QUANTUM SHANNON THEORY WITH SUPERPOSITION OF CAUSAL ORDERS

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Causality in a Quantum World September 17-20 2019, Capri

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(CLASSICAL) SHANNON THEORY:

Claude E. Shannon

The carriers of information are classical:
classical states, classical channels,
classical configuration of the channels

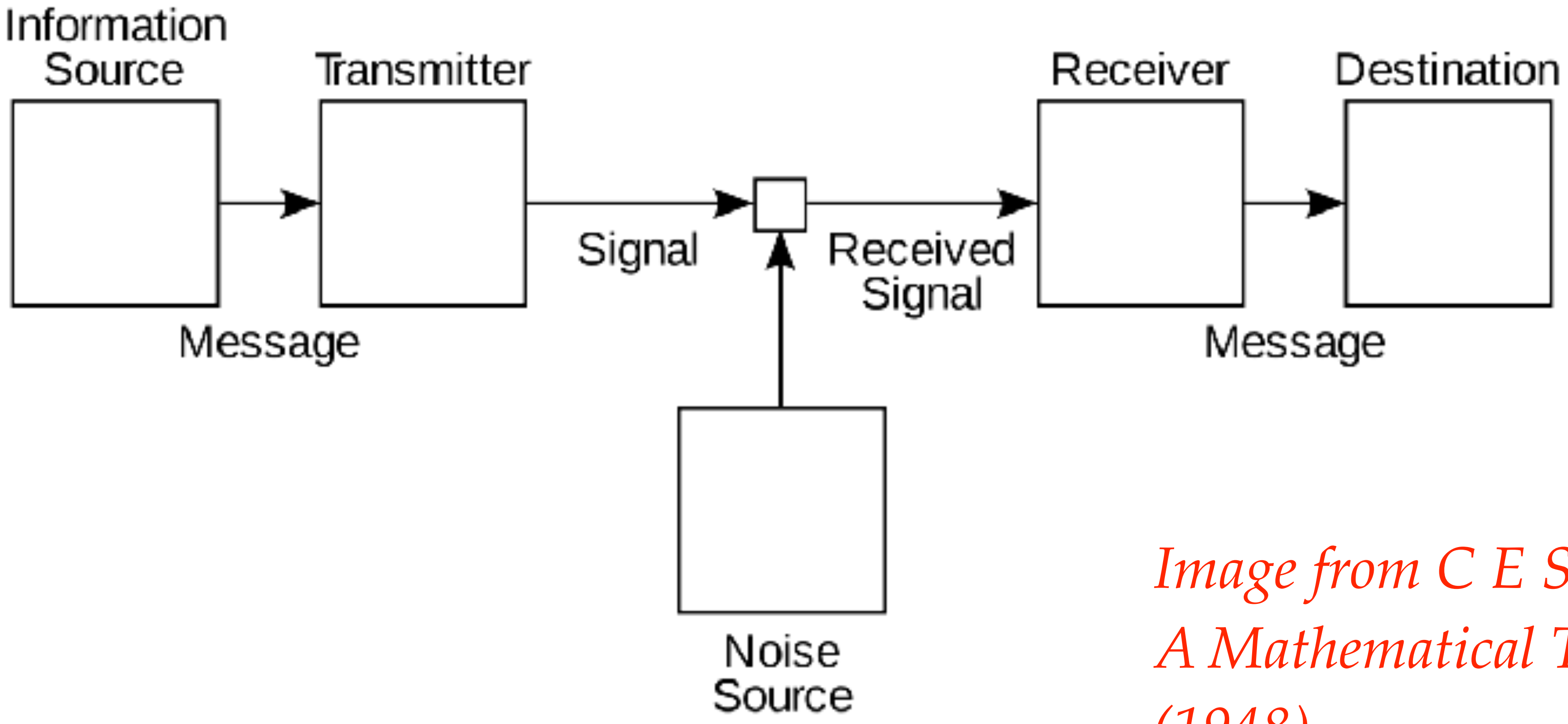


Image from C E Shannon,
A Mathematical Theory of Communication
(1948)

QUANTUM SHANNON THEORY

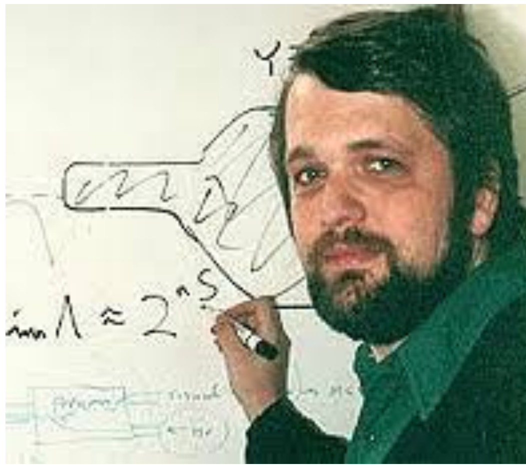
Allows the state of the information carriers and the channels be quantum.

Messages can be quantum:
not just strings of bits, like 0010110111,
but also quantum superpositions, like

$$|\Psi\rangle = \frac{|0010110111\rangle + |1010100011\rangle}{\sqrt{2}}$$

Still, the configuration of the communication channels is fixed.

