Quantum mechanics and the covariance of physical laws in quantum reference frames

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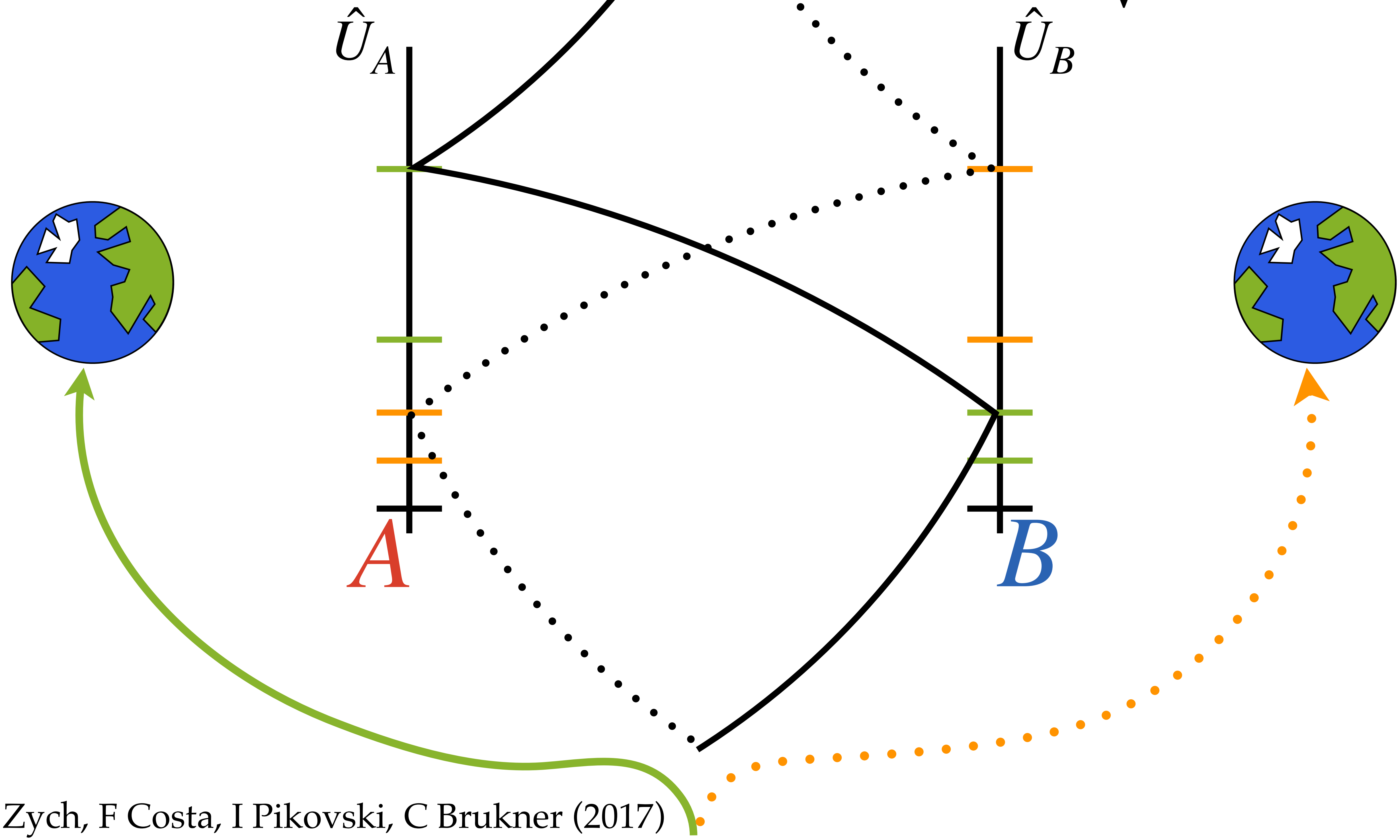
F. Giacomini, E. Castro Ruiz, Č. Brukner, Nat. Commun. **10**, 494 (2019)
F. Giacomini, E. Castro Ruiz, Č. Brukner, Phys. Rev. Lett. **123**, 090404 (2019)

Causality in the quantum world
Anacapri, 20 September 2019

The gravitational switch

$$\frac{\hat{U}_A \hat{U}_B |\psi\rangle_S |L\rangle_E + \hat{U}_B \hat{U}_A |\psi\rangle_S |R\rangle_E}{\sqrt{2}}$$



M Zych, F Costa, I Pikovski, C Brukner (2017)

