



# Quantum non-locality in Networks

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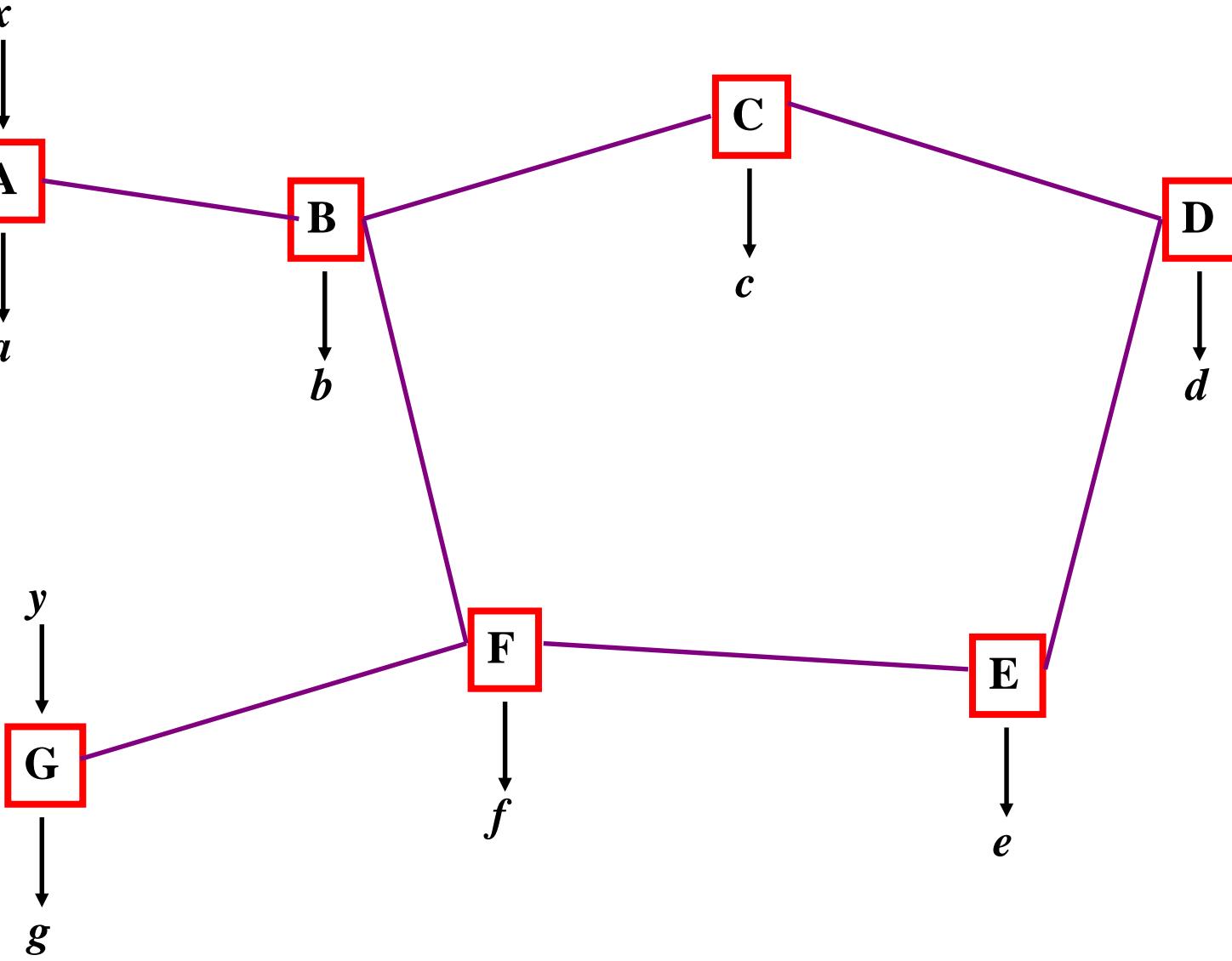
How to test quantumness in networks ?

1. For 2-party scenarios it doesn't matter whether the shared randomness is God given or carried by particles from common sources.
2. In realistic networks there are many independent sources.
3. The “God-given” view is the common view, but not the realistic view.





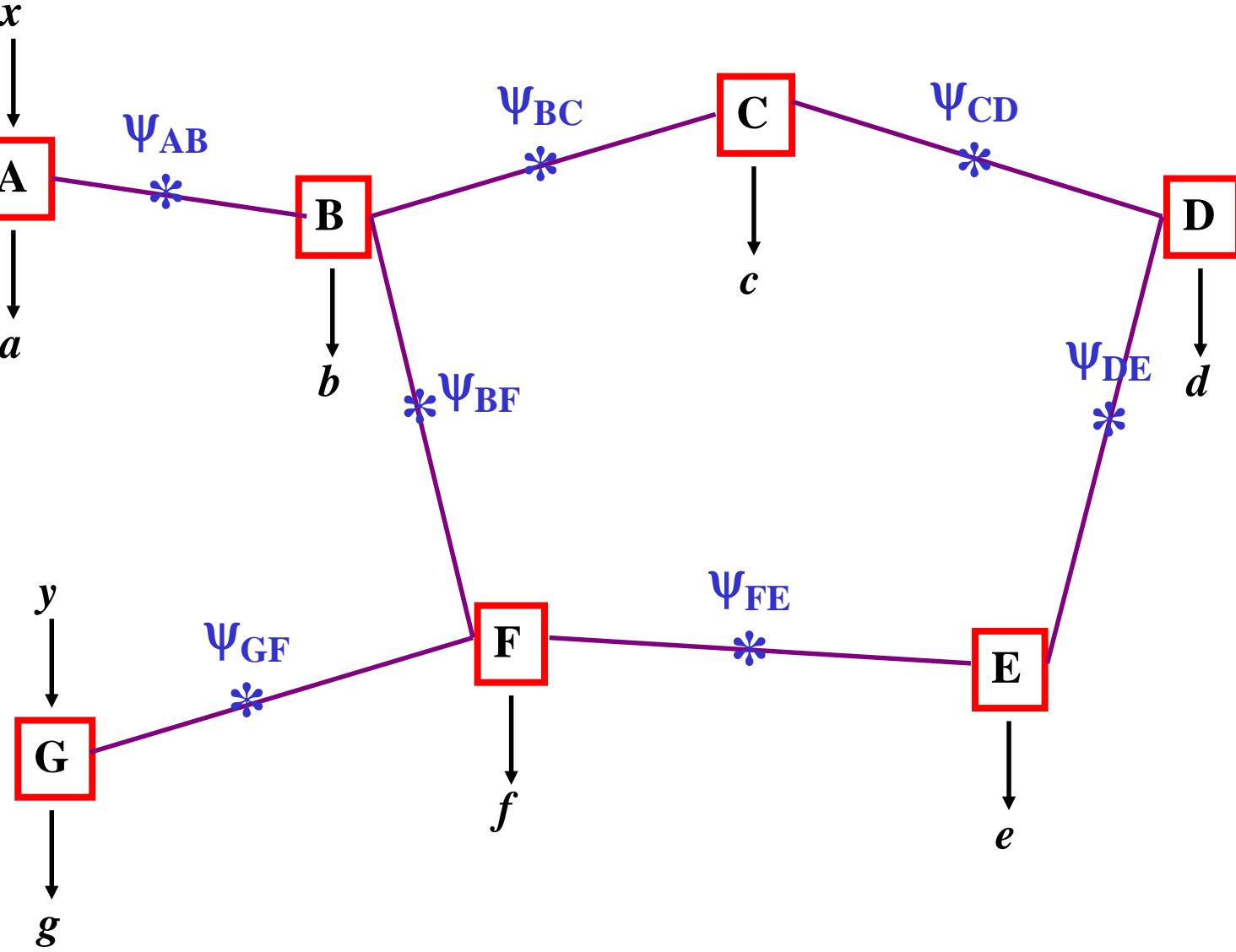
# Example of a Networks





# Quantum Networks

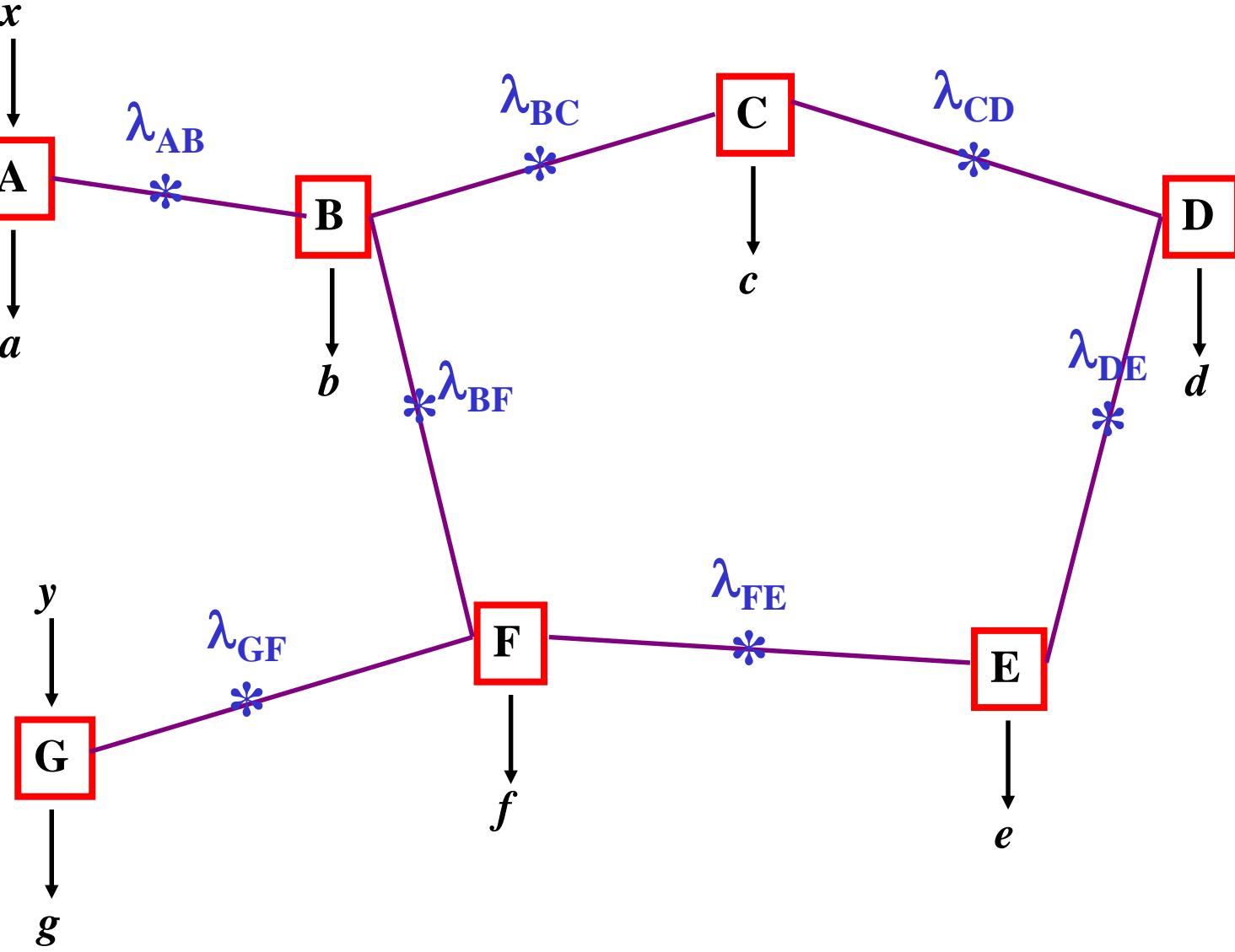
With independent quantum states  $\Psi_{ij}$





