THE UNIVERSITY OF QUEENSLAND
AUSTRALIA

EQUS

# Communication through Quantum Causal Structures

Fabio Costa

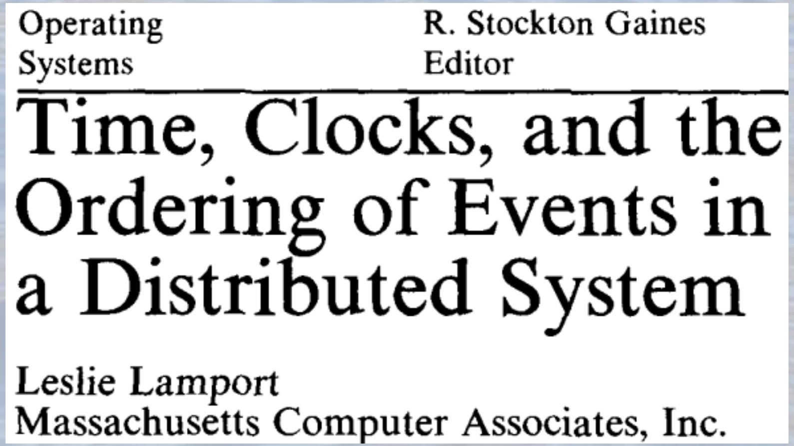
Causality in the Quantum World, Anacapri

- D. Jia, F. C., “Causal order as a resource for quantum communication”, **arXiv:1901.09159** [quant-ph] (2019).
- K. Goswami, F.C., in progress.

17 Sep 2019

# Causal order can be unknown

Distributed systems, quantum networks

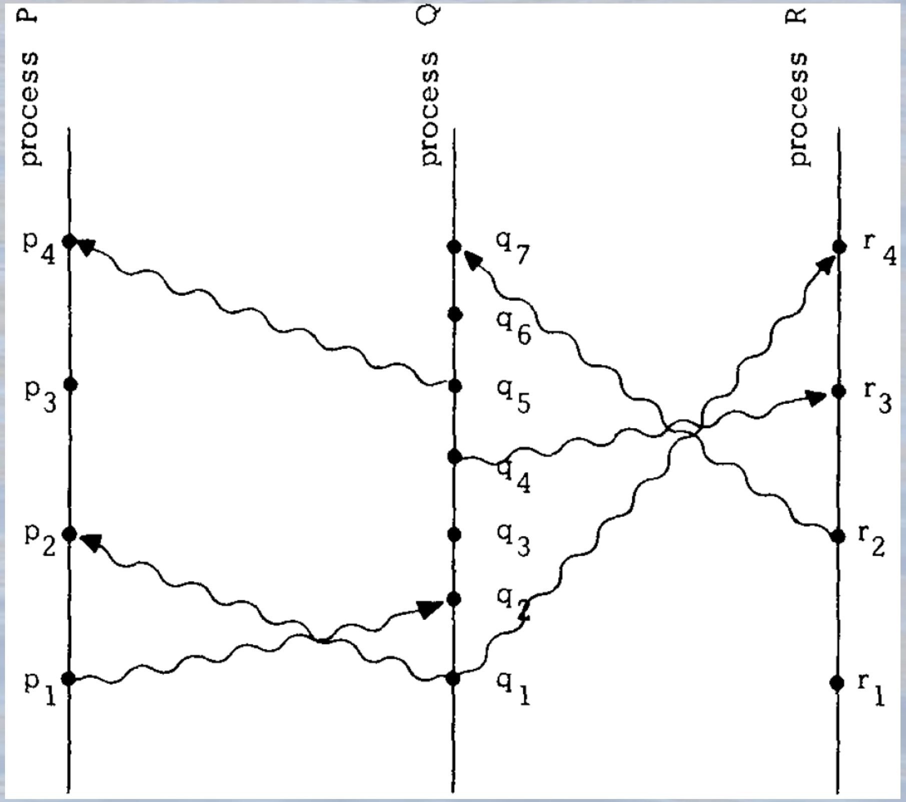


Operating Systems

R. Stockton Gaines, Editor

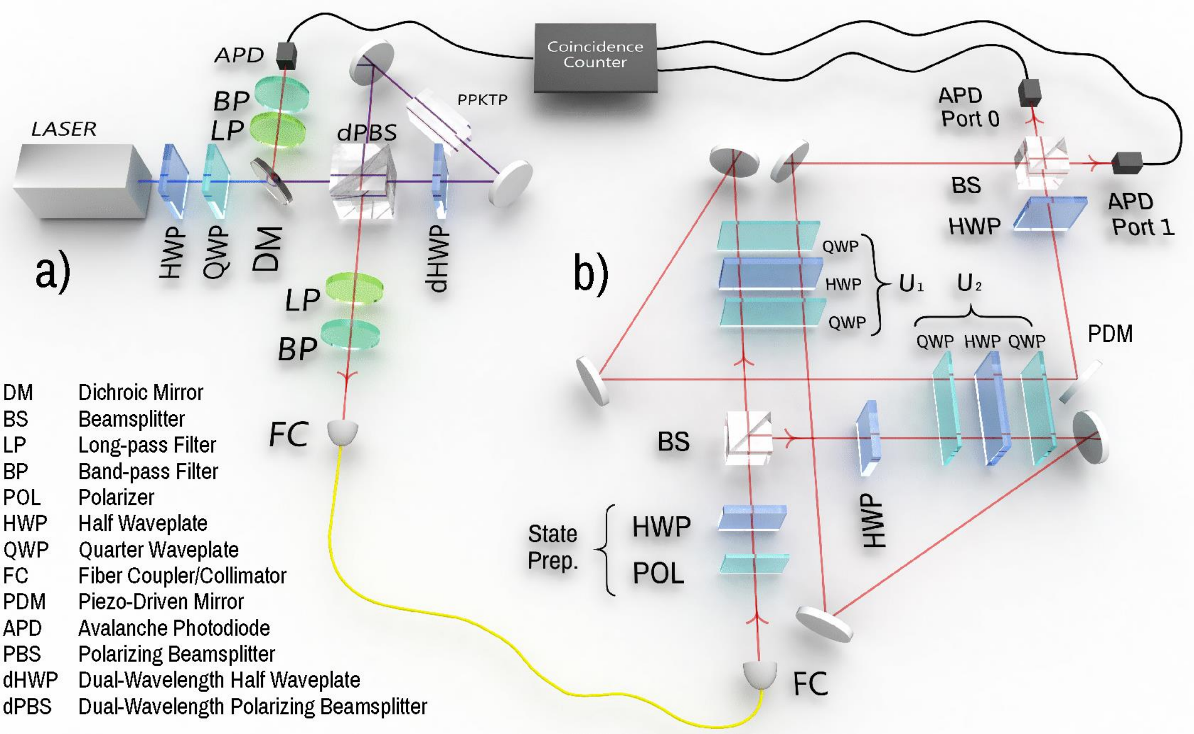
**Time, Clocks, and the Ordering of Events in a Distributed System**