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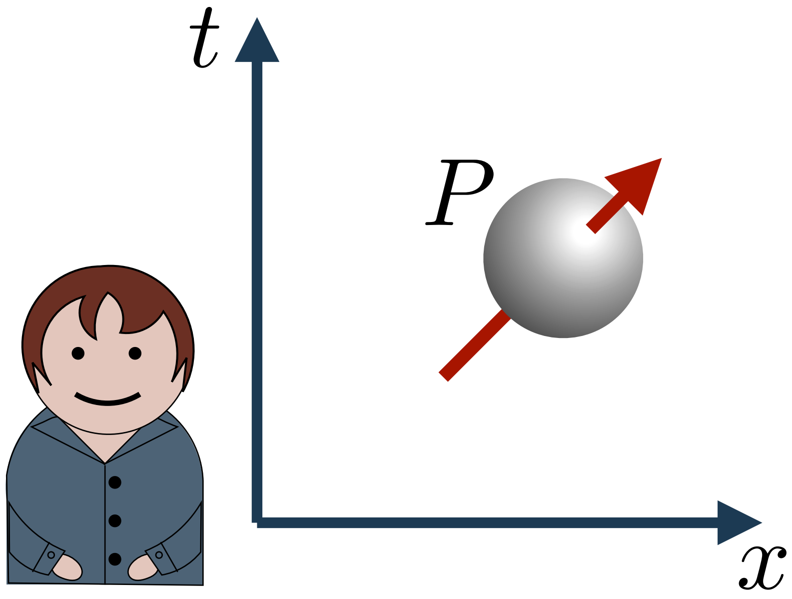
INTRODUCTION

What is a reference frame?

Reference frames are abstract entities, used to fix the point of view from which observations are carried out.



The laws of physics are the same regardless of the choice of the reference frame.
(**Principle of covariance**).

Translation	$\hat{U}_T = e^{\frac{i}{\hbar} X_0 \hat{p}}$	
Galilean boost	$\hat{U}_B = e^{\frac{i}{\hbar} v \hat{G}}$	$\hat{G} = \hat{p}t - m\hat{x}$
⋮		

The reference frame enters the transformation as a **parameter**.

Covariance of physical laws

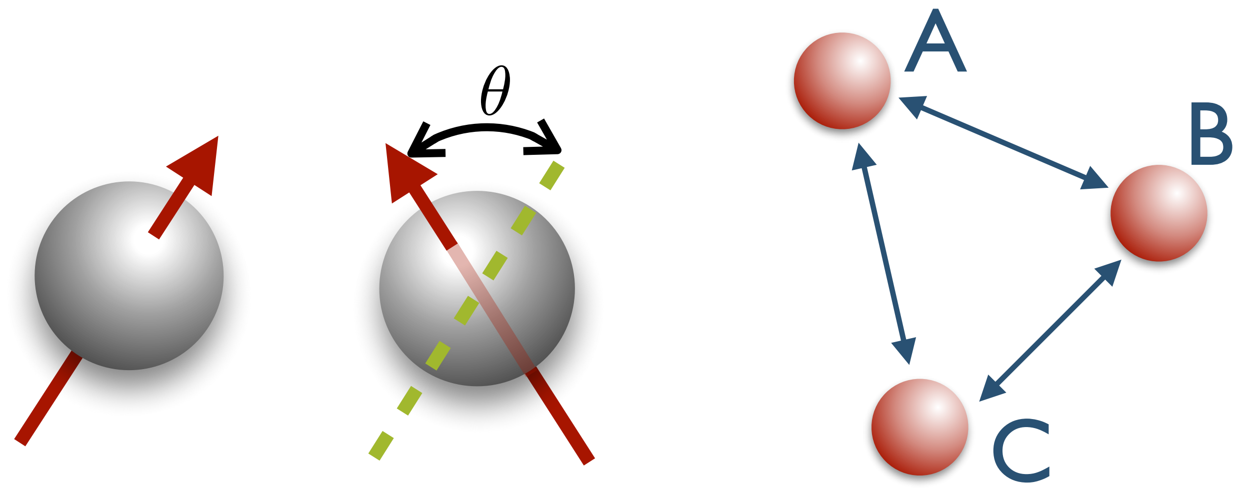
$$\hat{H}' = \hat{U}\hat{H}\hat{U}^\dagger + i\hbar \frac{d\hat{U}}{dt}\hat{U}^\dagger$$