# Classical Networks

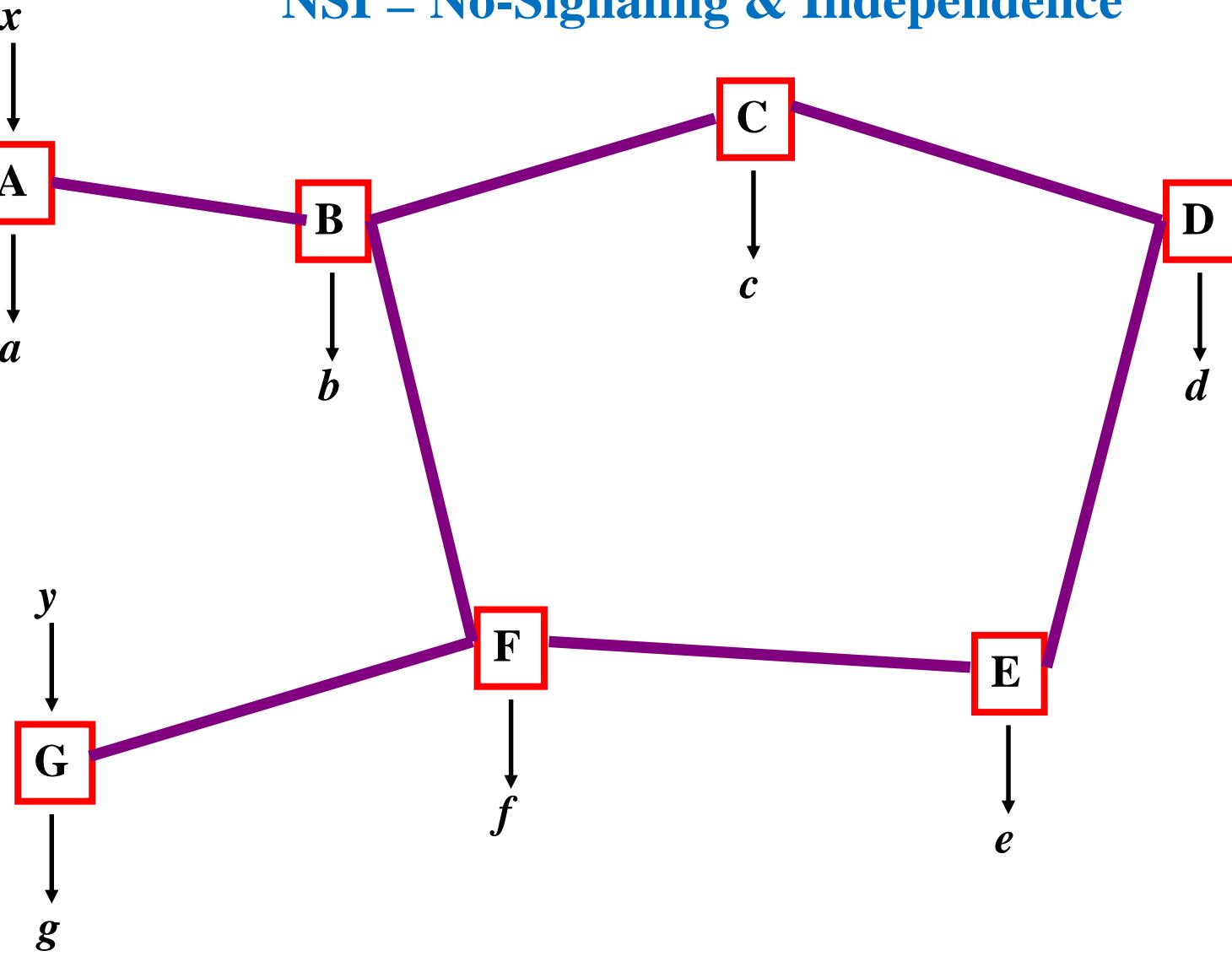
With independent shared randomness  $\lambda_{ij}$





# Non-Local Networks

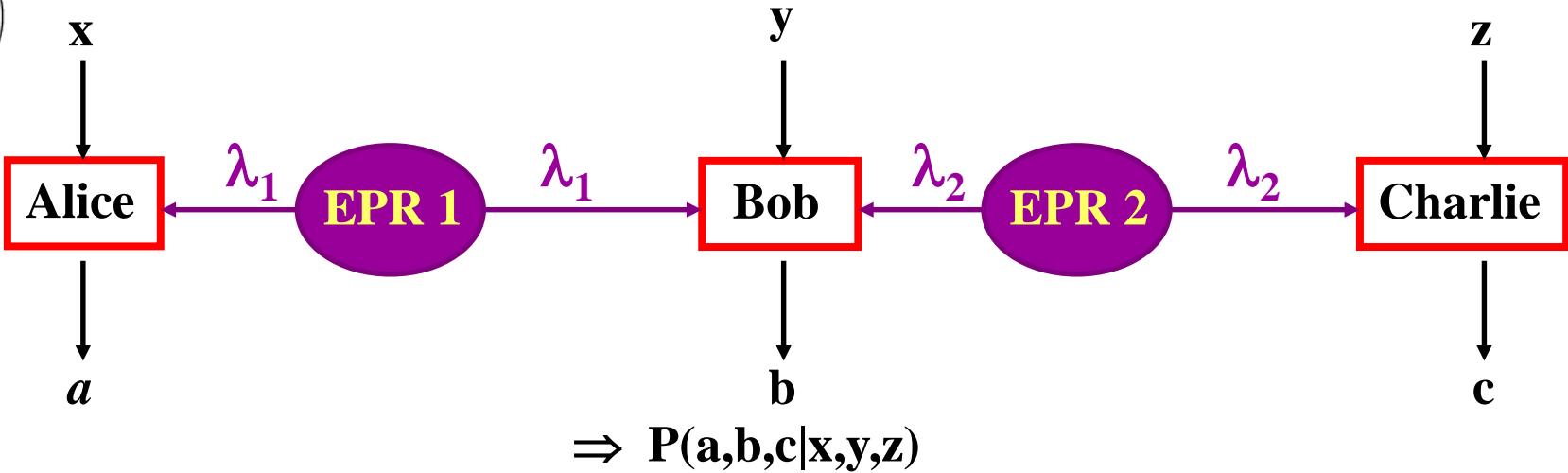
With non-local boxes satisfying the NSI principle:  
NSI = No-Signaling & Independence





# Bi-locality

Branciard, NG, Pironio  
PRL 104, 170401-1/4, 2010



Independent locality (bi-locality) :  $P(\lambda_1, \lambda_2) = P(\lambda_1) \cdot P(\lambda_2)$

$$P_{\text{biloc.}}(a,b,c|x,y,z,\lambda_1,\lambda_2) = p(a|x, \lambda_1) \cdot p(b|y, \lambda_1, \lambda_2) \cdot p(c|z, \lambda_2)$$

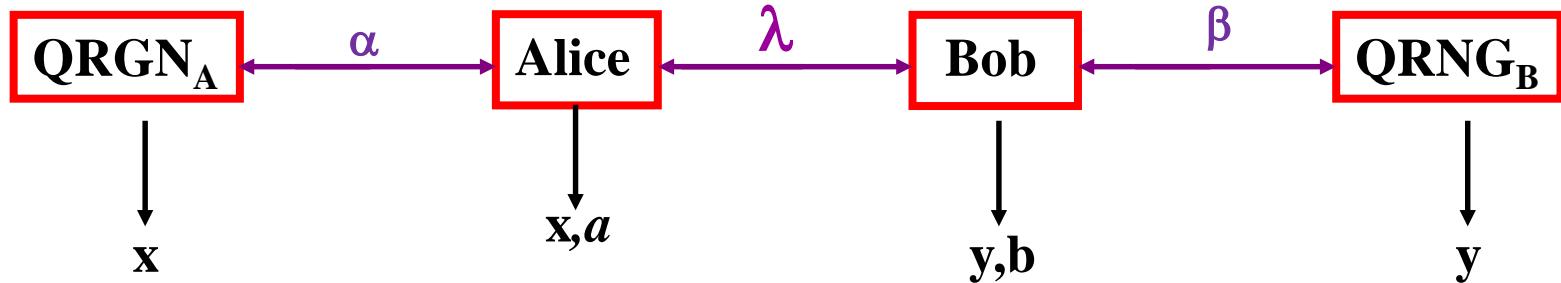
$\Rightarrow$  ??? Bell-like inequalities ???

