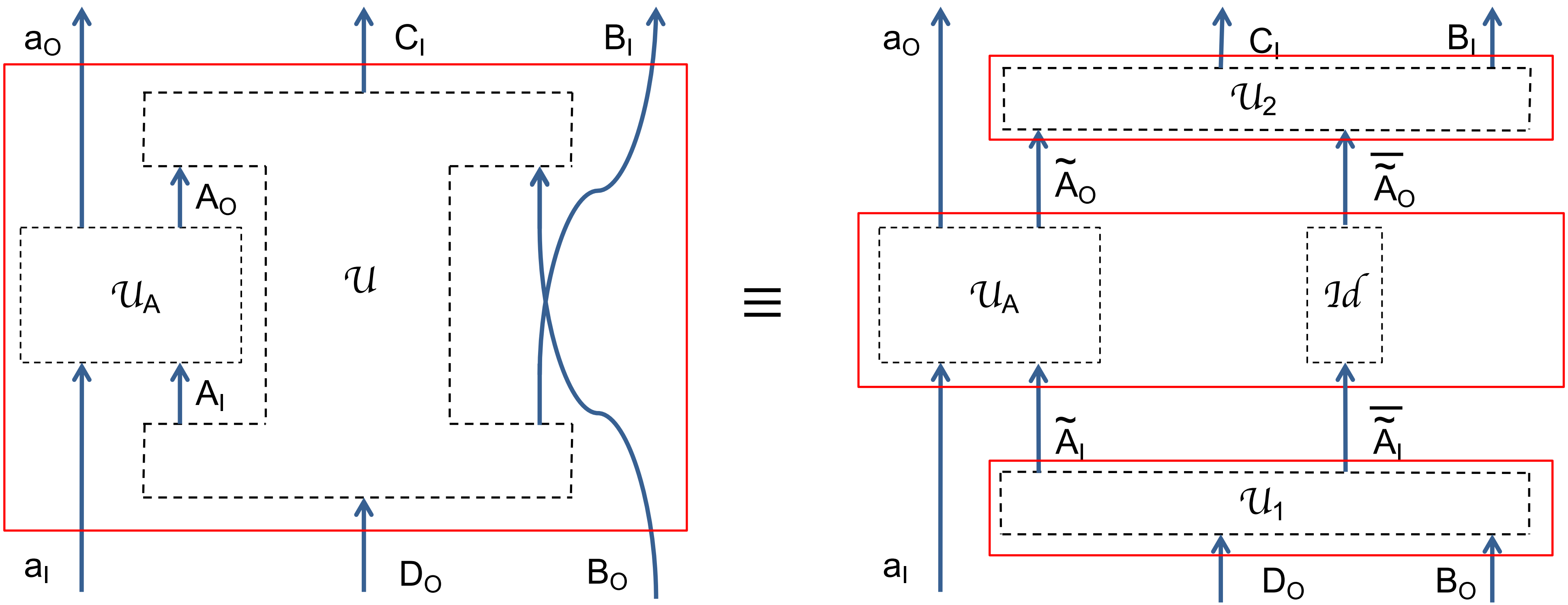
Proof:



Proof:

$\tilde{A}_I$ – a subsystem of $D_O B_O$.

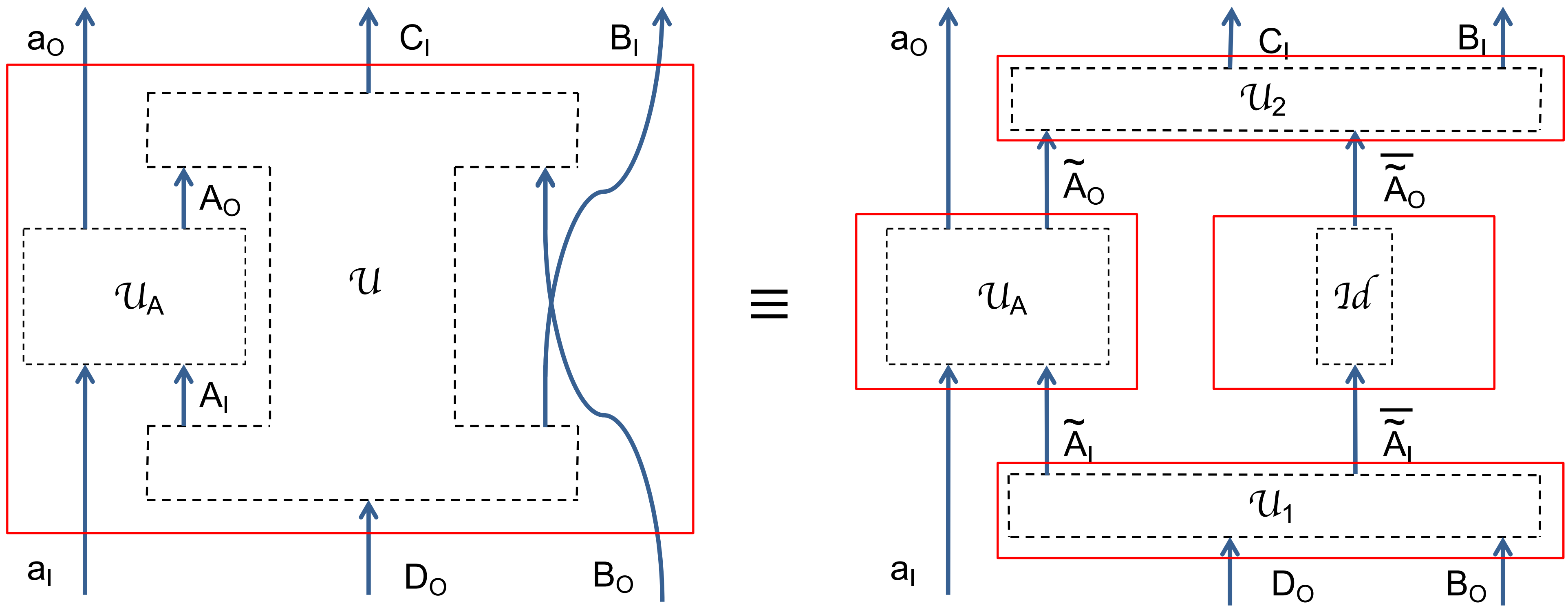
$\tilde{A}_O$ – a subsystem of $C_I B_I$.