Leslie Lamport
Massachusetts Computer Associates, Inc.



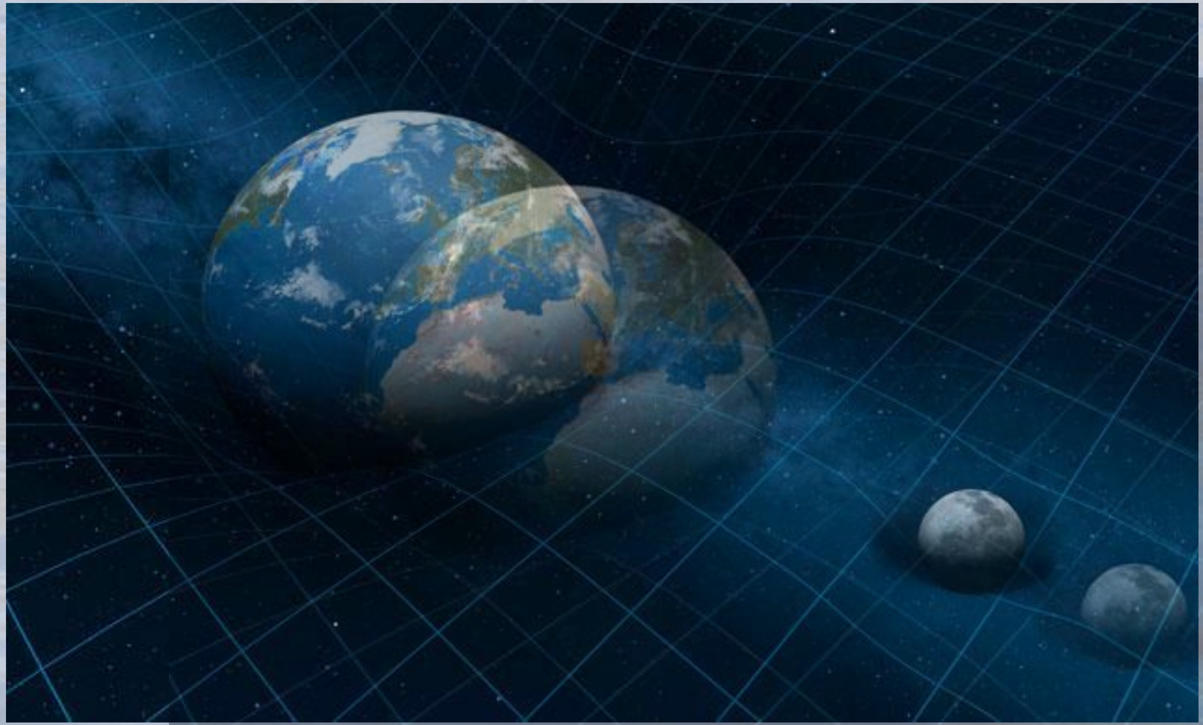
# Causal order can be indefinite

## In the lab




Procopio et al., *Nat. Comm.* **6**, 7913 (2015)

## Quantum + gravity



Zych et al., *Nat Comm.* **10**, 3772 (2019)

# Communication in unknown/indefinite causal order?

- Communication settings
  - Single shot/asymptotic

- Classical communication
  - Generalised Holevo bound
  - Entropic causal inequalities

- Quantum communication (definite, unknown order)
  - Bound on quantum capacity
  - No two-way quantum communication

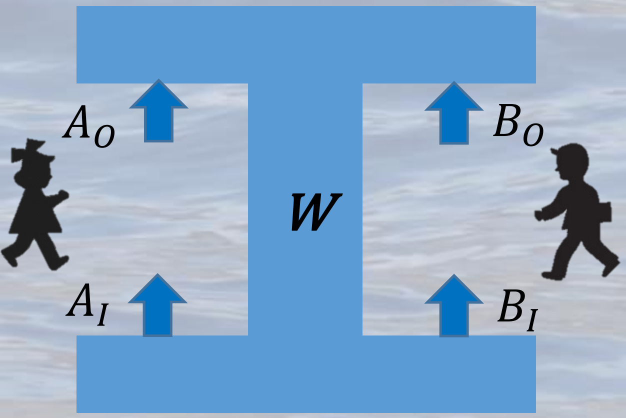
# Quantum processes

General quantum communication resource

Each party can perform *one* operation on a quantum system

**input**: system `just before' operation

**output**: system `just after' operation

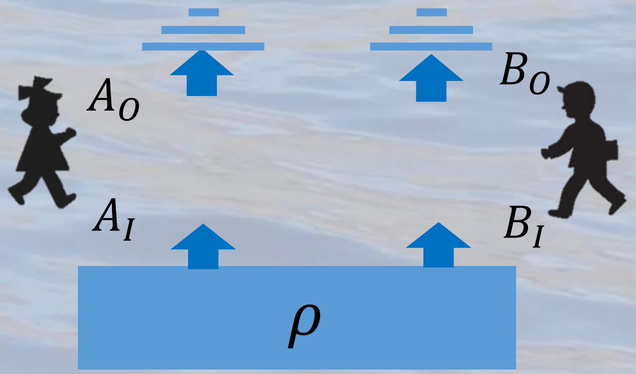



Example:

Shared state, no communication

Choi:

$$W^{A_I A_O B_I B_O} = \rho^{A_I B_I} \otimes \mathbb{I}^{A_0 B_O}$$



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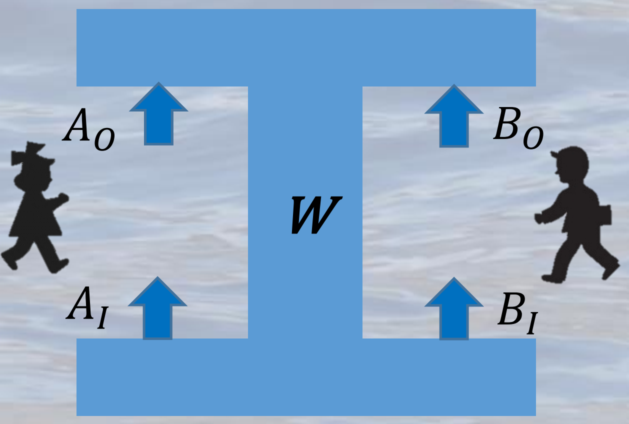
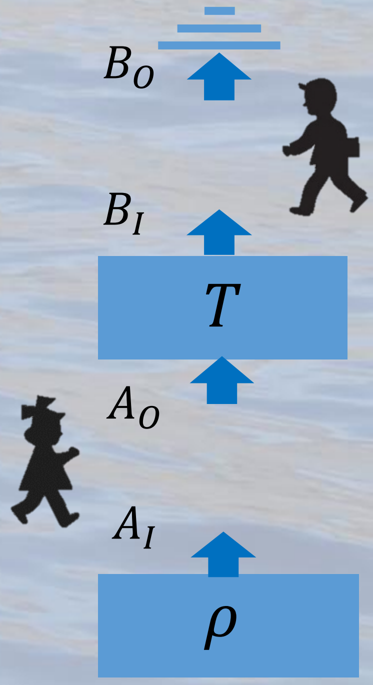



Example:

One way, $A \to B$, Markov

Choi:

$$W^{A_I A_O B_I B_O} = \rho^{A_I} \otimes T^{A_0 B_I} \otimes \mathbb{I}^{B_O}$$



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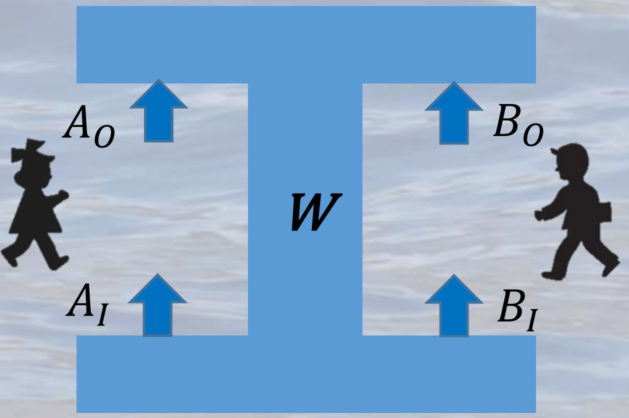
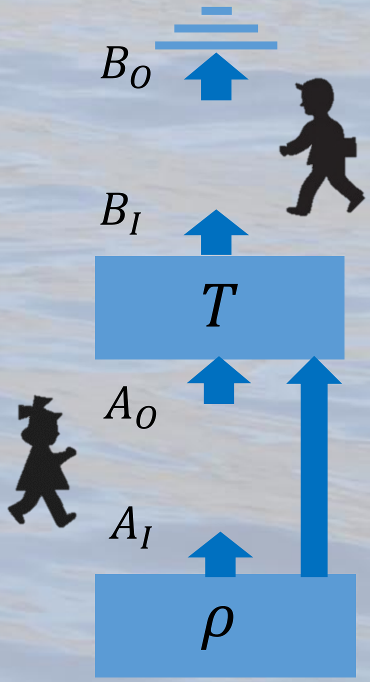



Example:

One way, $A \to B$, memory

Choi:

$W^{A_I A_O B_I B_O} = \widetilde{W}^{A_I A_0 B_I} \otimes \mathbb{I}^{B_O}$



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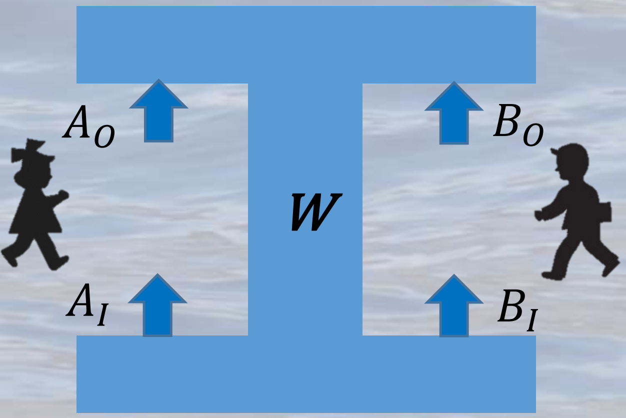



Example:

*Causally separable* process

Classical uncertainty of causal order

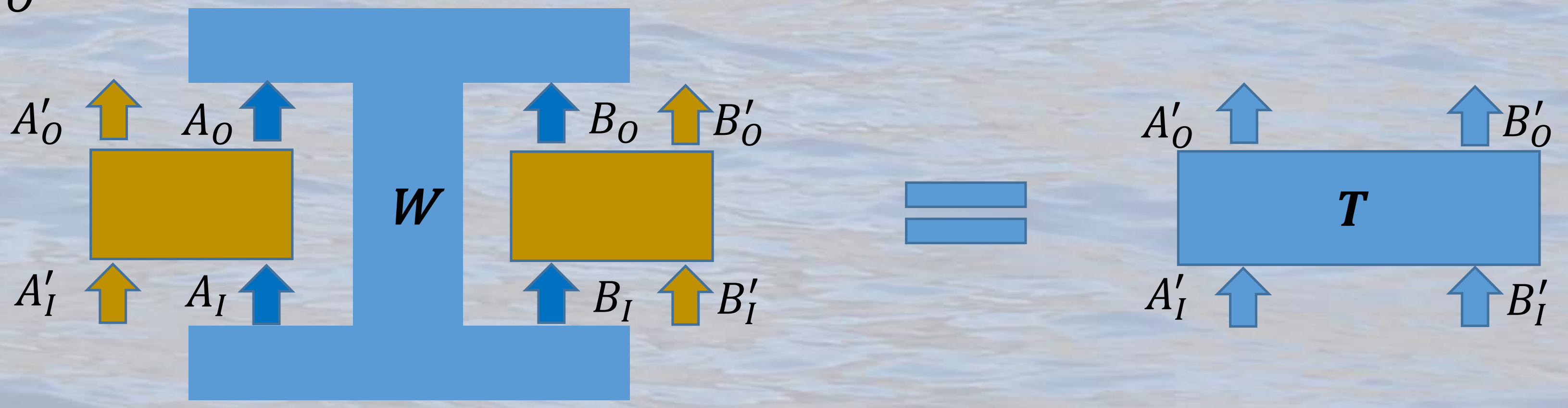
Choi:

$$W = p\, W^{A \prec B} + (1-p)\, W^{B \prec A}$$

# Communication through a process

Local operations with additional
quantum **inputs** $A_I', B_I'$
quantum **outputs**, $A_O', B_O'$

**Turn process into bipartite channel**




**Use channel for communication $\boldsymbol{A \leftrightarrow B}$**

Choi:

$$T^{A_I' B_I' A_O' B_O'} = \mathrm{Tr}[(M^{A_I A_I' A_O A_O'} \otimes M^{B_I B_I' B_O B_O'}) \cdot W^{A_I A_O B_I B_O}]$$

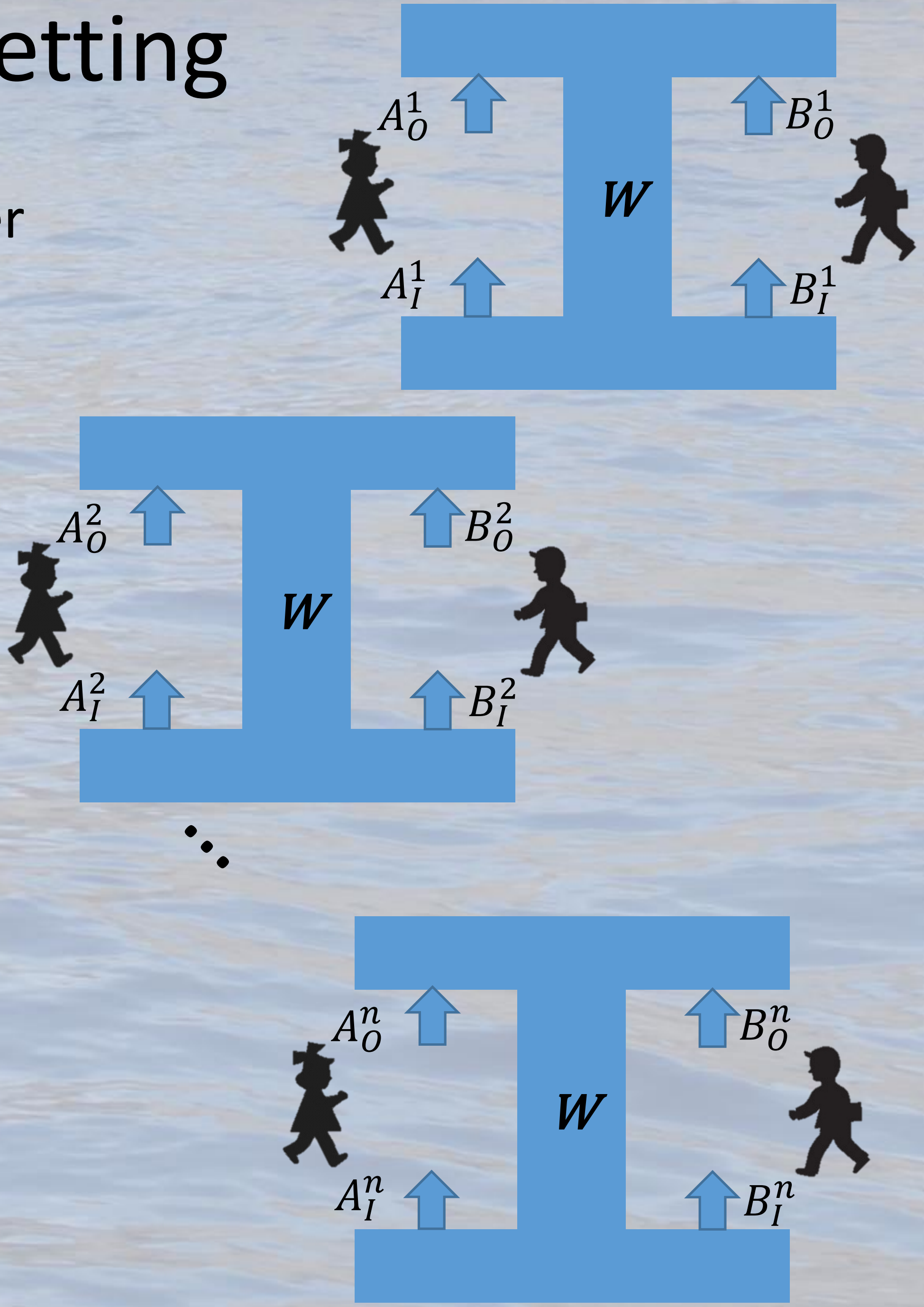
# Asymptotic setting

Multiple uses of resource without assigned causal order

Independent, identical processes:

$$W = W^1 \otimes \cdots \otimes W^n$$

**“party”**: single access to individual copy
**“agent”**: coordinated action of multiple parties



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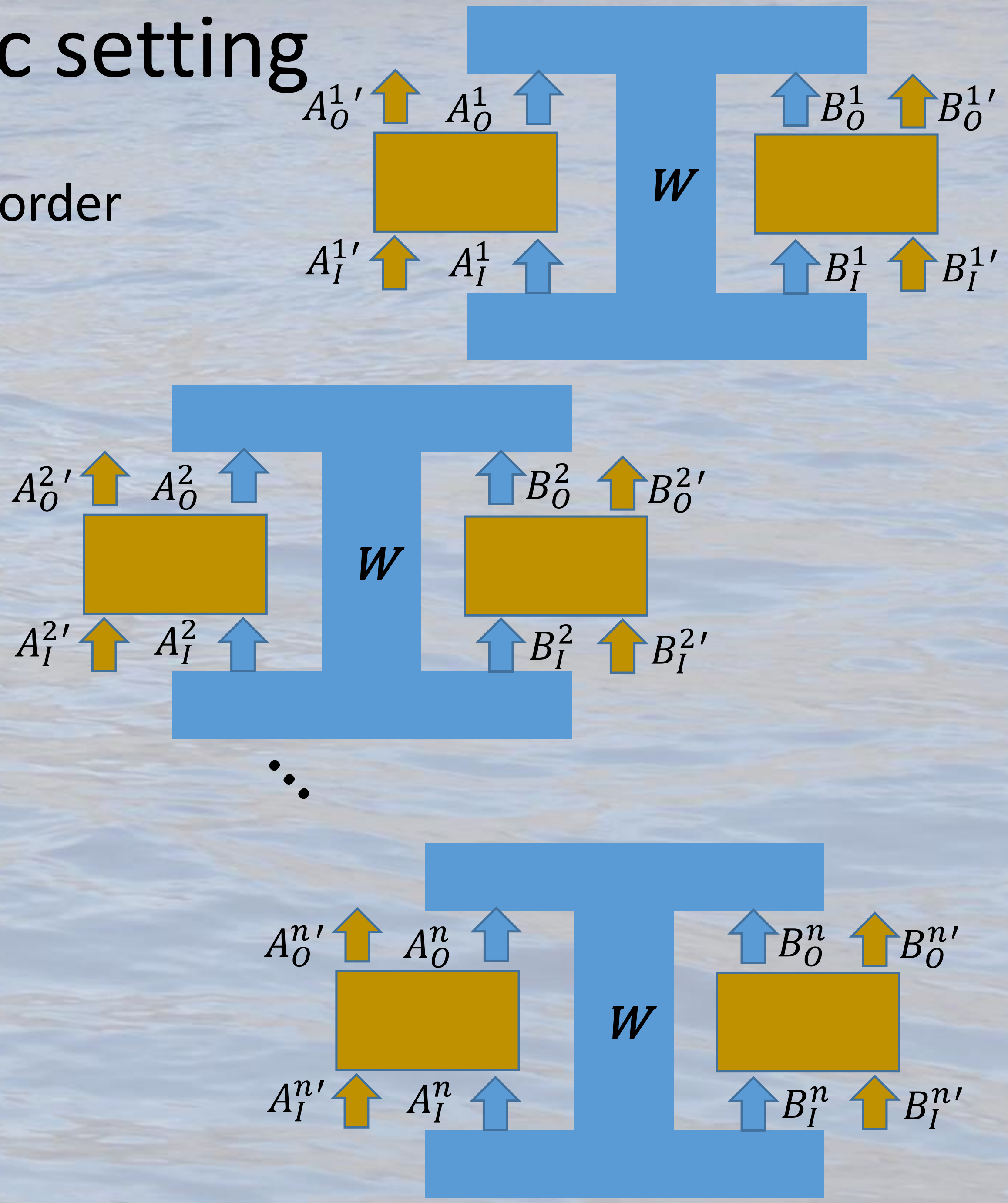
“basic” setting:

No prior causal relation among parties

$$M^A = M^{A^1} \otimes \cdots \otimes M^{A^n}$$

$$M^B = M^{B^1} \otimes \cdots \otimes M^{B^n}$$



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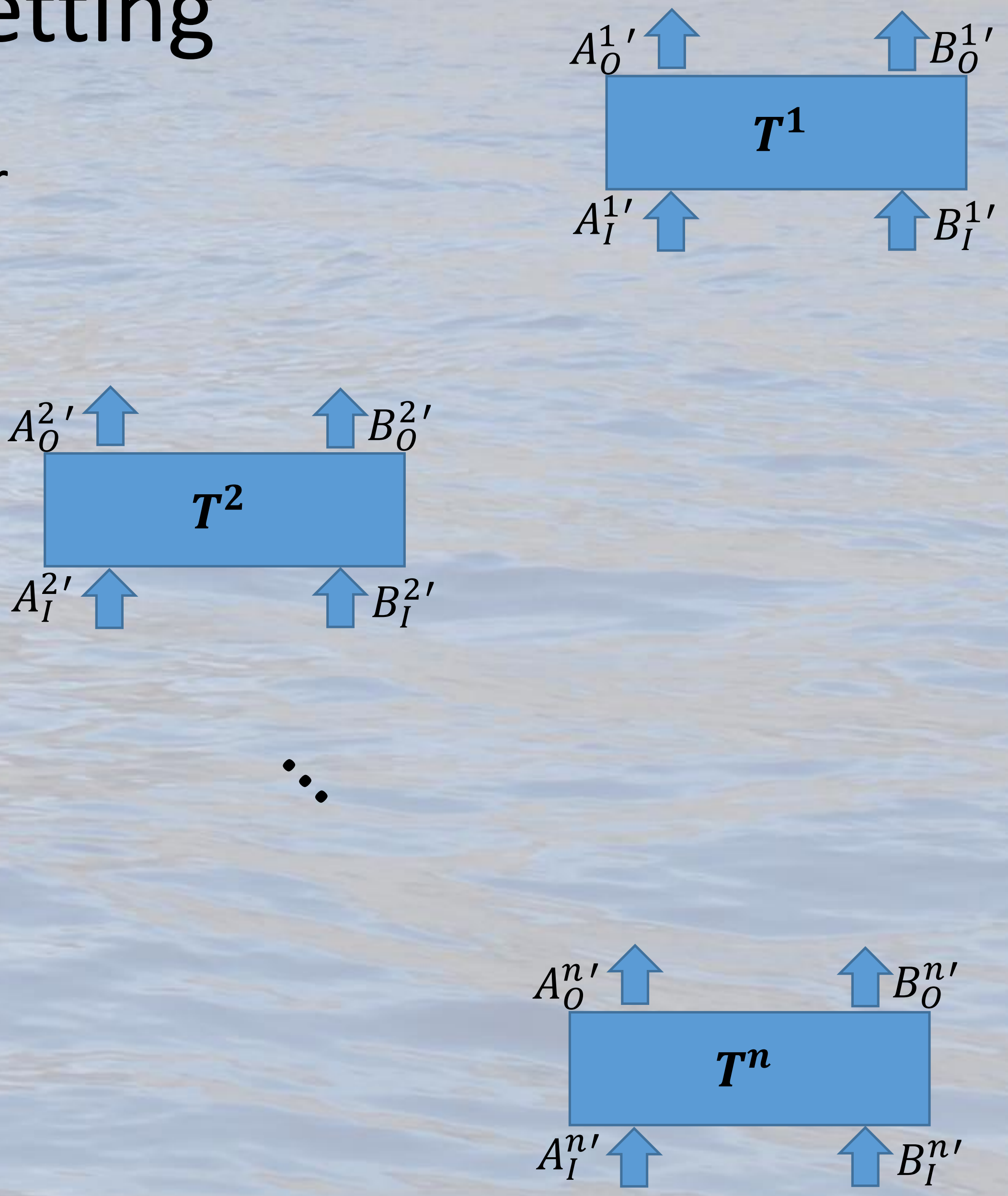
(Other, “causal order assisted” settings possible)