The **NSI** assumption when the sources EPR1 and EPR2 are independent is as natural as Bell's locality assumption.

It is implicitly assumed in all Bell test with Quantum Random Number Generators (QRNG).



# Bell inequality without inputs



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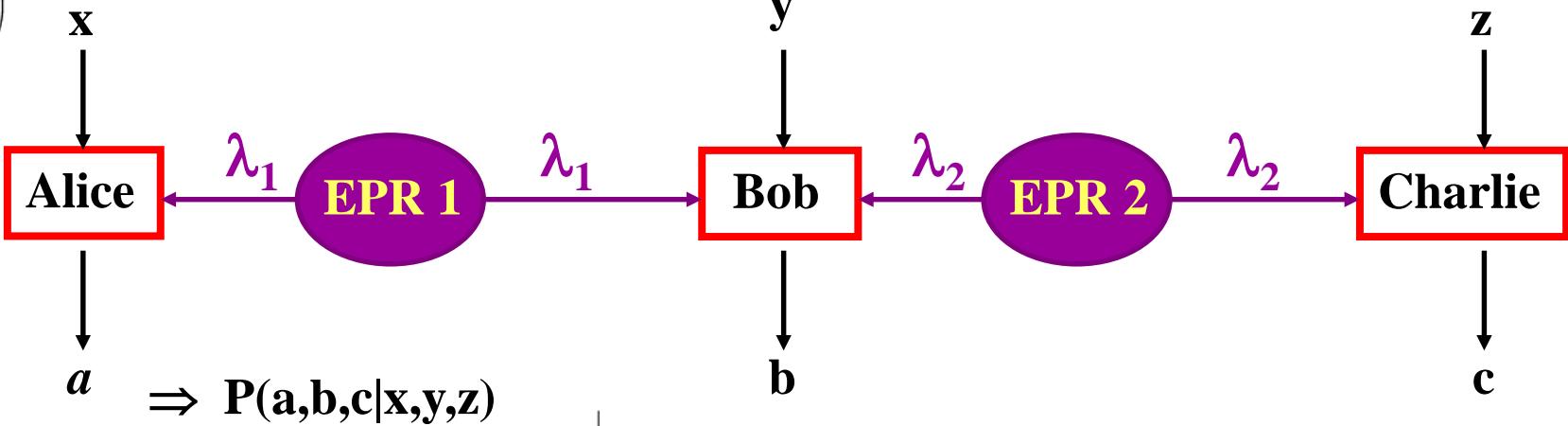
It is implicitly assumed in all Bell test with Quantum Random Number Generators (QRNG).



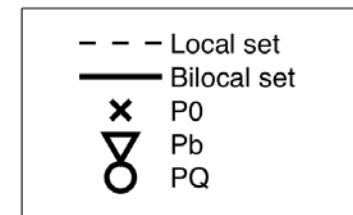
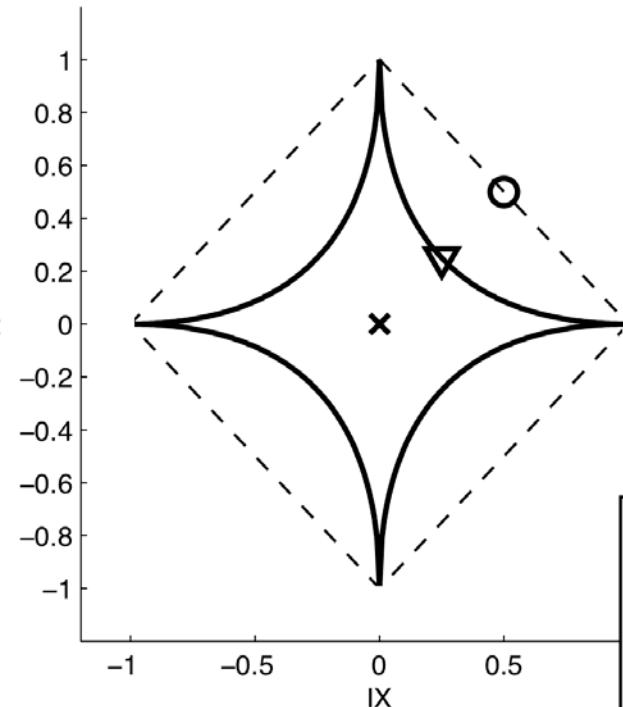
# Bi-locality

$a, b, c, x, y, z$  are all binary

Branciard, NG, Pironio  
PRL 104, 170401-1/4, 2010

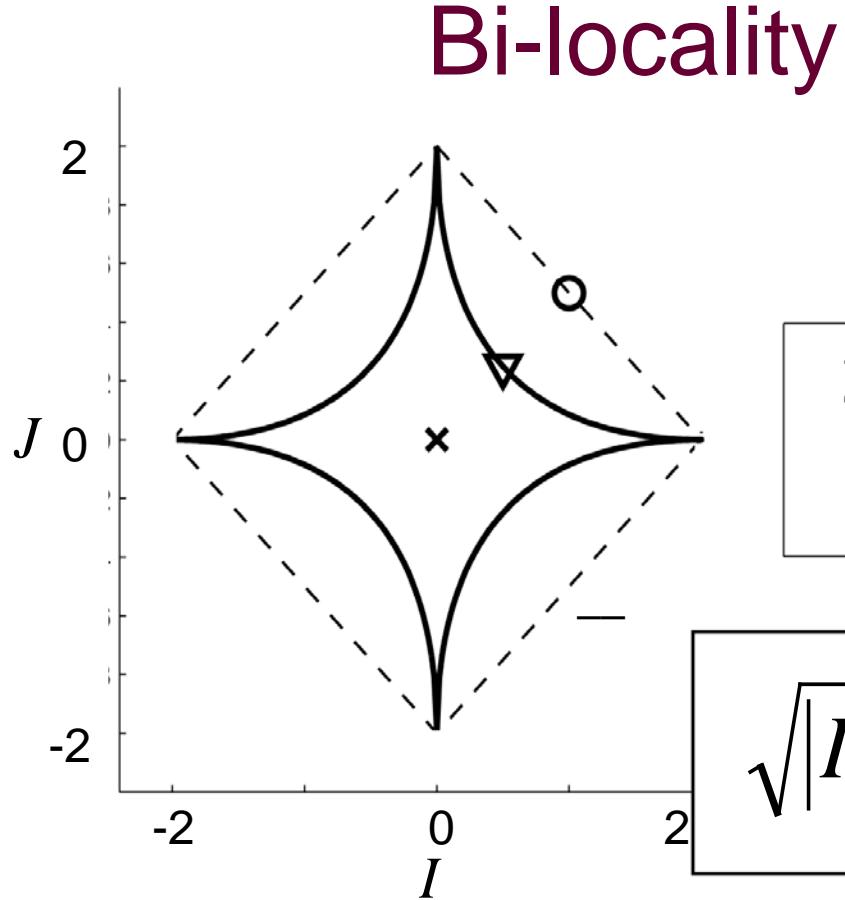


The set of bilocal  
 $p_{\text{biloc}}(a,b,c | x,y,z)$   
is not convex:



**Bilocal set**

$$\sqrt{|I_X|} + \sqrt{|I_Y|} \leqslant 1$$



— Local set  
— Bilocal set  
✖ P0  
▽ Pb  
○ PQ

$$\sqrt{|I|} + \sqrt{|J|} \leq 1$$

Branciard, NG, Pironio  
PRL 104, 170401-1/4, 2010

Rosset +Branciard et al.  
PRL 116, 010403, 2016

$$I = \frac{1}{4} \sum_{x,z} \langle a_x b_0 c_z \rangle$$

$$J = \frac{1}{4} \sum_{x,z} (-1)^{x+z} \langle a_x b_1 c_z \rangle$$

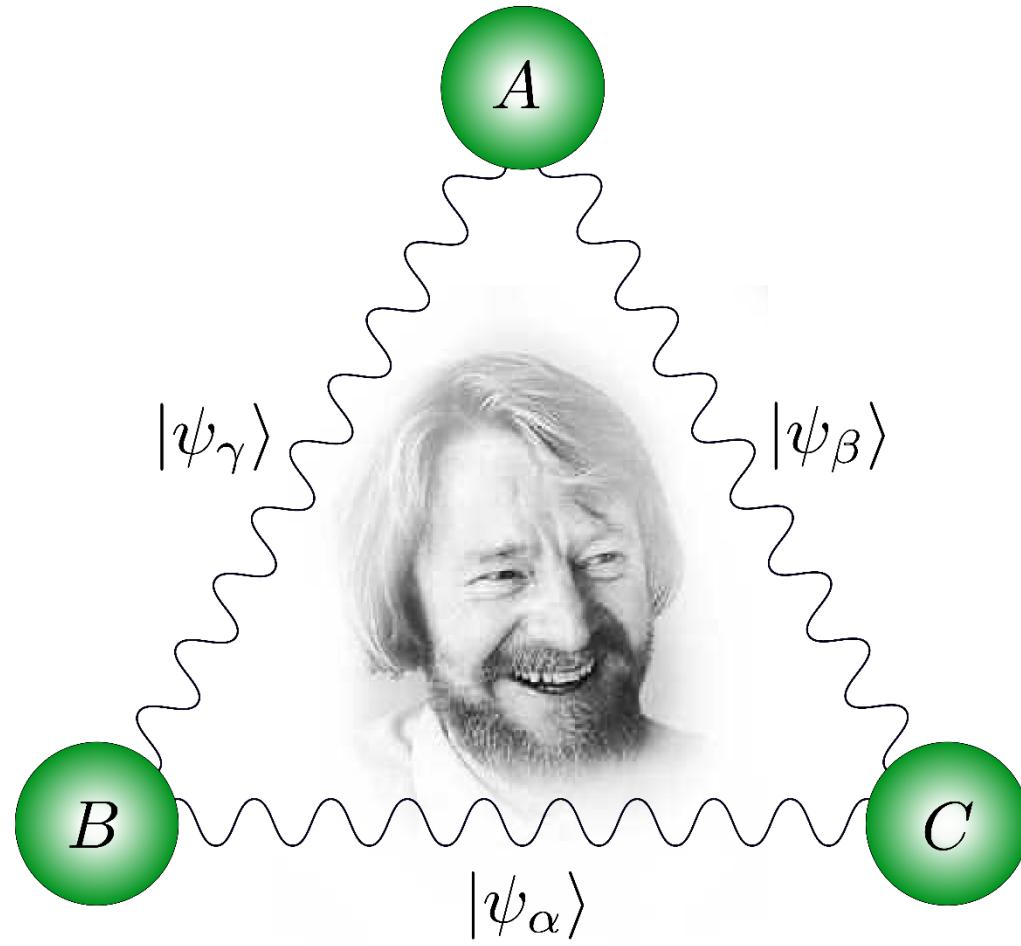
Critical visibility  $V_{\text{th}}=1/2$ , significantly lower than for CHSH, hence it is easier to experimentally demonstrate quantumness.