Proof:

$\tilde{A}_I$ – a subsystem of $D_O B_O$.

$\tilde{A}_O$ – a subsystem of $C_I B_I$.



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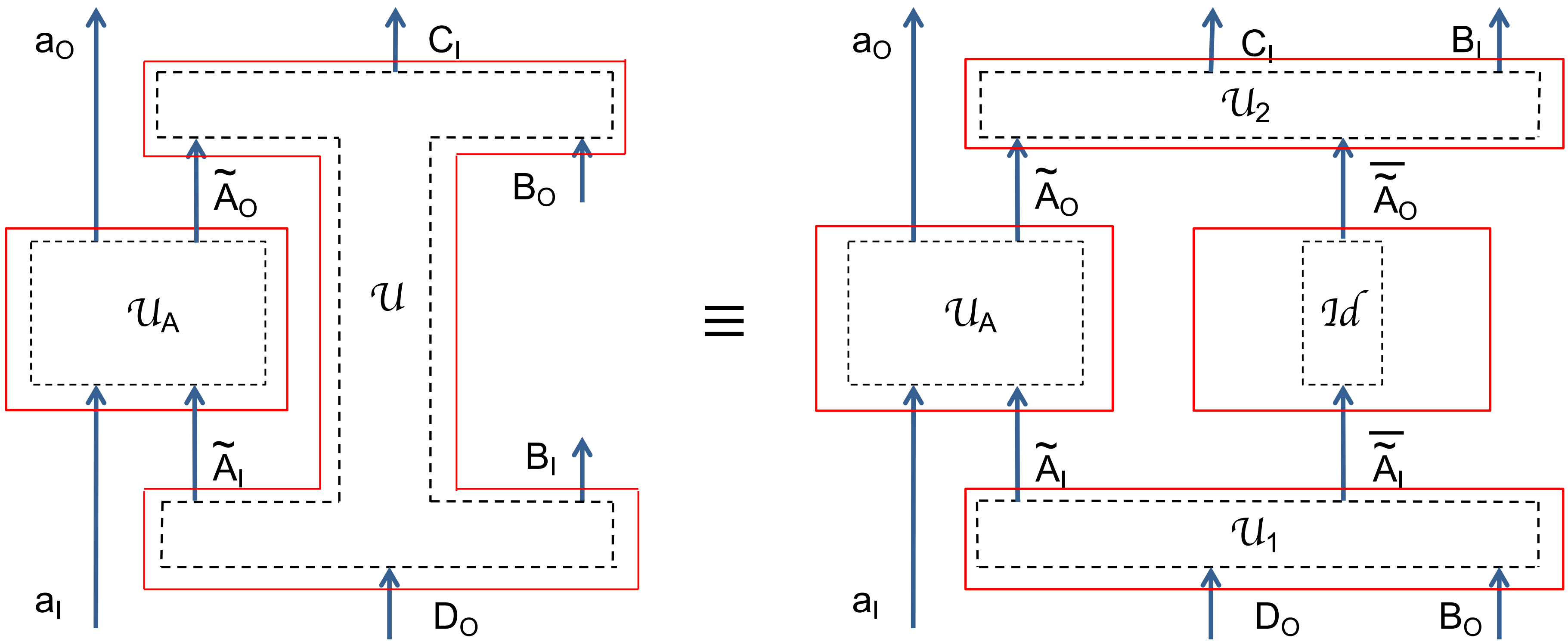