Symmetry

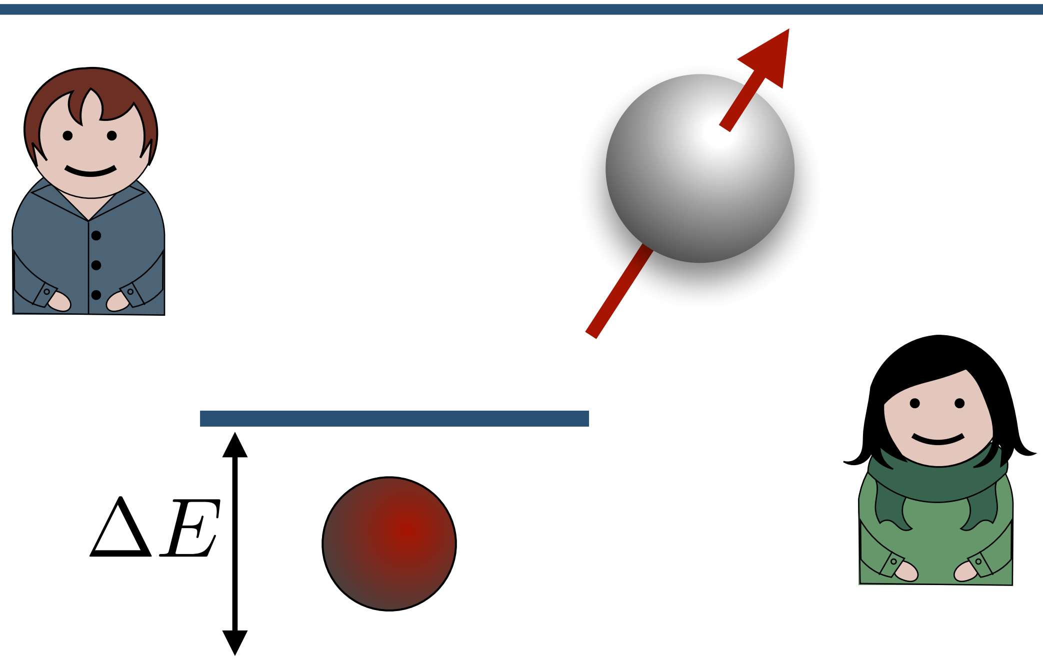
$$\hat{H}' = \hat{H}$$

Quantum reference frames

A reference frame is a **physical system** and obeys the laws of physics.



The description of the quantum state is given in terms of **relative** quantities.



Only relational degrees of freedom: no degrees of freedom of the reference frame itself.

No absolute reference frame needed.

Covariance in QRFs

The goal is the **generalisation** of the **principle of covariance.**

Covariance in quantum reference frames

The laws of physics are the same irrespective of the choice of the quantum RF.

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TRANSFORMATION OF THE QUANTUM STATE