But the critical visibility per singlet is still  $1/\sqrt{2}$ . Disappointing !

Challenge 1: find a bi-locality scenario with critical visibility per singlet  $< 1/\sqrt{2}$

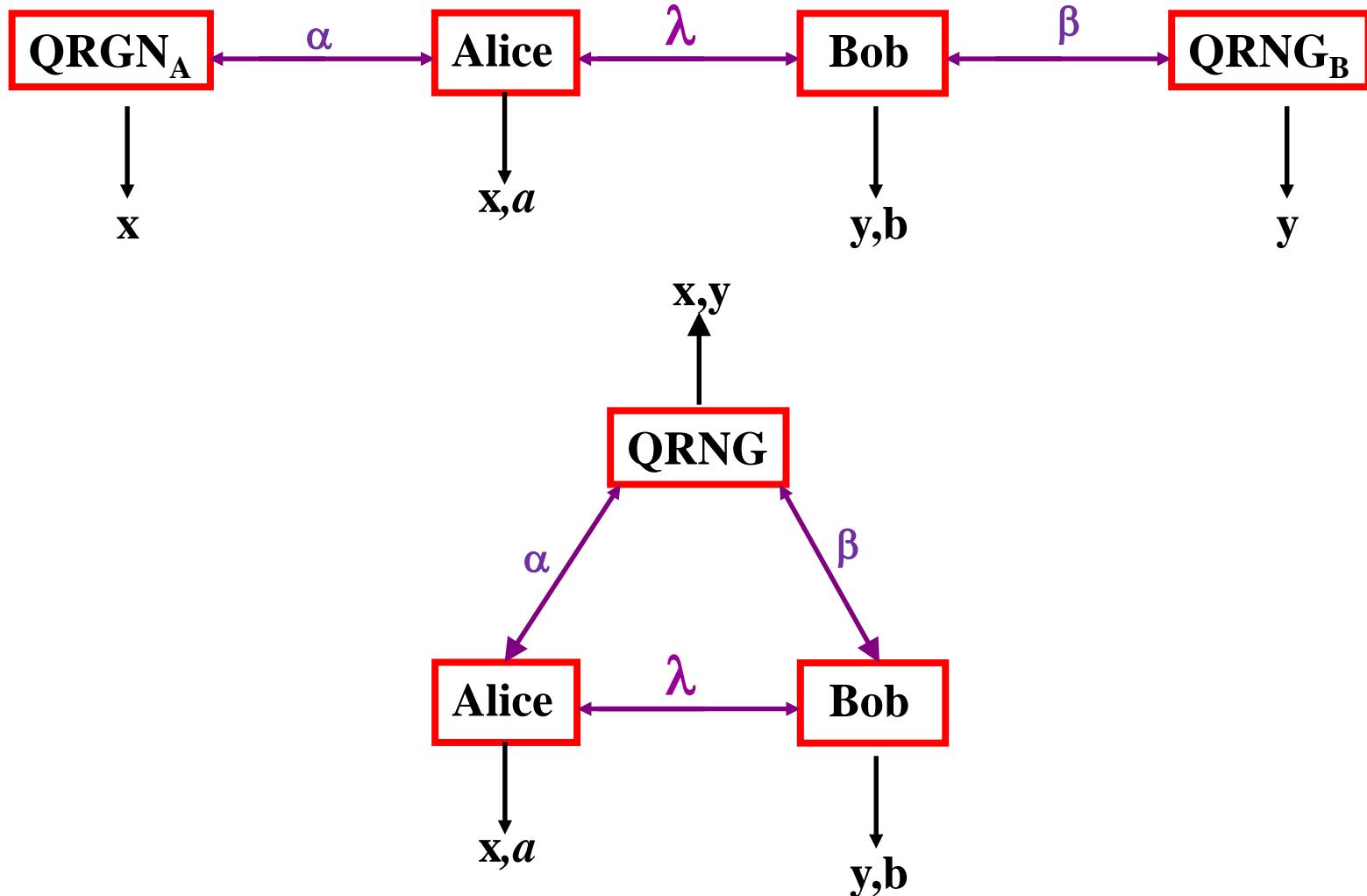


# Quantum Networks with a Loop



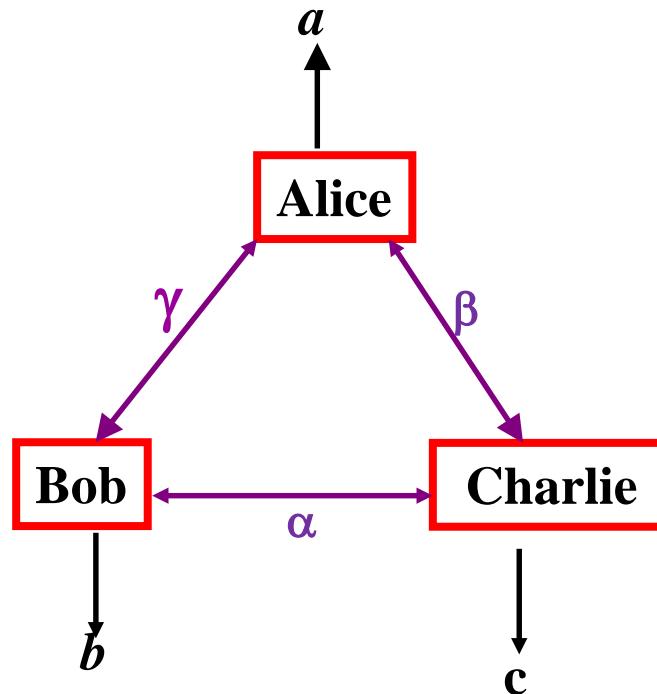


# Bell inequality without inputs





# The – genuine – Triangle



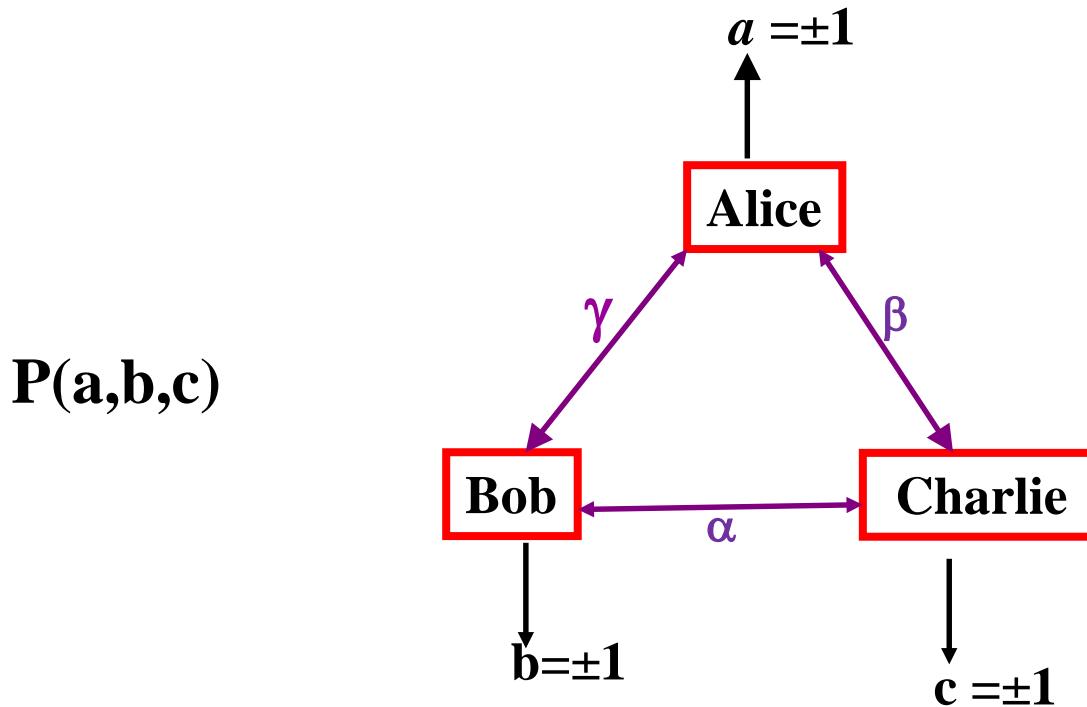
- Is there a quantum case (i.e. feasible with quantum states and measurements) which is not 3-local ?
- Can the local variables be discrete? How many symbols?