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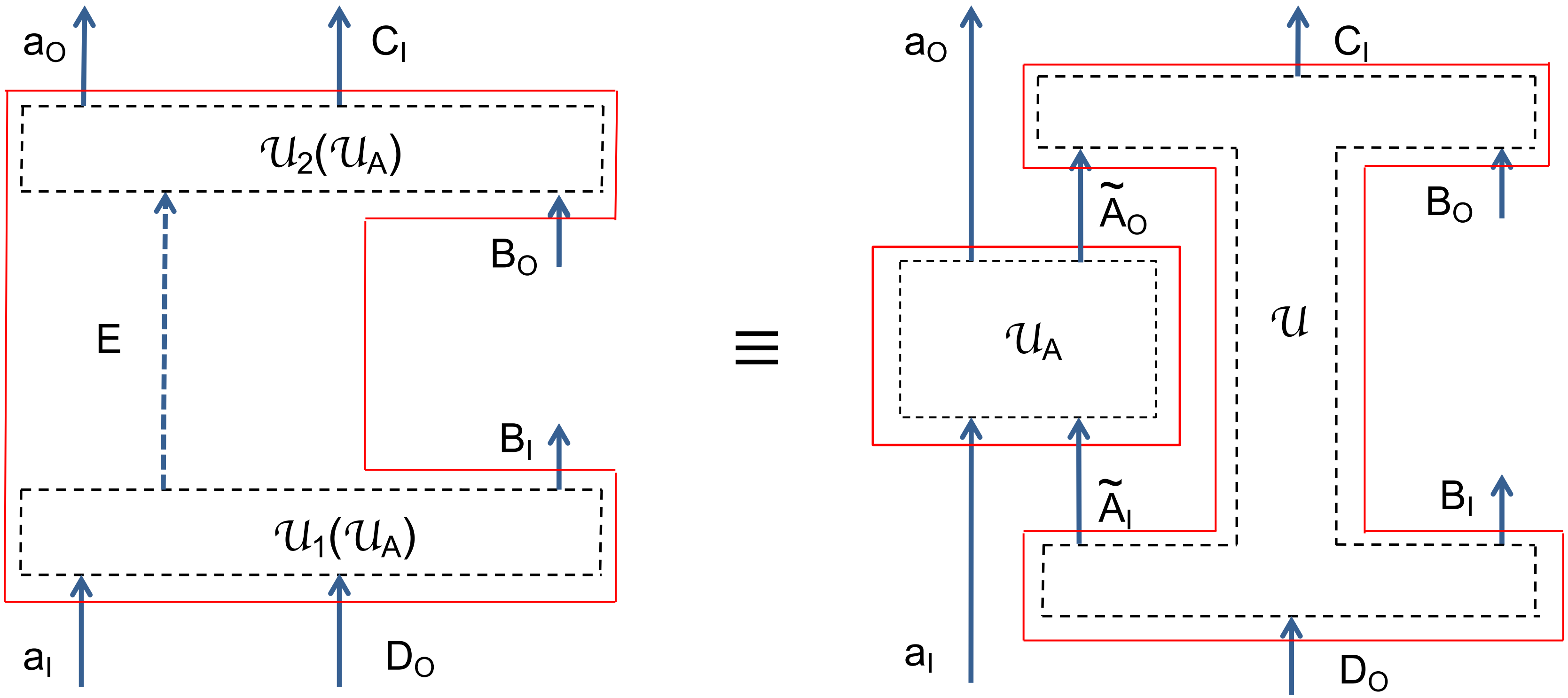
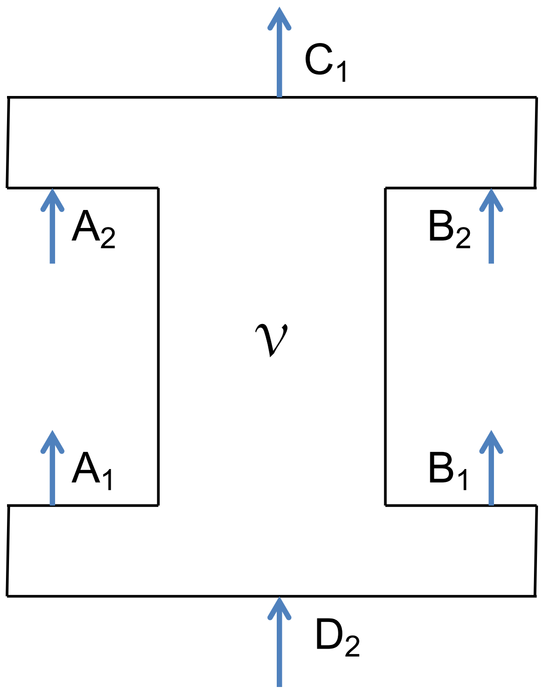


a_O
C_I
$\mathcal{U}_2(\mathcal{U}_A)$
E
B_O
B_I
$\mathcal{U}_1(\mathcal{U}_A)$
a_I
D_O
$\equiv$
a_O
C_I
$\tilde{A}_O$
$\mathcal{U}_A$
$\mathcal{U}$
B_O
$\tilde{A}_I$
B_I
a_I
D_O

Similarly can prove realizability of the following class:



$\mathcal{V}$ is an isometric channel that maps a tensor factor of D_2B_2 onto A_1.