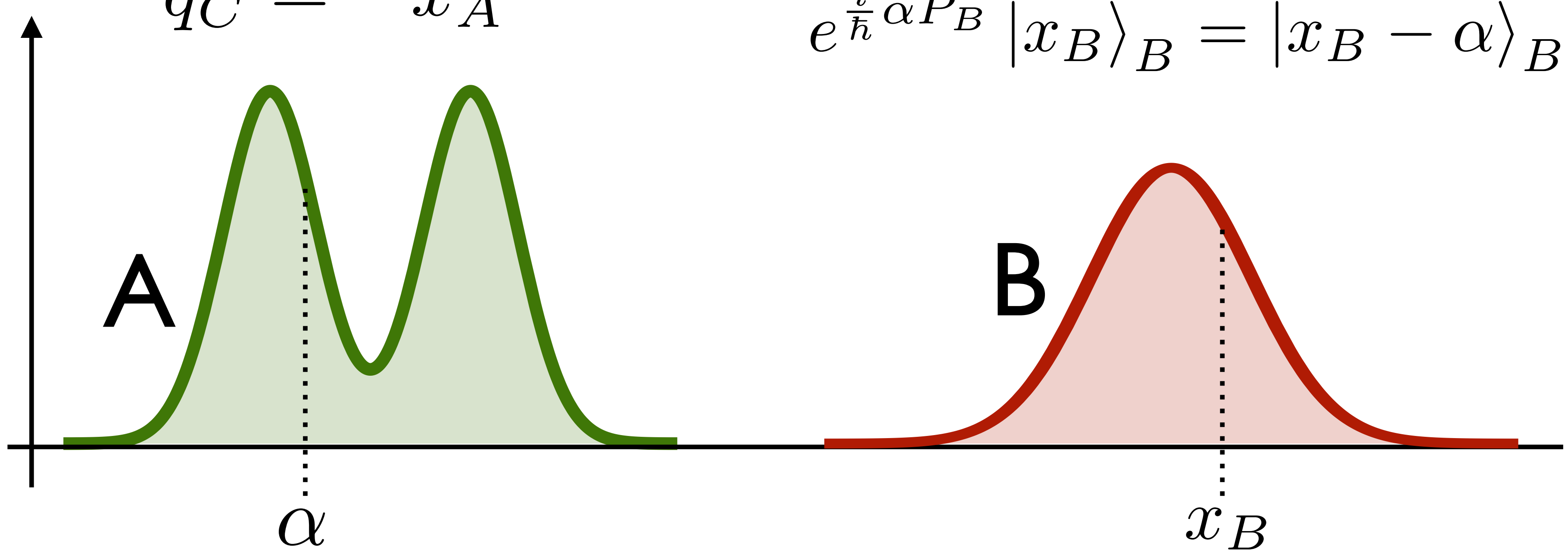
Transformation of the state

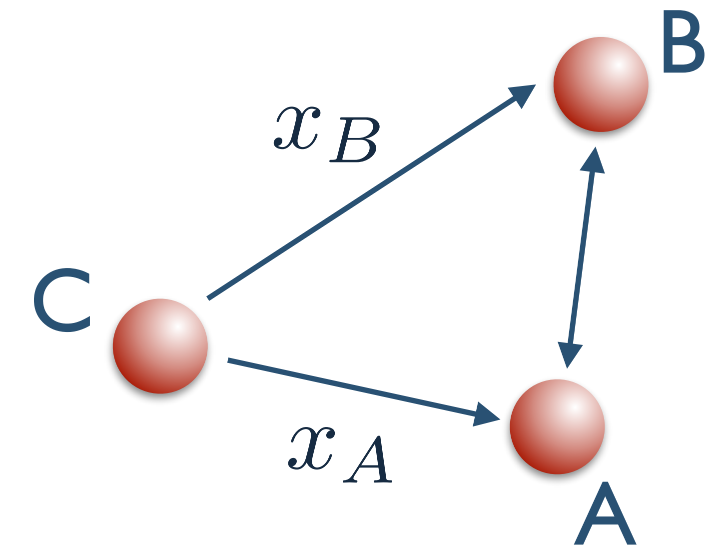
Simplest case: transformation to relative coordinates

$$q_B = x_B - x_A$$

$$q_C = -x_A$$

$$e^{\frac{i}{\hbar}\alpha \hat{P}_B} \left| x_B \right\rangle_B = \left| x_B - \alpha \right\rangle_B$$





For the whole state:

$$e^{\frac{i}{\hbar}\hat{X}_A \hat{P}_B} \left| \psi \right\rangle_A \left| \phi \right\rangle_B$$

usually a parameter of the group!

- Wavepackets instead of sharp position/velocities
- Quantum superposition, entanglement

$$\boxed{\hat{S}_x = \hat{\mathcal{P}}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}}$$

$$\rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^{\dagger}$$

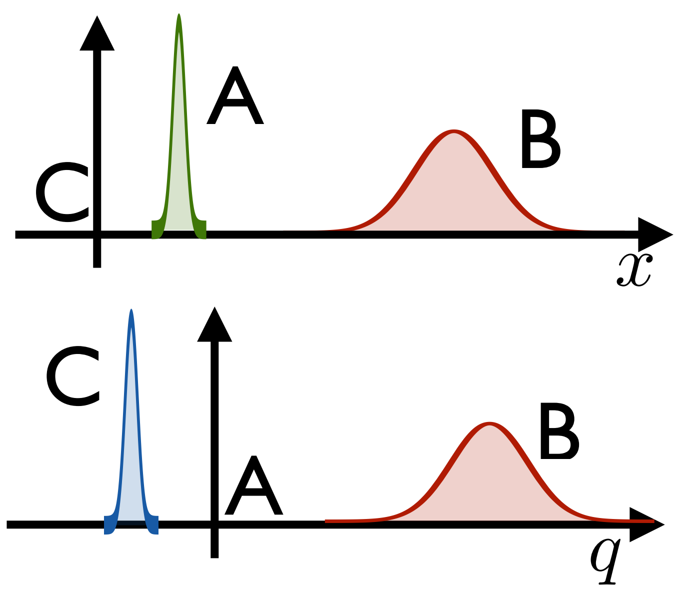
$\hat{\mathcal{P}}_{AC}$: parity operator + swap between A and C.

Example: Relative states

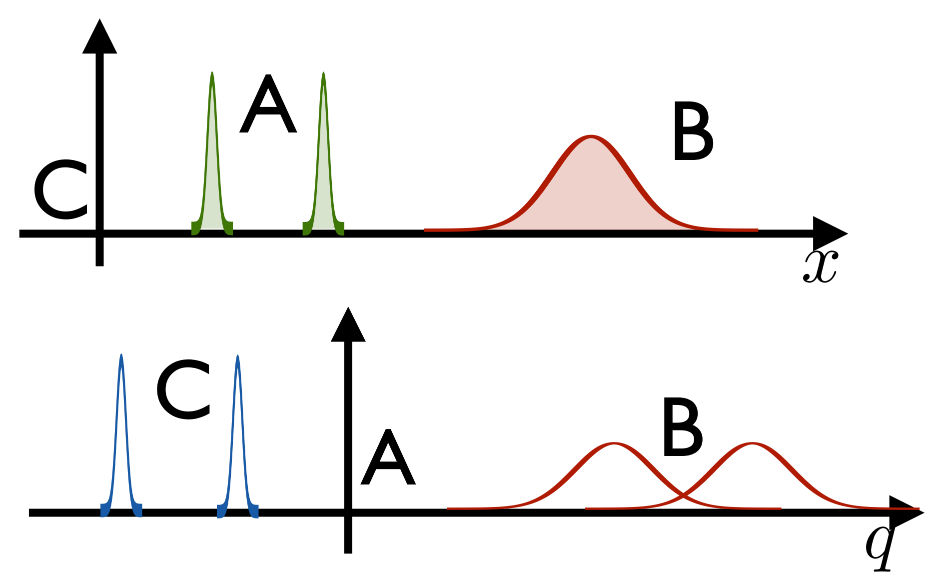
$$\hat{S}_x = \hat{\mathcal{P}}_{AC} e^{\frac{i}{\hbar}\hat{x}_A \hat{p}_B}$$