Example:  $\alpha, \beta, \gamma$  uniformly random in  $[0,1]$ ;  $a = (\beta \geq \gamma)$ ,  $b = (\gamma \geq \alpha)$ ,  $c = (\alpha \geq \beta)$   
 $\Rightarrow p(a=b=c)=0$  and all other  $p(a,b,c)=1/6$

Impossible to reproduce with finite alphabets and identical output fcts.  
 But feasible with trits and different output fcts. [QIC 18, 910-926 \(2018\)](#)



# The Triangle with binary outcomes



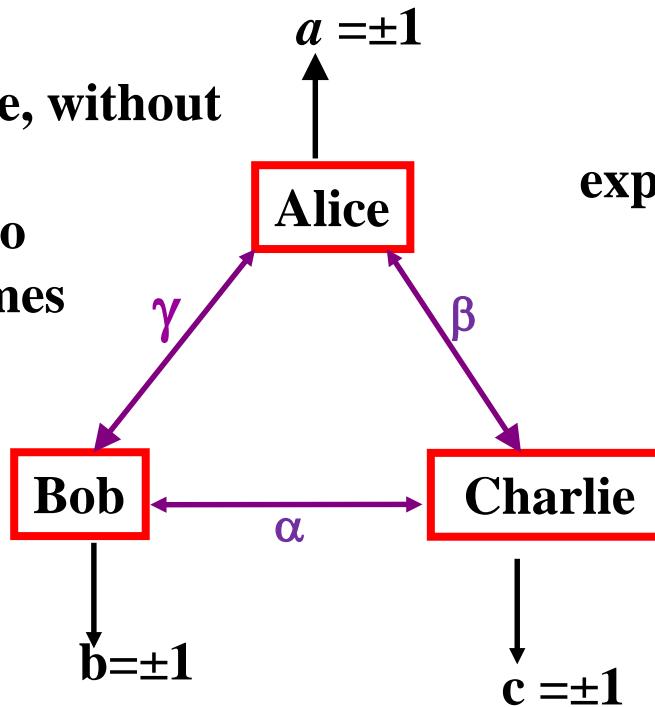
- Denis Rosset: QIC 18, 910-926 (2018)  
⇒ Alphabets of 6 letters suffices.
- How to characterize the  $p(a,b,c)$  that satisfy the NSI principle ?  
NSI = No-Signaling and Independence.



# The Triangle with binary outcomes

A 6 qubit toy universe, without input. How to prove quantumness? How to prove that the outcomes contain some randomness?

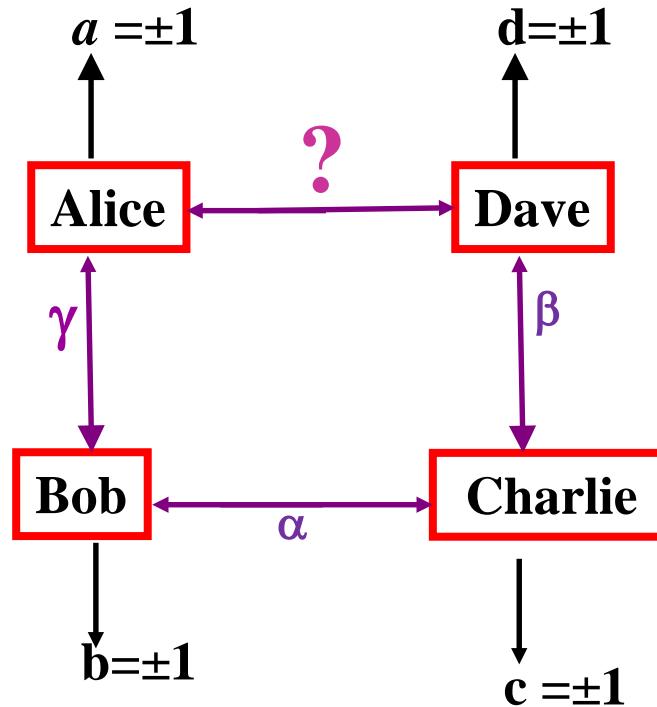
How to formalize and exploit the NSI principle?  
One has only access to  $P(a,b,c)$ .



There are no inputs. However, Alice can decide not to perform any measurement, or to change the local topology,  $\Rightarrow$  that should not affect the B-C correlation: no-signaling.



# The Triangle with binary outcomes



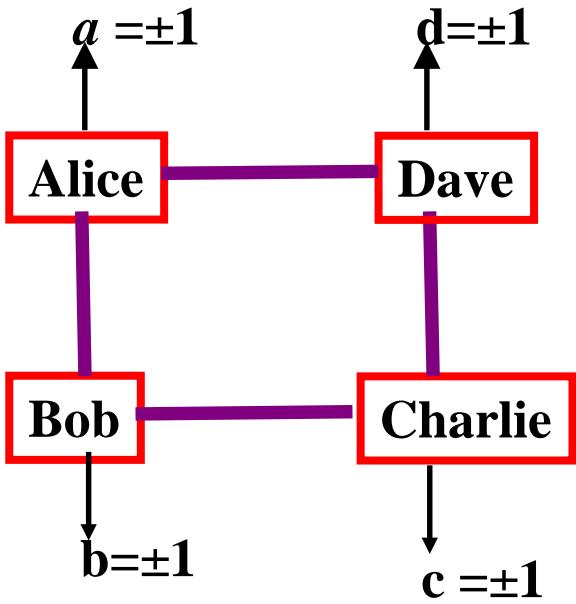
There are no inputs. However, Alice can decide not to perform any measurement, or to change the local topology,  
 $\Rightarrow$  that should not affect the B-C correlation: no-signaling.

Alice and Charlie are independent. Similarly,  
 Dave and Bob are independent.

↑  
← NSI



# NSI for the triangle with symmetric non-local boxes inflated to a square



$E_1 = 0$  (locally random)

$E_{AB} = E_{BC} = E_{CD} = E_{DA} = E_2$

No-signaling  $\Rightarrow$

$E_2$  square =  $E_2$  triangle

$E_{ABC} = E_{BCD} = E_{CDA} = E_{DAB} = E_3$

$E_{AC} = E_{BD} = 0$  Independence

$$P(a, b, c, d) = \frac{1}{16} \begin{pmatrix} 1 + (ab + bc + cd + da) \cdot E_2 \\ + (abc + bcd + cda + dab) \cdot E_3 \\ + abcd \cdot E_4 \end{pmatrix}$$

$$P(+-+-) = \frac{1}{16} (1 - 4 \cdot E_2 + E_4) \geq 0$$

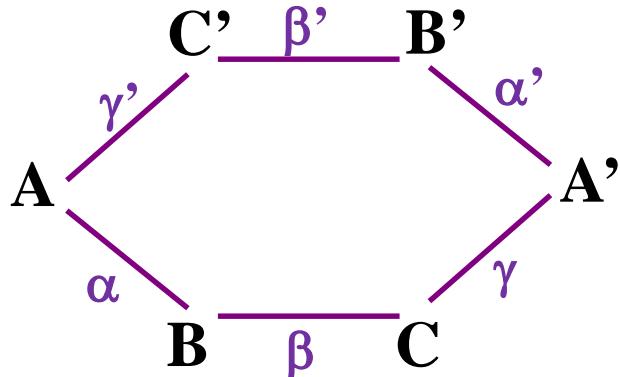
$$\Rightarrow 1 \geq E_4 \geq 4 \cdot E_2 - 1$$

$$\Rightarrow E_2 \leq \frac{1}{2}$$



# Inflation to Hexagon

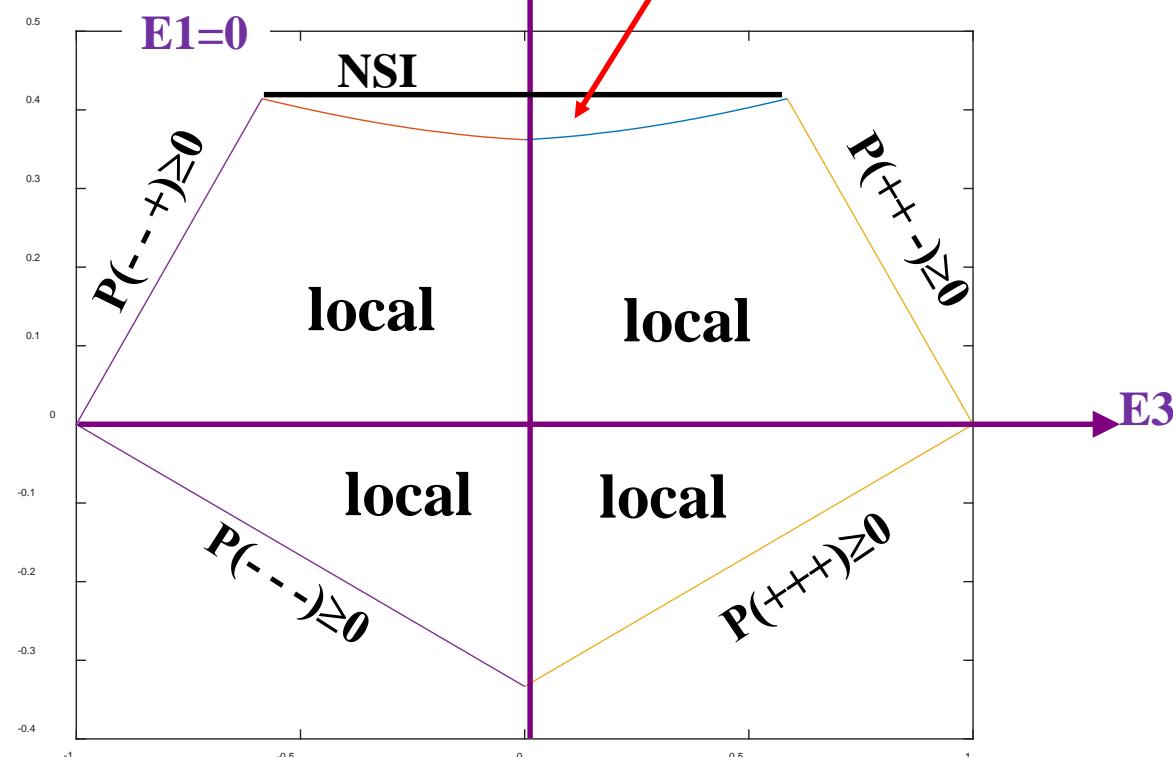
E.Wolfe et al.,  
arXiv:1609.00672



$$E1 = 0 \Rightarrow E2 \leq \sqrt{2} - 1$$

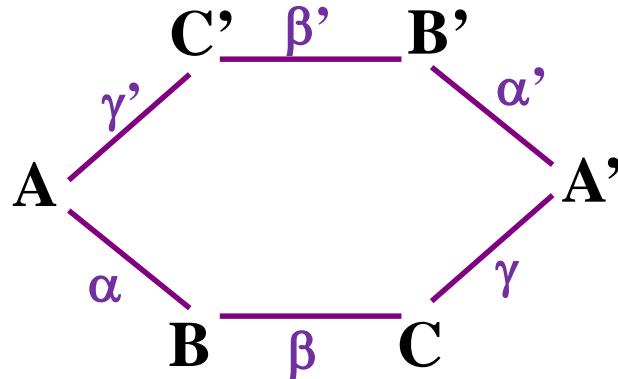
Challenge 2:  
is this local?  
Quantum?