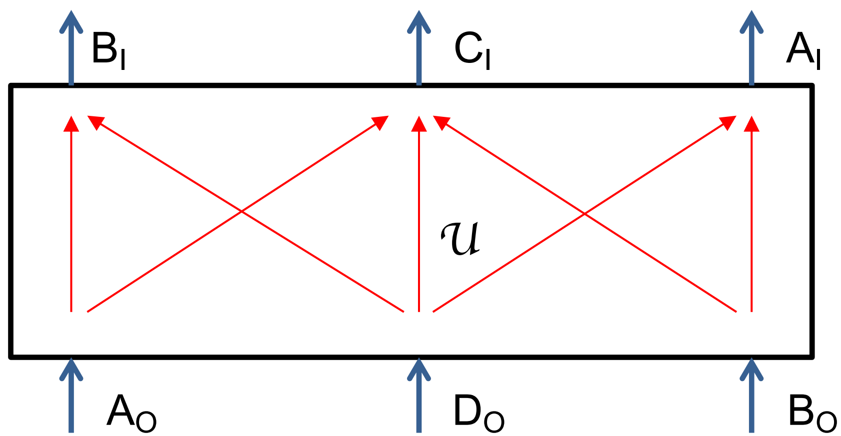
Theorem: all bipartite processes that obey the unitary extension postulate are causally separable.

Hence, the unitary extensions are just variations of the quantum SWITCH.

With R. Lorenz and J. Barrett, *Cyclic Quantum Causal Models*, in preparation.

Idea behind proof

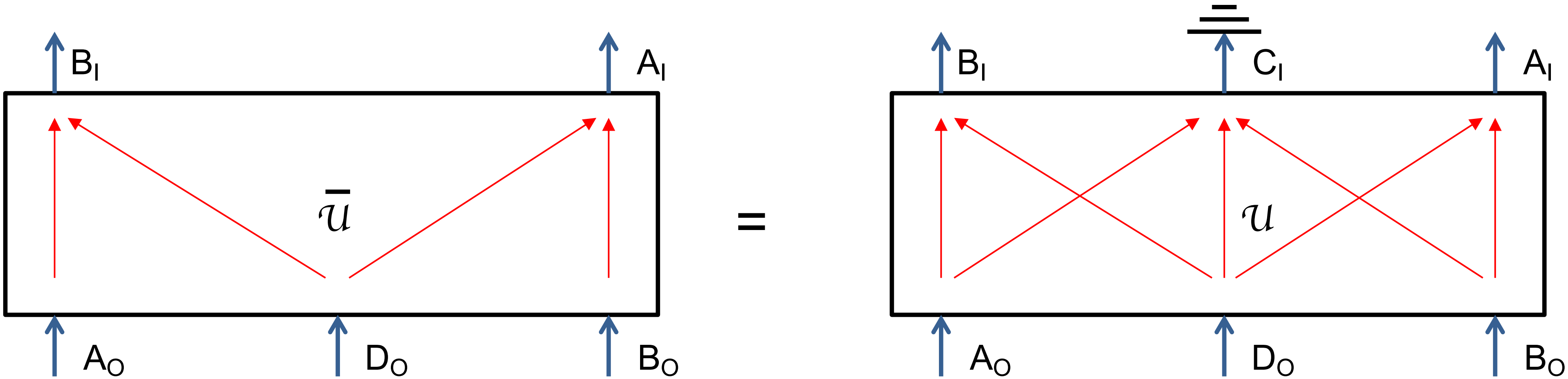
We have the following no-signaling constraints for the unitary channel of the 4-partite process:



(lack of arrow means no signaling)

This follows from the types of terms permitted in a process matrix.

Consider the reduced process obtained by tracing our Charlie:



Let $\overline{U}$ denote the process matrix (the transposed Choi state of the channel $\overline{\mathcal{U}}$).

Then, there exists a decomposition $$\mathcal{H}^{D_O} = \bigoplus_i \mathcal{H}^{L_i} \otimes \mathcal{H}^{R_i}$$

such that $$\overline{U}^{A_I A_O B_I B_O D_O} = \bigoplus_i \rho^{A_O B_I L_i} \otimes \rho^{A_I B_O R_i}$$

[J.-M. Allen et al., PRX 7, 031021 (2017); J. Barrett, R. Lorenz, O. O., in preparation]

Bad news?

Not at all.