$$\rho_{BC}^{(A)} = \hat{S}_x \rho_{AB}^{(C)} \hat{S}_x^\dagger$$

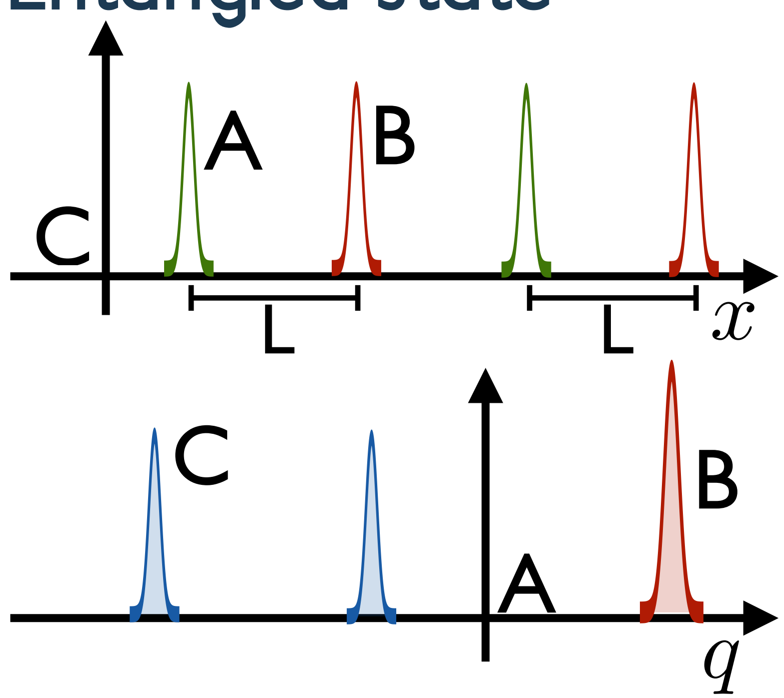
Localised state of A



Product state and spatial superposition

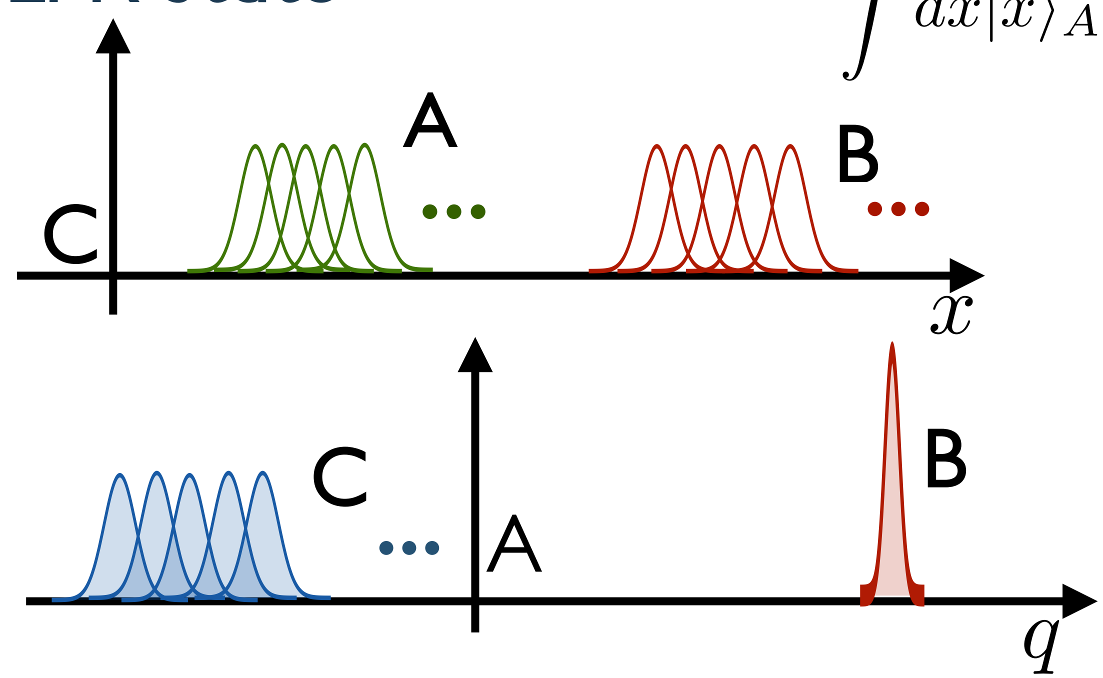


Entangled state



EPR state

$$\int dx |x\rangle_A |x + X\rangle_B$$



A: new reference frame; B: quantum system; C: old reference frame

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TEMPORAL EVOLUTION: DYNAMICS