Triangle :





# With no symmetry assumption



Challenge 3:  
find a triangle  
NSI-inequality that  
involves 3-party  
correlators.

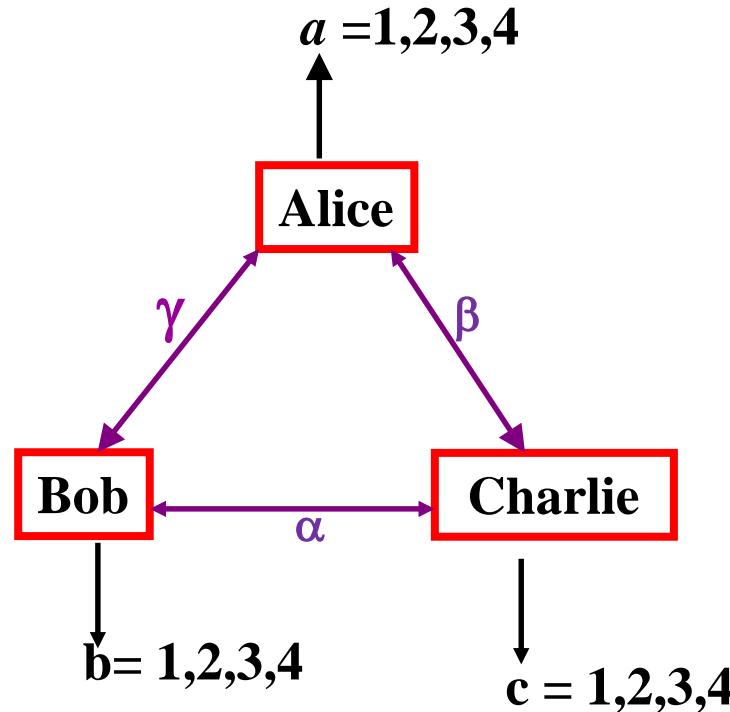
Inequalities for the triangle:

$$E_A = E_B = E_C = 0 \Rightarrow (1 + E_{AB})^2 + (1 + E_{BC})^2 + (1 + E_{CA})^2 \leq 6$$

$$\begin{aligned} & (1 + 2|E_A E_B| + E_{AB})^2 + (1 + 2|E_B E_C| + E_{BC})^2 + (1 + 2|E_C E_A| + E_{CA})^2 \\ & \leq 6(|E_A| \cdot |E_B| \cdot |E_C|) \end{aligned}$$



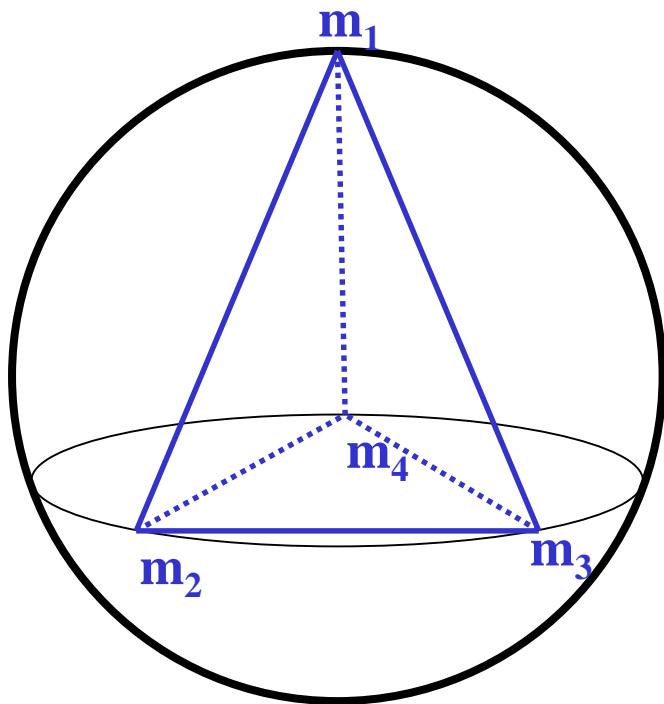
# 4-outcome Triangle



- $\alpha, \beta, \gamma = \phi^+$  and Bell State Measurements  $\Rightarrow p(a,b,c)$  is local
- Are there other «natural» 2-qubit measurements ?



# The Elegant Joint Measurement (EJM)



$$Tr_B(|\Phi_j\rangle\langle\Phi_j|) = \frac{1}{2} \left( 1 + \frac{1}{2} \vec{m}_j \right)$$

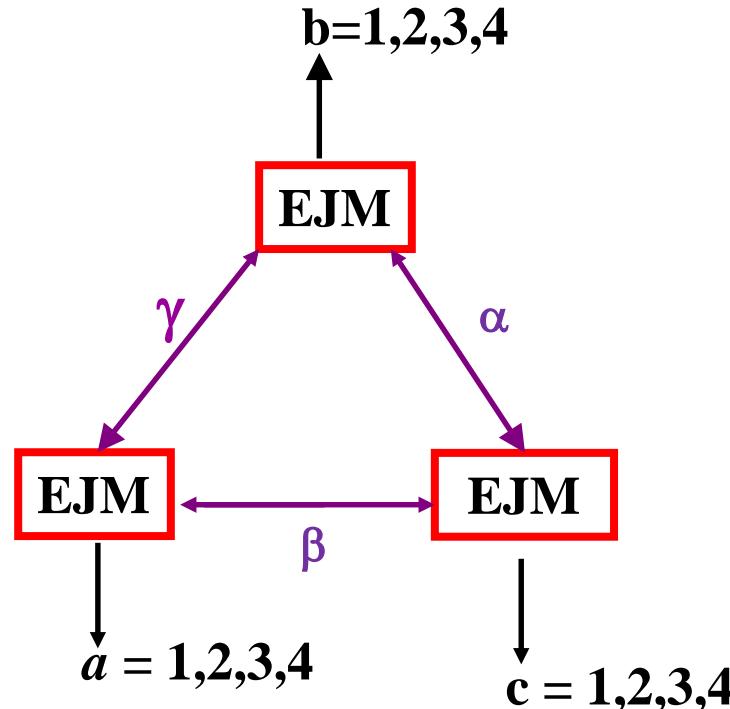
Look for 4 partially entangled and mutually orthogonal states with same degrees of entanglement and with partial states along the vertices of the tetrahedron.

$$\begin{aligned} |\Phi_j\rangle &= c_0 |\vec{m}_j, -\vec{m}_j\rangle + q_0 |-\vec{m}_j, \vec{m}_j\rangle \\ &= \frac{\sqrt{3}+1}{2\sqrt{2}} |\vec{m}_j, -\vec{m}_j\rangle + \frac{\sqrt{3}-1}{2\sqrt{2}} |-\vec{m}_j, \vec{m}_j\rangle \\ \langle\Phi_j|\Phi_i\rangle &= \delta_{ji} \end{aligned}$$

$$Tr_A(|\Phi_j\rangle\langle\Phi_j|) = \frac{1}{2} \left( 1 - \frac{1}{2} \vec{m}_j \right)$$



# The Elegant Distribution



$$\Rightarrow p(a,b,c) = \begin{cases} \frac{25}{256} & \text{if } a = b = c \\ \frac{1}{256} & \text{if } a = b \neq c, \quad a = c \\ \frac{5}{256} & \text{if } a \neq b \neq c \neq a \end{cases}$$

**Challenge 4:**  
**prove that the elegant distribution is local / non-local.**



# Distributions invariant under output permutations: $\pi(0,1,2,3)$

$$p(a,b,c) = \begin{cases} p_1 & \text{if } a = b = c \\ p_2 & \text{if } a = b \neq c, \quad a = c \neq b, \quad a \neq b = c \\ p_3 & \text{if } a \neq b \neq c \neq a \end{cases}$$

Normalization  $\Rightarrow 4p_1 + 36p_2 + 24p_3 = 1$

Write the outcomes as 2 bits, e.g.  $a=a_0+2a_1$  and look for the correlators:

Switching to "physicists" bits  $\pm 1$  all correlators vanish except

$$\langle a0b0 \rangle = \langle a1b1 \rangle = 4p_1 + 4p_2 - 8p_3 = E2$$

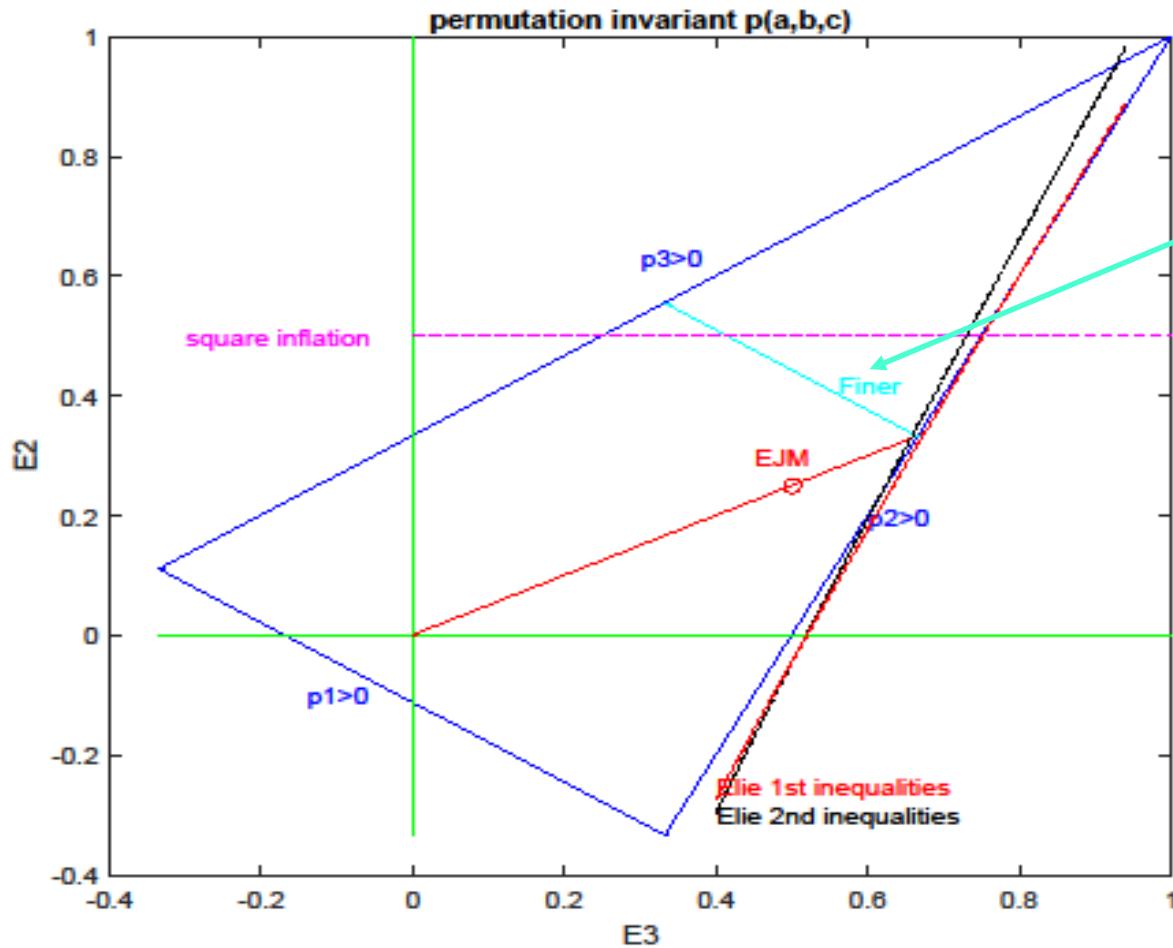
$$\langle a0a1b0b1 \rangle = 4p_1 + 4p_2 - 8p_3 = E2$$

$$\langle a0a1b0c1 \rangle = \langle a0a1b1c0 \rangle = 4p_1 - 12p_2 + 8p_3 = E3$$

Only correlators with an even number of bits and an even number of indices 1 do not vanish.



# Distributions invariant under output permutations: $\pi(0,1,2,3)$



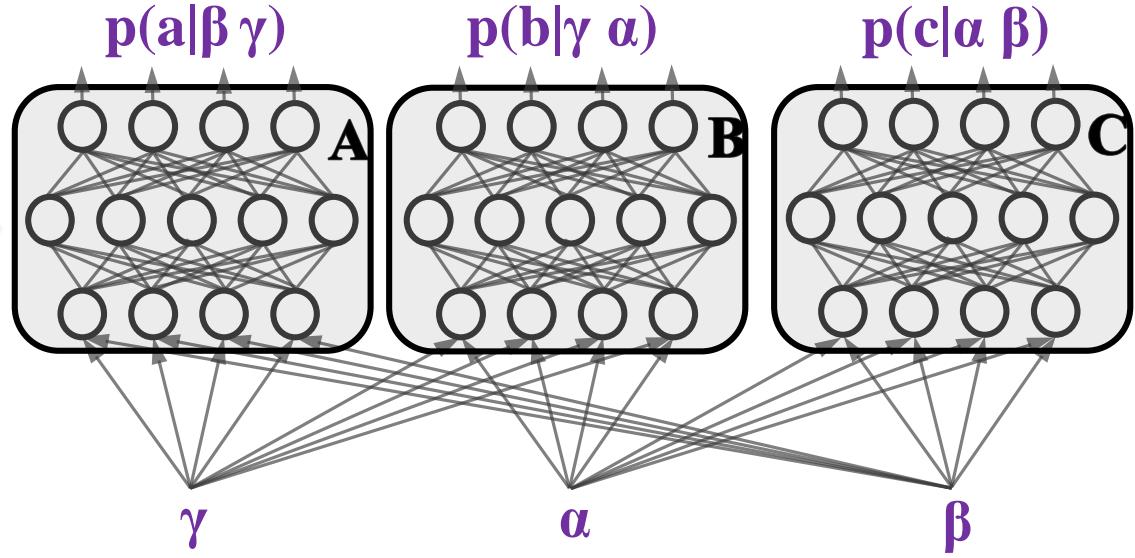
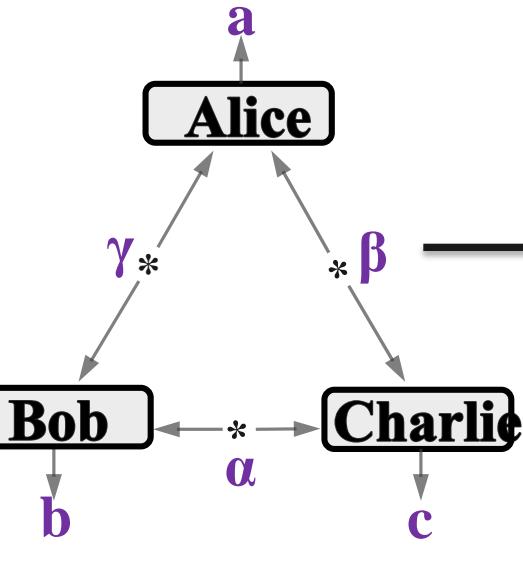
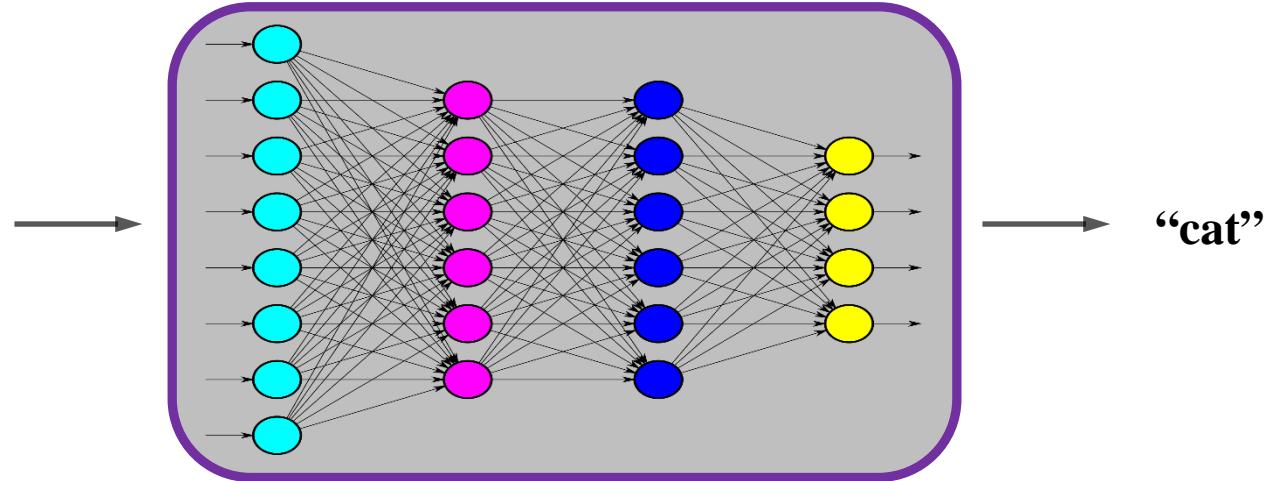
See Marco  
Renou's poster



Tamas Krivachy's poster

# Machine Learning

## Neural networks



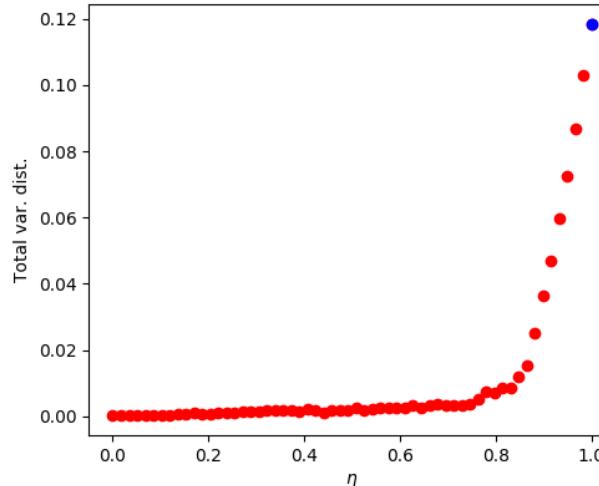
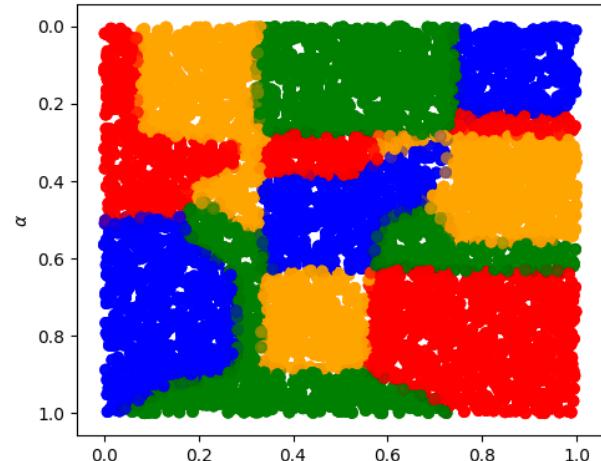
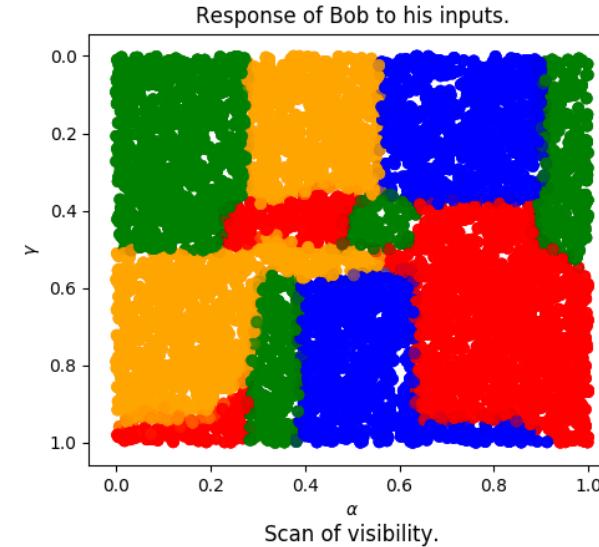
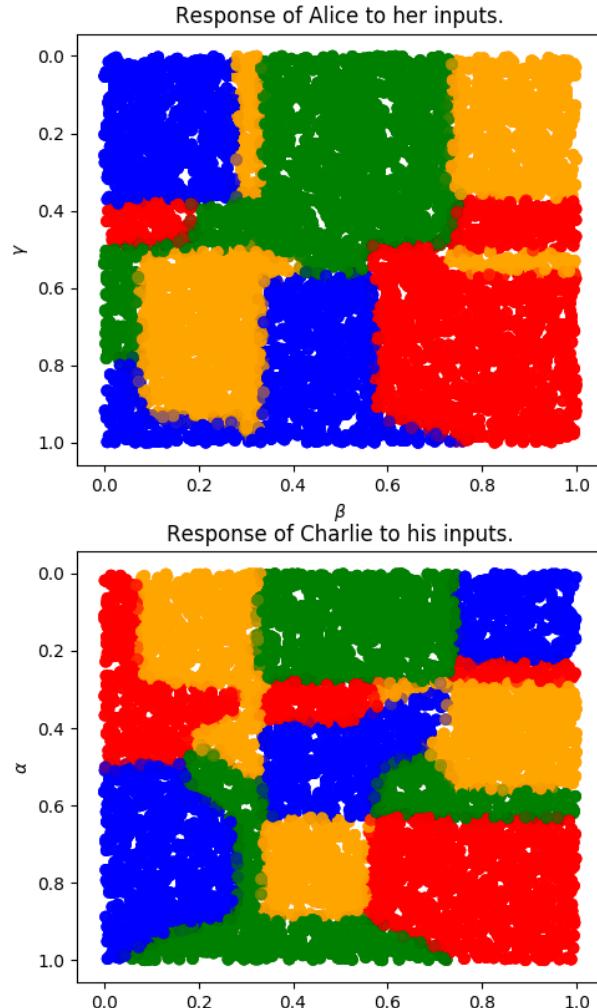