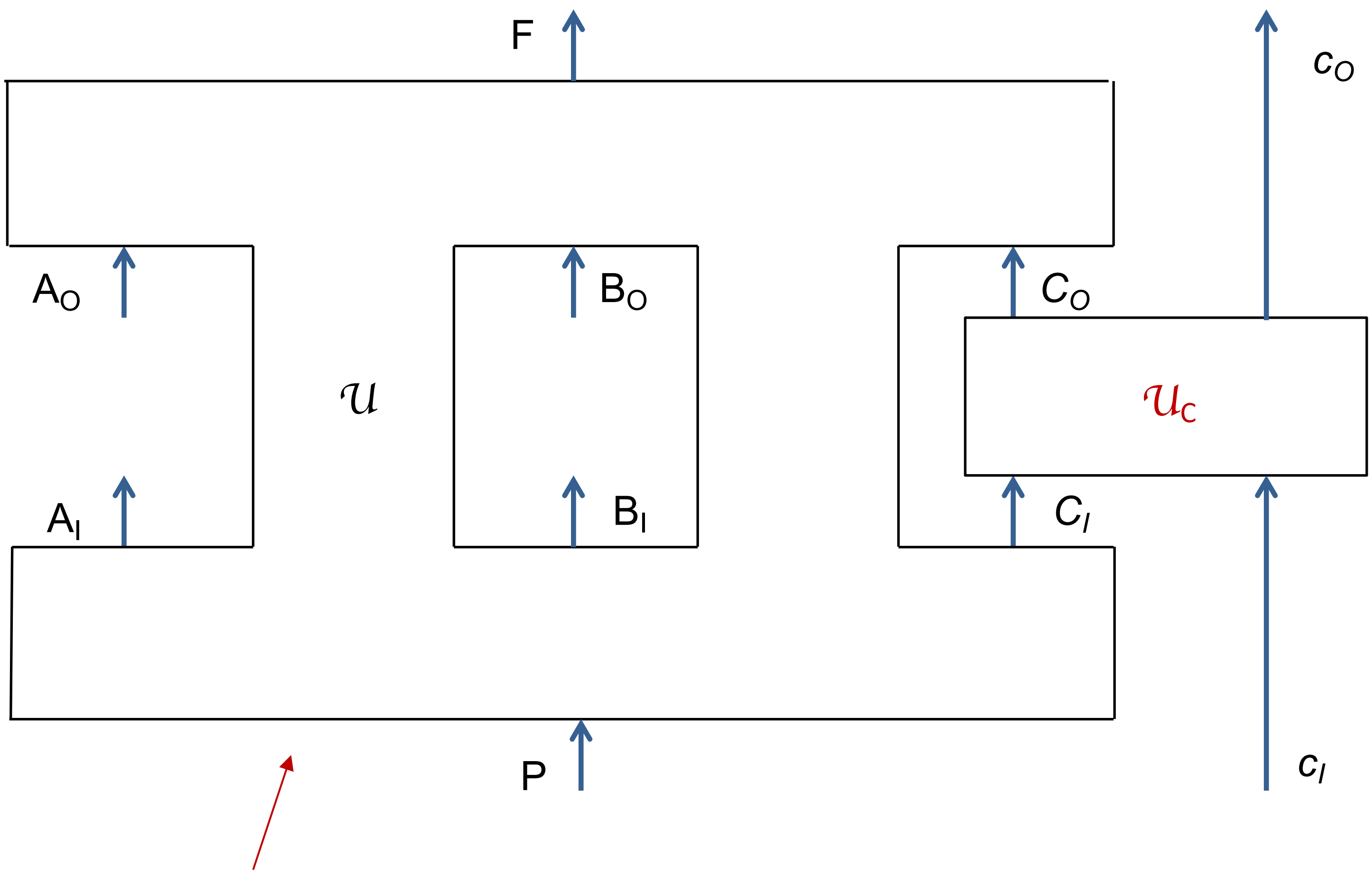
Theorem! All tripartite unitarily extendible tripartite processes, including their unitary extensions, have realizations on time-delocalized systems.

This includes processes violating causal inequalities!

[E.g., the classical noncausal process by Baumeler and Wolf, NJP (2016)]