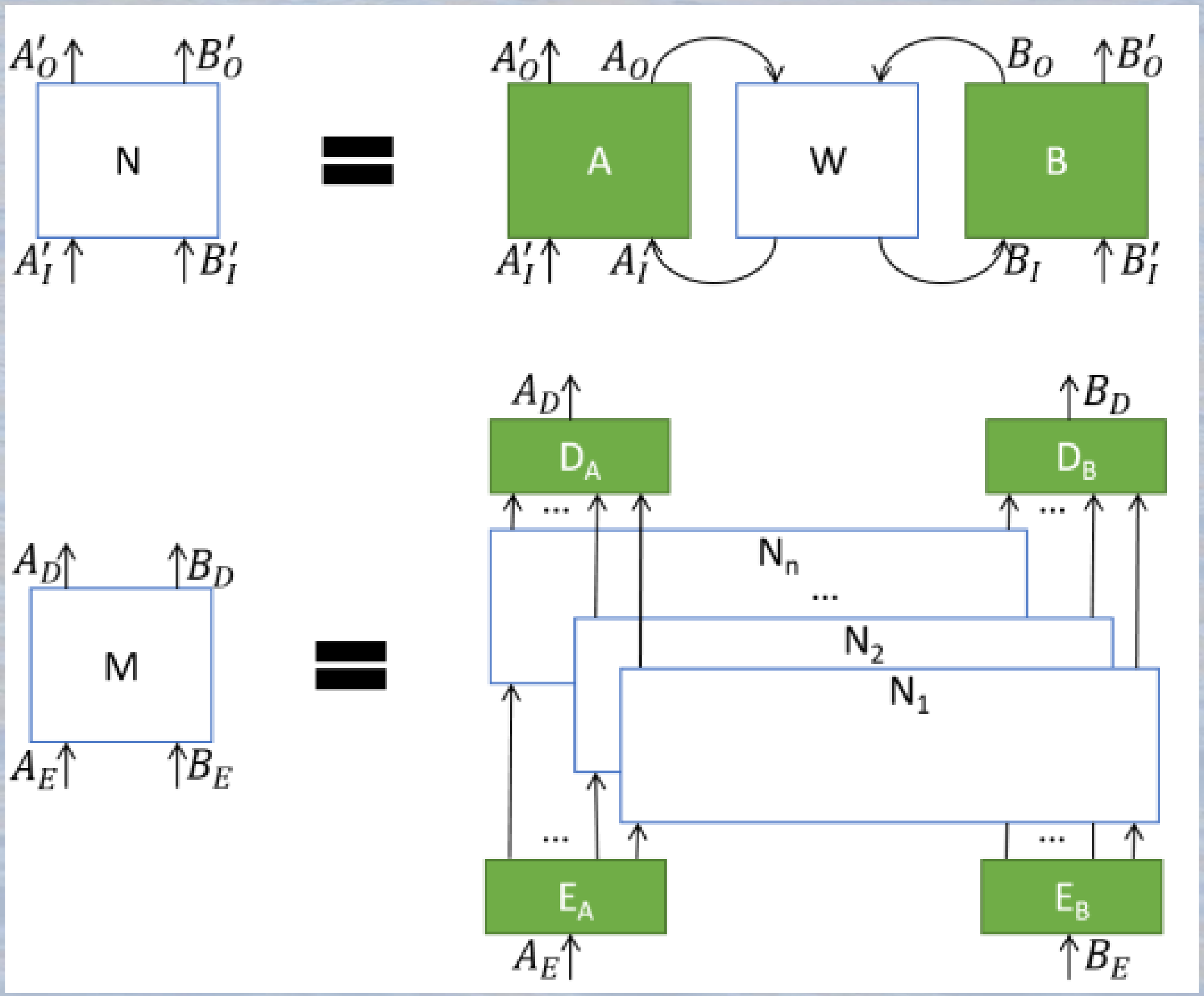
# Asymptotic setting

For communication, include global encoding/decoding within agents (not among agents)