The Schrödinger equation

Schrödinger equation in C's reference frame

$$i\hbar\frac{d\rho_{AB}^{(C)}}{dt} = \left[H_{AB}^{(C)}, \rho_{AB}^{(C)}(t)\right]$$

A: new reference frame
B: quantum system
C: old reference frame

To change to the frame of A we apply the transformation $\hat{S}$

$$\boxed{i\hbar\frac{d\rho_{BC}^{(A)}}{dt} = \left[H_{BC}^{(A)}, \rho_{BC}^{(A)}(t)\right]}$$

$$\hat{H}_{BC}^{(A)} = \hat{S}\hat{H}_{AB}^{(C)}\hat{S}^\dagger + i\hbar\frac{d\hat{S}}{dt}\hat{S}^\dagger$$

$$\hat{\rho}_{BC}^{(A)} = \hat{S}\hat{\rho}_{AB}^{(C)}\hat{S}^\dagger$$

The evolution in the new reference frame is unitary.

We define a symmetry transformation as:

$$\hat{S}\hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=A,B}\right)\hat{S}^\dagger + i\hbar\frac{d\hat{S}}{dt}\hat{S}^\dagger = \hat{H}\left(\{m_i, \hat{x}_i, \hat{p}_i\}_{i=B,C}\right)$$

General transformation

Hamiltonian of the old reference frame C as seen from A

Hamiltonian of the new reference frame A as seen from C

$$\hat{S} = e^{-\frac{i}{\hbar}\hat{H}_C t}\hat{\mathcal{P}}^{(i)}_{AC}\Pi_n e^{\frac{i}{\hbar}\hat{f}^n_A(t)\hat{O}^n_B} e^{\frac{i}{\hbar}\hat{H}_A t}$$

generalised parity operator (switches the equations of motion of A and C)

generalisation of the standard change of reference frame

$$\hat{U}_i = \Pi_n e^{\frac{i}{\hbar}f^n(t)\hat{O}^n_B}$$

function describing the relationship between old and new reference frame

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SUPERPOSITION OF SPATIAL TRANSLATIONS

Translations in QRFs

The new QRF is described by system A at time 0.

$$|\Psi_0\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|x_1\rangle_A + |x_2\rangle_A\right)|\phi_0\rangle_B \qquad \hat{H}^{(C)}_{AB} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

We want to jump
to the QRF of A

$$\hat{S}_T = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\underbrace{\hat{\mathscr{P}}^{(x)}_{AC}\exp\left(\frac{i}{\hbar}\hat{x}_A\hat{p}_B\right)}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

$\hat{S}_x$ translation to a reference frame which is frozen in time.

$$\hat{H}^{(A)}_{BC} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The free hamiltonian is symmetric under generalised translations.

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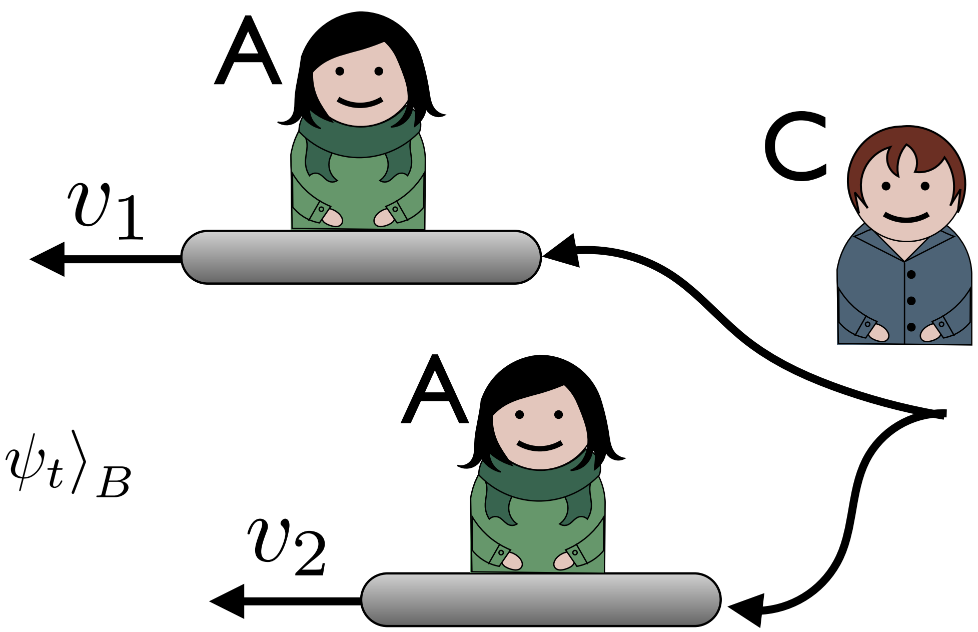
SUPERPOSITION OF GALILEAN BOOSTS

The boost in QRFs

We want to boost to a QRF A moving in a superposition of velocities from the point of view of C.