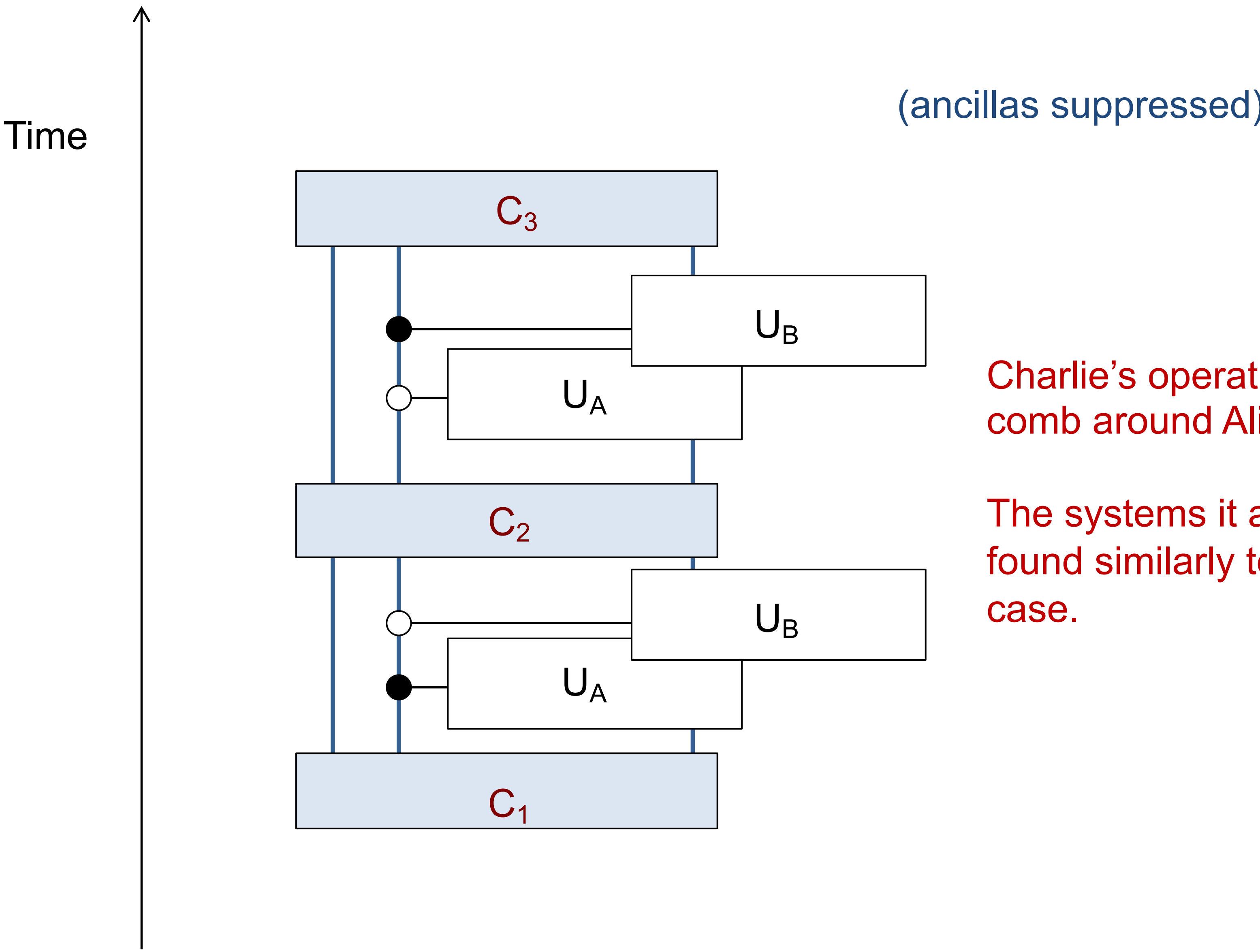
With J. Wechs, C. Branciard, … , in preparation.

Idea of proof



variation of the quantum SWITCH on Alice and Bob

There is a ***universal*** way of implementing all such processes, such that Alice and Bob act on **fixed** time-delocalized systems:



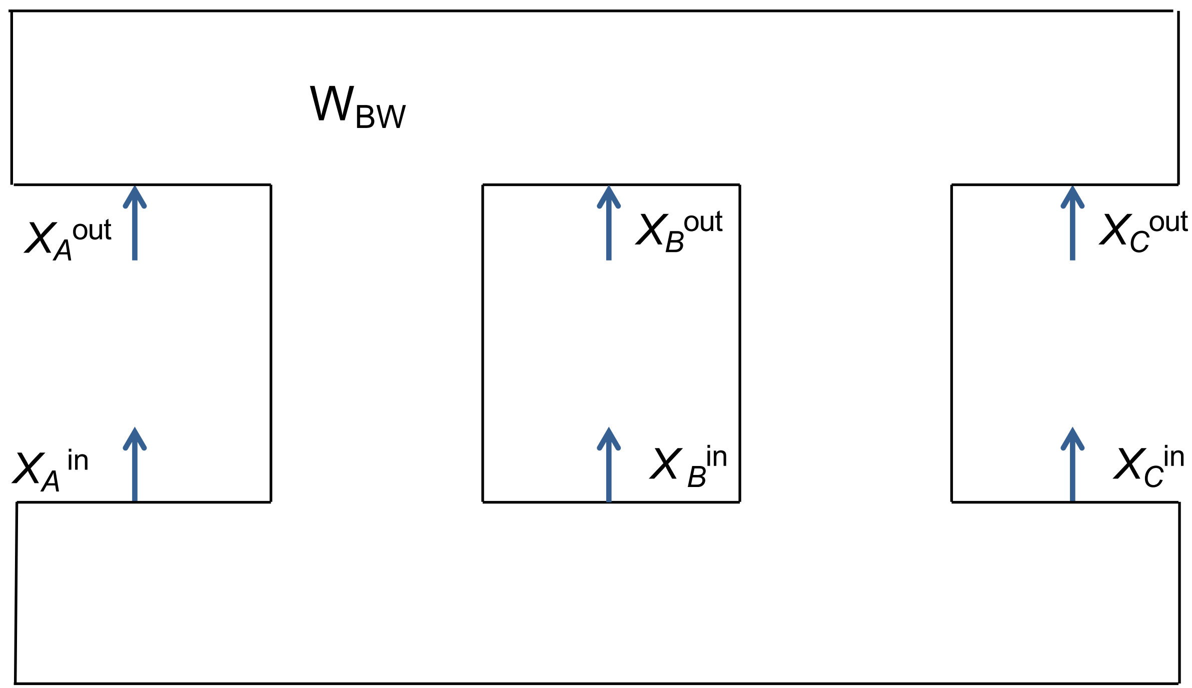
Charlie's operation is within the comb around Alice and Bob.