Goal, $A \to B$ :
Approximate identity channel $A_E \to B^D$

Channel capacity:
Asymptotic rate #qubits/n

For classical capacity:
Replace
$A_E \to x, B^D \to b$
qubits $\to$ bits



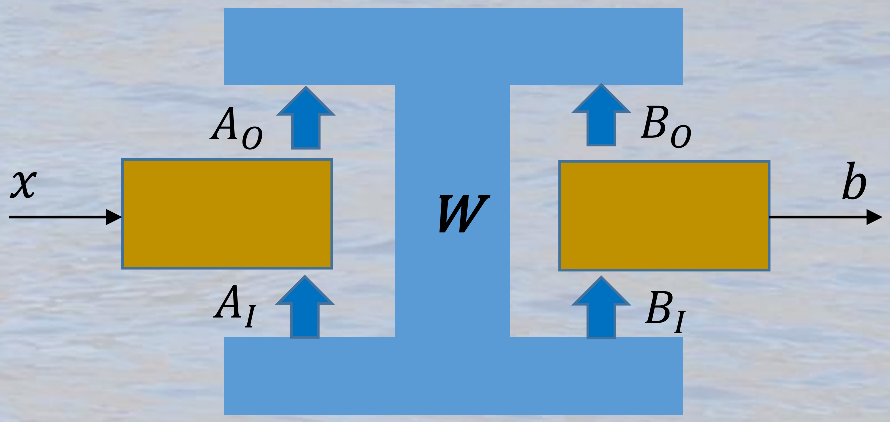
# Classical communication

**Single shot**

Encoding with probability $p_x$,

$P(x,b) = p_x \mathrm{Tr}\left[\left({M_{|x}}^{A_I A_O} \otimes {M_b}^{B_I B_O}\right) \cdot W^{A_I A_O B_I B_O}\right]$

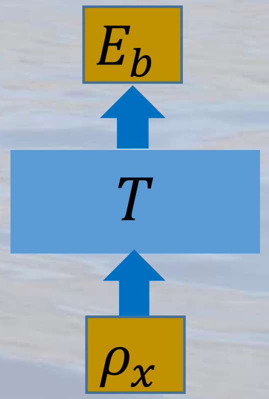
$M_{|x}, \sum_b M_b$ CPTP




Compare with

$P(x,b) = p_x \mathrm{Tr}[E_b\, \mathcal{T}(\rho_x)]$

$\rho_x$ states, $\sum_b E_b = \mathbb{I}$




Holevo bound on mutual information:

$$I(B{:}X) \leq S\left(\sum_x p_x \mathcal{T}(\rho_x)\right) - \sum_x p_x\, S(\mathcal{T}(\rho_x)) \leq \log d$$

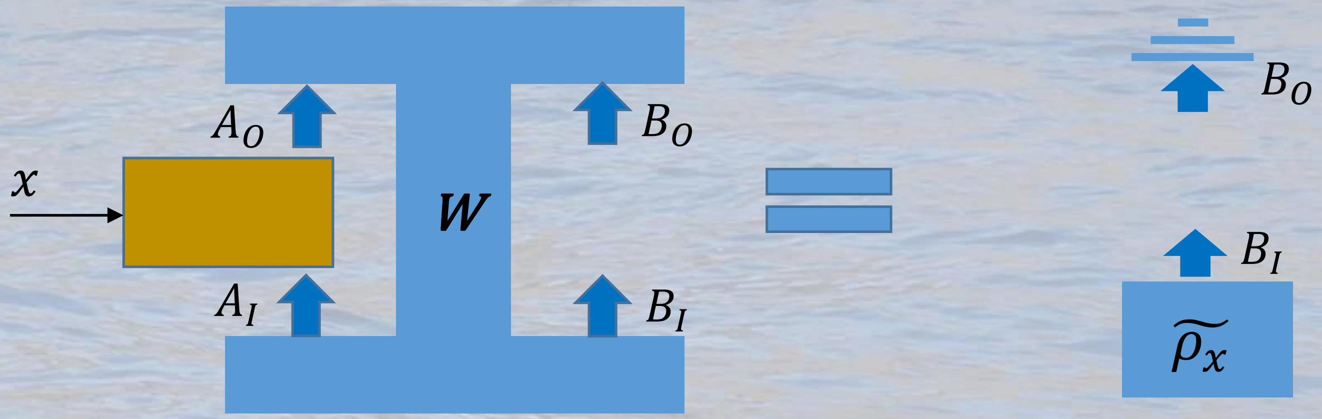
Can we beat it with general processes?

(note: include memory—shared entanglement)

$S(\rho) = -\mathrm{Tr}\rho \log \rho$

# Generalised Holevo bound

**Result:**

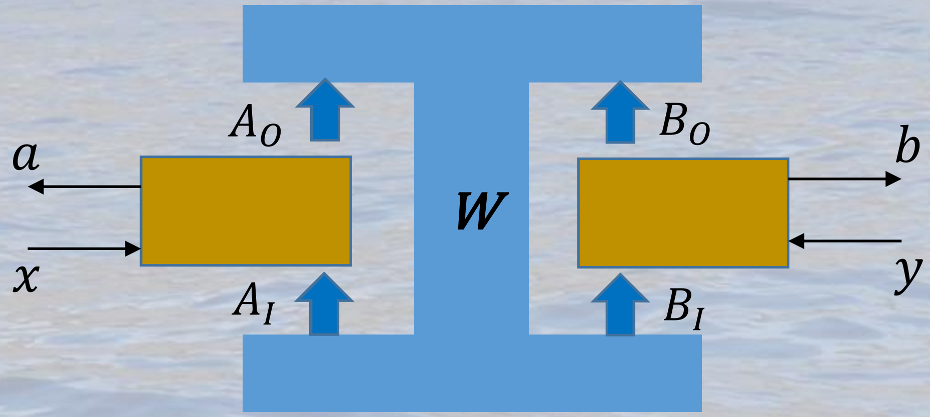



$$I(B:X) \leq S\left(\sum_x p_x \widetilde{\rho_x}\right) - \sum_x p_x \, S(\widetilde{\rho_x}) \leq \log d_{B_I}$$