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

QRF of A



$$|\Psi_t\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|m_A v_1\rangle_A + |m_A v_2\rangle_A\right)|\psi_t\rangle_B$$

$$\hat{S}_b = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}_{AC}^{(v)}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A}{m_A}\hat{G}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$

- parity and swap
- sets velocity of C to the opposite of velocity of A

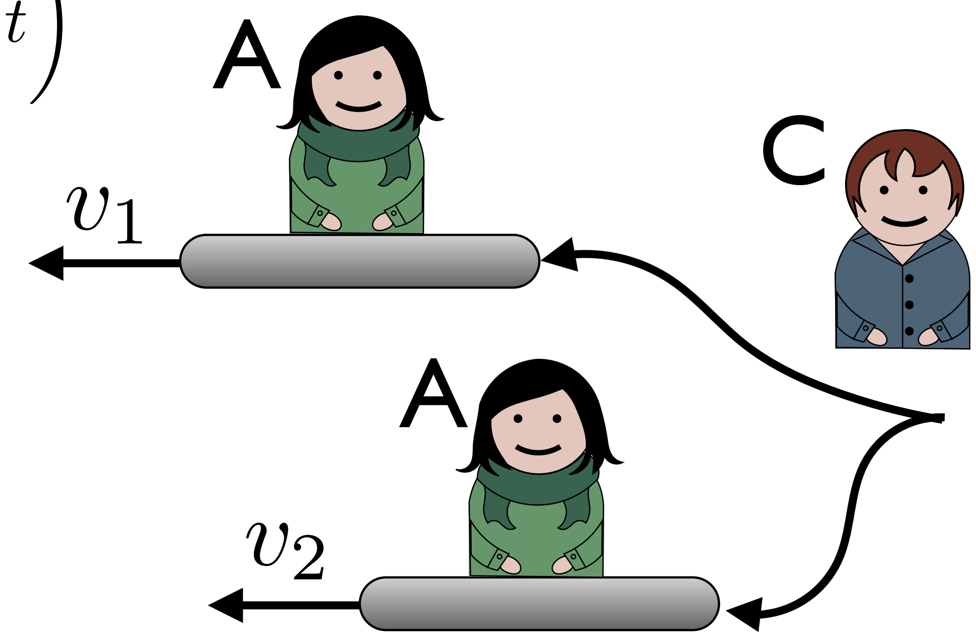
$$\pi_C = -\frac{m_C}{m_A}p_A$$

Operator replacing v!

Galilean boost on B controlled by the velocity of A

The boost in QRFs

$$\hat{S}_b = \exp\left(-\frac{i}{\hbar}\frac{\hat{\pi}_C^2}{2m_C}t\right)\hat{\mathcal{P}}_{AC}^{(v)}\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A}{m_A}\hat{G}_B\right)\exp\left(\frac{i}{\hbar}\frac{\hat{p}_A^2}{2m_A}t\right)$$



From the QRF of C

$$\hat{H}_{AB}^{(C)} = \frac{\hat{p}_A^2}{2m_A} + \frac{\hat{p}_B^2}{2m_B}$$

$$|\Psi_t\rangle_{AB} = \frac{1}{\sqrt{2}}\left(|m_A v_1\rangle_A + |m_A v_2\rangle_A\right)|\psi_t\rangle_B$$

From the QRF of A

$$\hat{H}_{BC}^{(A)} = \frac{\hat{\pi}_B^2}{2m_B} + \frac{\hat{\pi}_C^2}{2m_C}$$

The free hamiltonian is symmetric under superposition of Galilean boosts.

$$|\Psi'_t\rangle_{BC} = \frac{1}{\sqrt{2}}\left(e^{\frac{i}{\hbar}v_1\hat{G}_B}|\psi_t\rangle_B|-m_C v_1\rangle_C + e^{\frac{i}{\hbar}v_2\hat{G}_B}|\psi_t\rangle_B|-m_C v_2\rangle_C\right)$$

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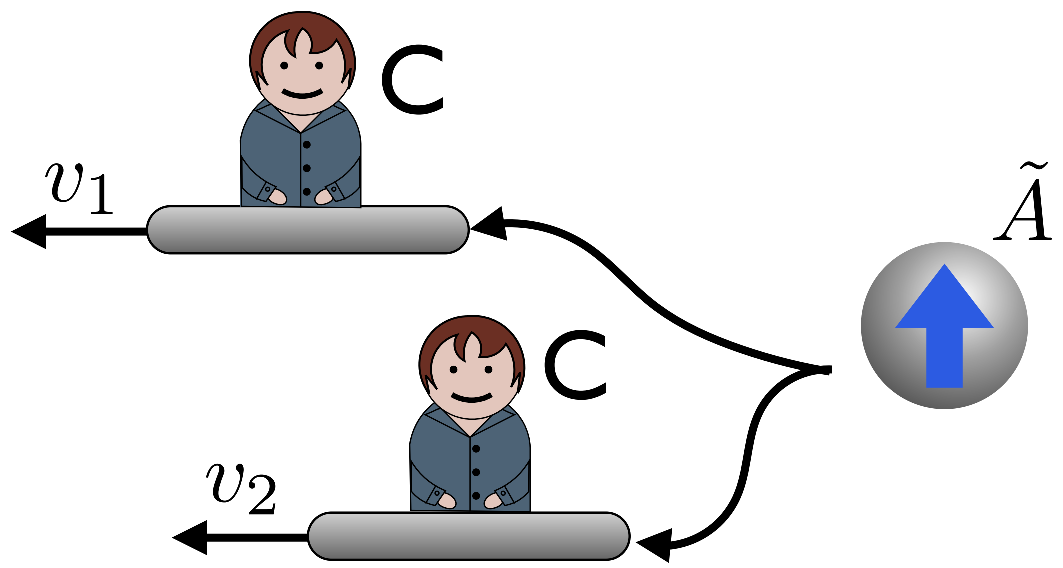
RELATIVISTIC QRF AND SPIN