Tamas Krivachy's  
poster

# Elegant distribution & Machine Learning

2-elegant-localnoise distribution with parameter=1.000

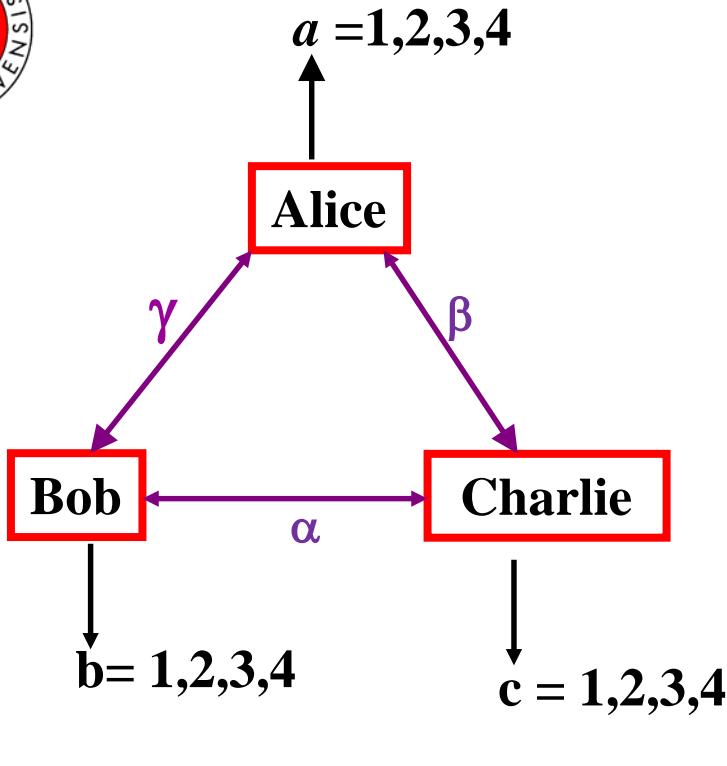
- 1
- 2
- 3
- 4





# The Salman Triangle

see MO Renou's poster



$$\alpha, \beta, \gamma = \phi^+$$

Eigenstates of measurements:

$$|\varphi_1\rangle = |0,1\rangle \quad |\varphi_2\rangle = |1,0\rangle$$

$$|\varphi_3\rangle = c_1|0,0\rangle + s_1|1,1\rangle$$

$$|\varphi_4\rangle = c_2|0,0\rangle + s_2|1,1\rangle$$

$$P(|0,1\rangle, |0,1\rangle, c) = 0$$

**Theorem:** For  $c_1$  large enough, the quantum distribution  $p(a,b,c)$  is non-local:

$$p(a,b,c) \neq \sum_{\alpha\beta\gamma} p(\alpha)p(\beta)p(\gamma)p(a|\beta\gamma)p(b|\gamma\alpha)p(c|\alpha\beta)$$



# The Salman Triangle see MO Renou's poster

Eigenstates of measurements:

$$|\varphi_1\rangle = |0,1\rangle, \quad |\varphi_2\rangle = |1,0\rangle$$

$$|\varphi_3\rangle = c_1|0,0\rangle + s_1|1,1\rangle$$

$$|\varphi_4\rangle = c_2|0,0\rangle + s_2|1,1\rangle \quad P(|0,1\rangle, |0,1\rangle, c) = 0$$

**Theorem:** For  $c_1$  large enough,  
the quantum distribution  
 $p(a,b,c)$  is non-local.

**Lemma 1:** Denote X,Y,Z the 3 sets of local variables  $\alpha, \beta, \gamma$ .

These sets partition in two:  $X=X_0 \cup X_1$ ,  $Y=Y_0 \cup Y_1$ ,  $Z=Z_0 \cup Z_1$

s.t.  $a(\gamma, \beta) = |0,1\rangle \Leftrightarrow \beta \in Y_0 \text{ & } \gamma \in Z_1$

$$a(\gamma, \beta) = |1,0\rangle \Leftrightarrow \beta \in Y_1 \text{ & } \gamma \in Z_0$$

**Lemma 2:** For  $c_1 > \frac{1}{2}\sqrt{2k - 6/k} \approx 0.886$ , where  $k = \sqrt[3]{9 + 6\sqrt{3}}$

no 3-local model reproduces  $p(a,b,c \in \{\varphi_3, \varphi_4\}^3)$

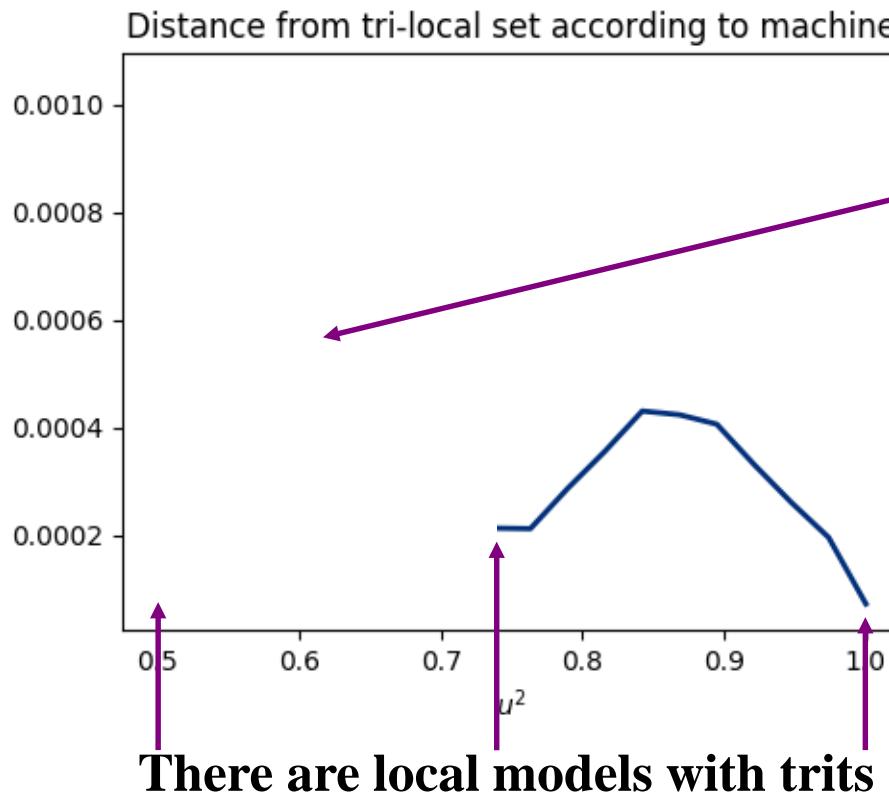
**Challenge 5:**

How robust – noise resistant – is the Salman distribution?



# The Salman Triangle

## Machine learning



**Challenge 6:**  
Prove Salman distribution  
non-local below  
the critical  $c^2$

**Challenge 7:**

- i) Is the Salman truly “genuinely triangle” ? Or can it be realized with only 1 or 2 entangled states ?
- ii) Define “genuinely triangle”: e.g.  
entangled states & measurements ?



# Conclusions - Challenges

1. Find a bi-locality scenario with critical visibility per singlet  $\geq 1/\sqrt{2}$ .
2. What is in the top of the E2 vs E3 set of the binary triangle: is this local? Quantum?
3. Find a triangle inequality that involves 3-party correlators.
4. Prove that the elegant distribution is local / non-local.
5. How robust – noise resistant – is the Salman distribution?
6. Prove Salman's distribution is non-local below the critical c.
7. Define «genuinely triangle» and apply to Salman's distr.

arXiv:1901.08287 (Finner ineq.), PRL 123, 070403 (2019)  
arXiv:1906.06495 (Salman Triangle), PRL in press (2019)  
arXiv:1905.04902 (NSI principle)  
arXiv:1907.10552 (Machine learning)

Thank you !