Asymptotic classical capacity:

$$C^{A\to B}(W) \leq \log d_{B_I}$$

# Two-way communication




Classically uncertain order:
$W = p\, W^{A \prec B} + (1-p)\, W^{B \prec A}$

Dimension-dependent entropic causal inequality:

$$I(B{:}X) + I(A{:}Y) \leq \log d$$

No violation known

Cf Miklin et al., *New J. Phys.* **19**, 113041 (2017)

# Quantum communication with uncertain order

$$W = p\, W^{A \prec B} + (1-p)\, W^{B \prec A}$$

Quantum capacity $A \rightarrow B$:
Asymptotic rate $d_{A_E}/n$

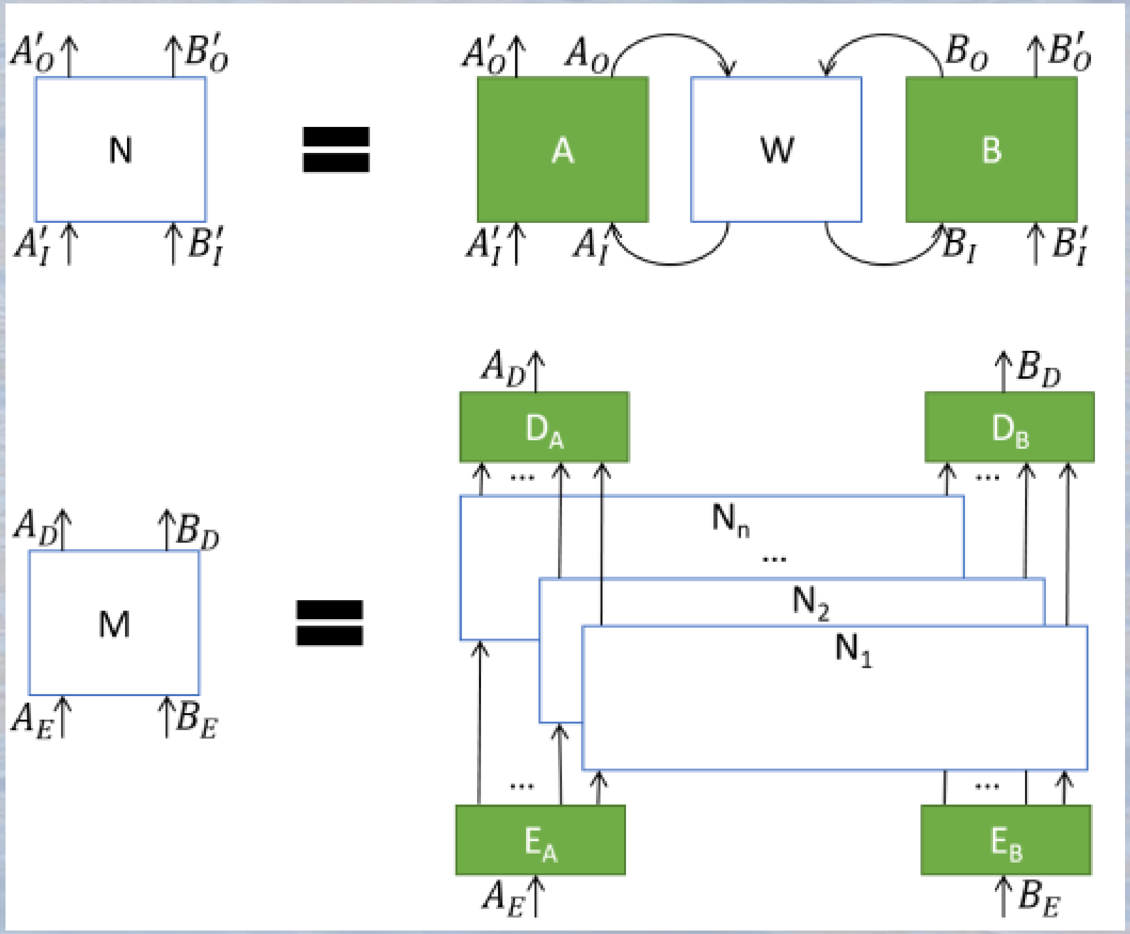
Results:

- $W^{B \prec A}$ can be replaced by white noise
- $W$ simulatable by erasure channel

$$\rho \rightarrow p\rho + (1-p)|e\rangle\langle e|$$

Quantum capacity known:

$$Q = \max\{0, (2p - 1)\log d\}$$



# Quantum communication with uncertain order

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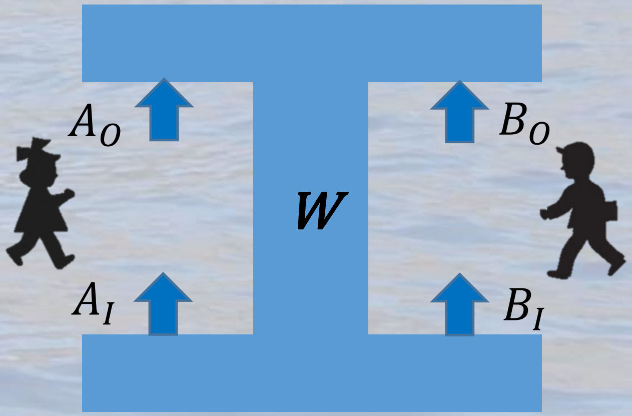


Quantum capacity $A \to B$:

$$Q \leq \max\{0, (2p-1)\log d_{B_O}\}$$

Consequences:

- $Q = 0$ if $p \leq \frac{1}{2}$
- Two-way quantum communication not possible

Includes all processes with physical interpretation (e.g., switch)

Unknown for more general processes

# Conclusions

- Communication tasks extend to non-fixed-order scenarios

- Classical communication:  no advantage in one-way capacity
  - Open: advantages for multipartite/combinations of capacities?

- Quantum communication requires $p > \frac{1}{2}$ "correct" order
  - Causal order resource for quantum communication
  - Open: two-way quantum communication using indefinite causal order?

- To explore: other communication settings
  (e.g., assisted by entanglement, causal order,…)