The systems it acts on can be found similarly to the bipartite case.

Example: Bameler and Wolf, NJP 18, 013036 (2016)

$$X_A^{in} = \neg X_B^{out} \wedge X_C^{out}$$
$$X_B^{in} = \neg X_C^{out} \wedge X_A^{out}$$
$$X_C^{in} = \neg X_A^{out} \wedge X_B^{out}$$

Violates causal inequalities.



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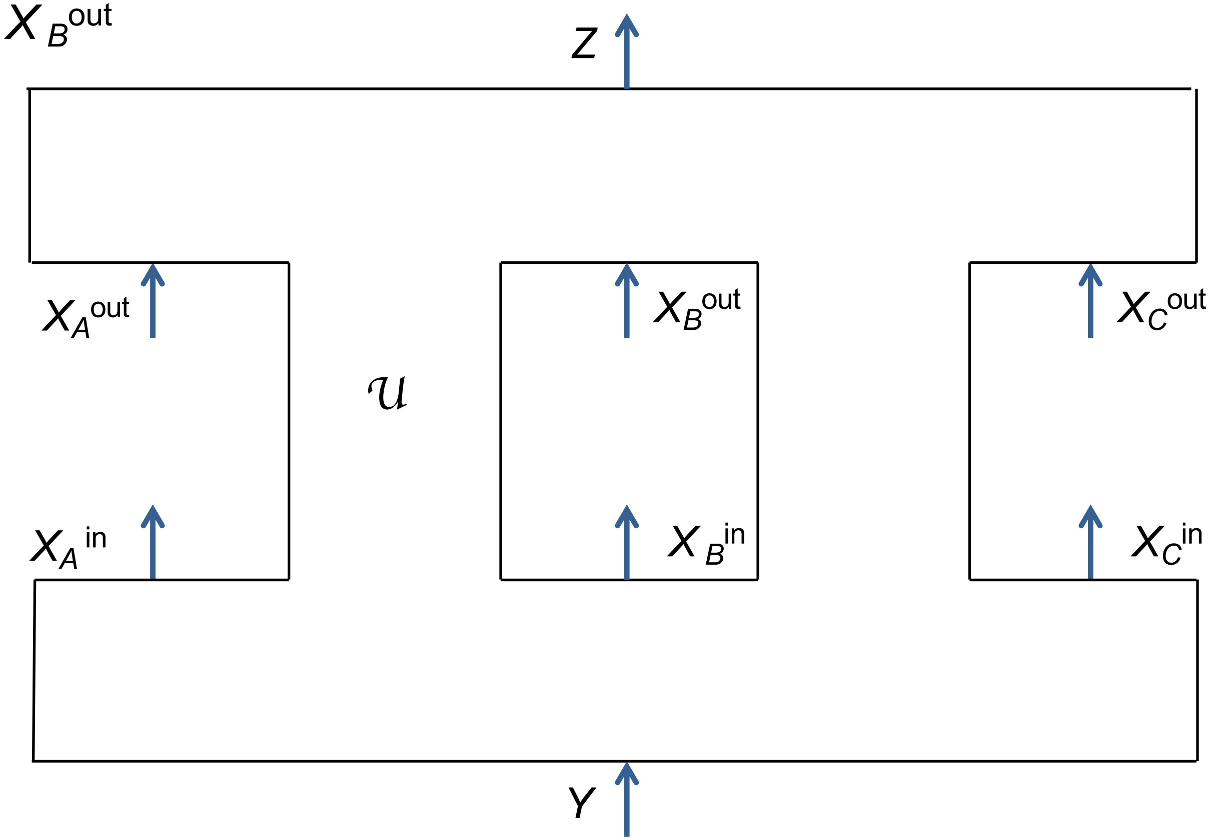
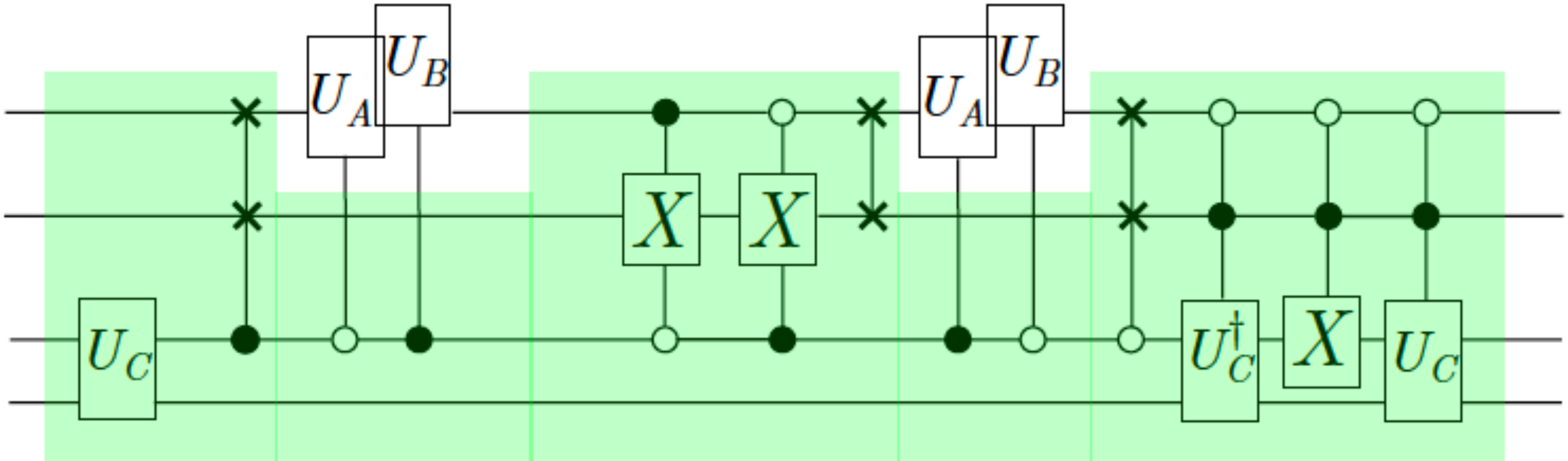
Violates causal inequalities.