Spin in special relativity

Lack of an operational definition of spin (Stern Gerlach experiment) in special relativistic quantum mechanics.

(Pauli-Lubanski, Wigner-Pryce, Foldy-Wouthuysen, Chakrabarti, Czachor, Fradkin-Good, Fleming,…)



We want to treat the spin as a QUBIT.

Spin is unambiguous in the rest frame

QRFs allow us to transform to the rest frame of a particle in a superposition of velocities.

QRF transformation to the rest frame of a quantum particle

$$\hat{S}_L = \mathcal{P}^{(v)}_{CA} U_{\tilde{A}}(\Lambda_{\pi_C})$$

Lorentz boost by each velocity of the laboratory relative to the particle

F. Giacomini, E. Castro Ruiz, Č. Brukner, arXiv:1811.08228 (2018)

Spin in special relativity

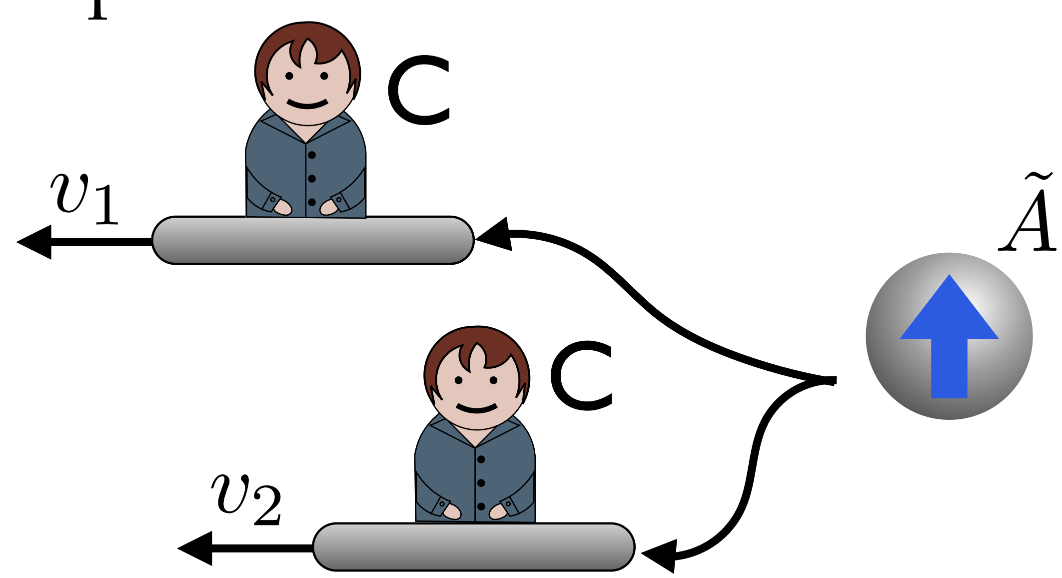
We find:

- A set of observables satisfying the spin su(2) algebra, having the correct eigenvalues and reducing to the Pauli operators in the non relativistic limit

$$\hat{\Xi}_i = \hat{S}_L(\mathrm{I}_C \otimes \hat{\sigma}_i)\hat{S}_L^\dagger$$

- A partition of the total Hilbert space which allows us to identify the qubit $|0\rangle$ and the qubit $|1\rangle$;

- A relativistic extension of the Stern-Gerlach experiment.



Summary

Quantum reference frames formalism: operational and relational.
Valid in both non relativistic and special-relativistic physics.

Question

How can we describe the world from the point of view of a non-idealised reference frame, i.e. associated to a quantum state and to a dynamical equation of motion?

Need to find a more general law to change the reference frame.

This leads to a **generalisation of the notion of covariance**, which has been explored in the two cases of the **superposition of spatial translations** and the **superposition of Galilean boosts**.

Can be applied also in special relativity to find an **operational definition of spin** and devise a relativistic Stern-Gerlach experiment.

Not covered: extension of the weak equivalence principle to QRFs.

THANK YOU