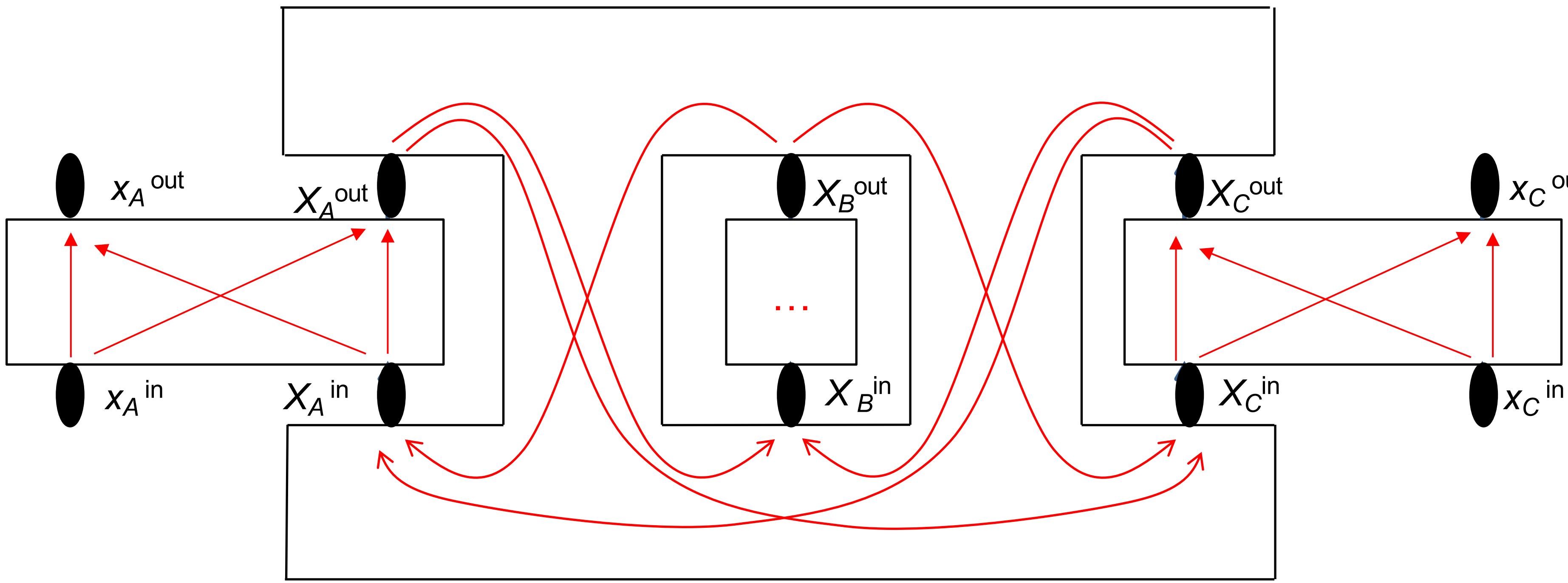
Admits unitary extension [Araujo et al., Quantum (2017)]

From Julian Wechs:



Closely linked to a circuit previously found by Allard Guérin and Brukner, NJP (2018)

There are (time-nonlocal) classical variables that violate causal inequalities.



1) They form a cyclic causal model as above, which can be tested.

2) Under the assumption of closed laboratories (no extra arrows apart from those shown), the correlations cannot be explained by dynamical causal order.

You can do it on your desk!