Alexander Holevo



Benjamin Schumacher

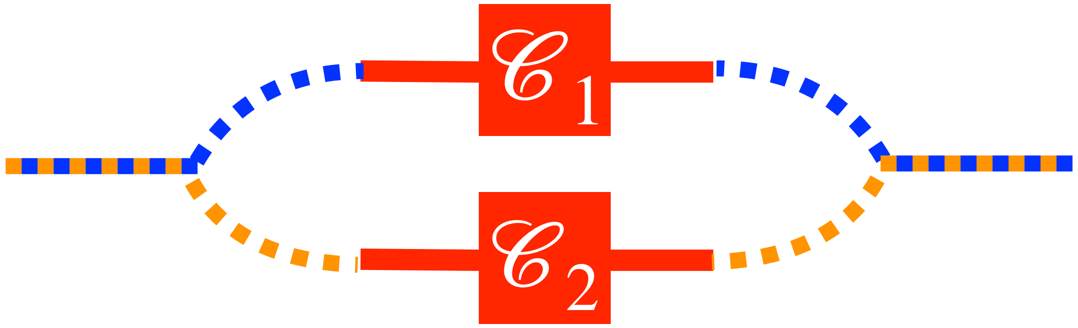
Charles Bennett

Gilles Brassard

QUANTUM CONFIGURATIONS

Quantum theory allows quantum configurations of the communication channels

- **Example 1**: message in a superposition of going through one communication channel or another

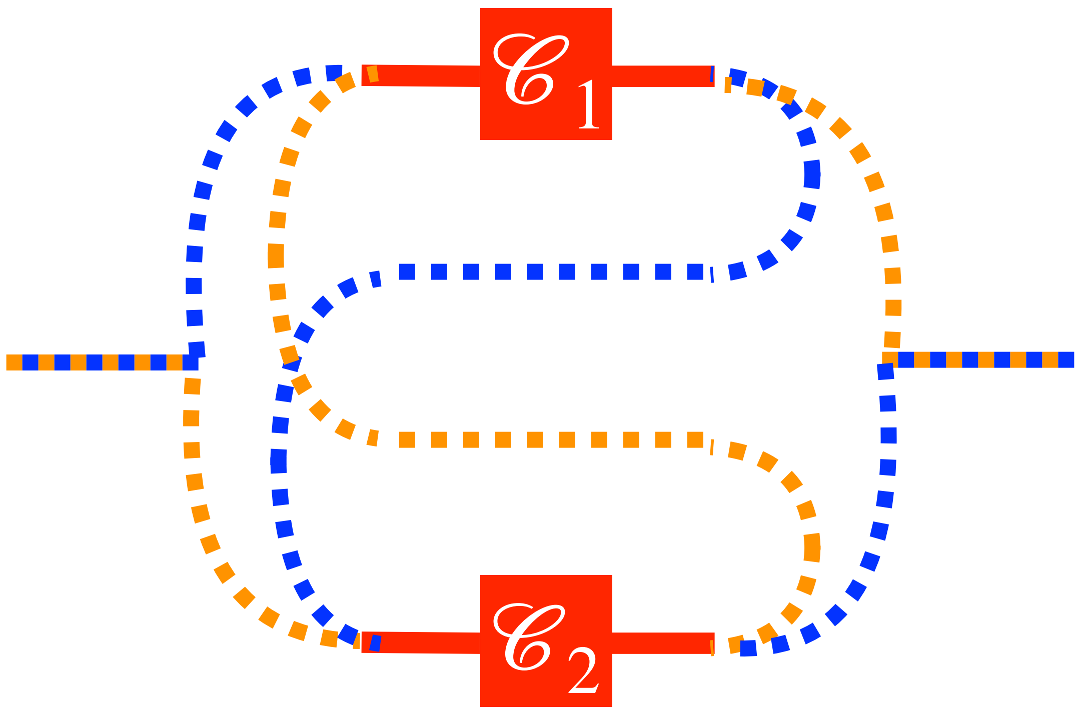


Resource: channels $\mathscr{C}_1$ and $\mathscr{C}_2$ that can act coherently on the message and on the vacuum

Aharonov, Anandan, Popescu, Vaidman, PRL 64, 2965 (1990).
Oi, PRL 91, 067902 (2003)
Gisin, Linden, Massar, Popescu PRA 72, 012338 (2005)
Abbott, Wechs, Horsman, Mhalla, Branciard, arXiv:1810.09826
Chiribella and Kristjánsson, Proc. Royal Soc. A 475, 20180903 (2019)

QUANTUM CONFIGURATIONS

- **Example 2 (this talk)**:
  message traversing two channels in a superposition of two alternative causal orders



Chiribella, D'Ariano, Perinotti, Valiron, arXiv:0912.0195; Phys. Rev. A 88, 022318 (2013)
Ebler, Salek, Chiribella, Phys. Rev. Lett. 120, 120502 (2018)
K. Goswami, J. Romero, A. White, arXiv:1807.07383
Salek, Ebler, Chiribella, arXiv:1809.06655
Chiribella, Banik, Bhattacharya, Guha, Alimuddin, Roy, Saha, Agrawal, Kar, arXiv:1810.10457

QUANTUM SHANNON THEORY
AS
A RESOURCE THEORY

RESOURCE THEORIES

General framework to capture various notions of resource

Mathematical framework set by Coecke, Fritz, and Spekkens:
- set of operations **Op**,
  closed under parallel and sequential composition
- subset of *free* operations **FreeOp**,
  also closed under parallel and sequential composition

What is not free is a resource.

Coecke, Fritz, Spekkens, Information and Computation 250, 59 (2016).

RESOURCE THEORIES OF COMMUNICATION

Objects of interest: communication devices, mathematically described by quantum channels

- Kraus representation: $$\mathcal{C}(\rho) = \sum_i C_i \, \rho \, C_i^\dagger$$

- Normalization: $$\sum_i C_i^\dagger \, C_i = I_A$$

Operations of interest: transformations of quantum channels

SUPERMAPS

Operations that transform *channels into channels* are known as **supermaps**

Chiribella, D'Ariano, Perinotti, EPL 83, 30004 (2008);
Phys. Rev. A 80, 022339 (2009);
Phys. Rev. A 88, 022318 (2013)

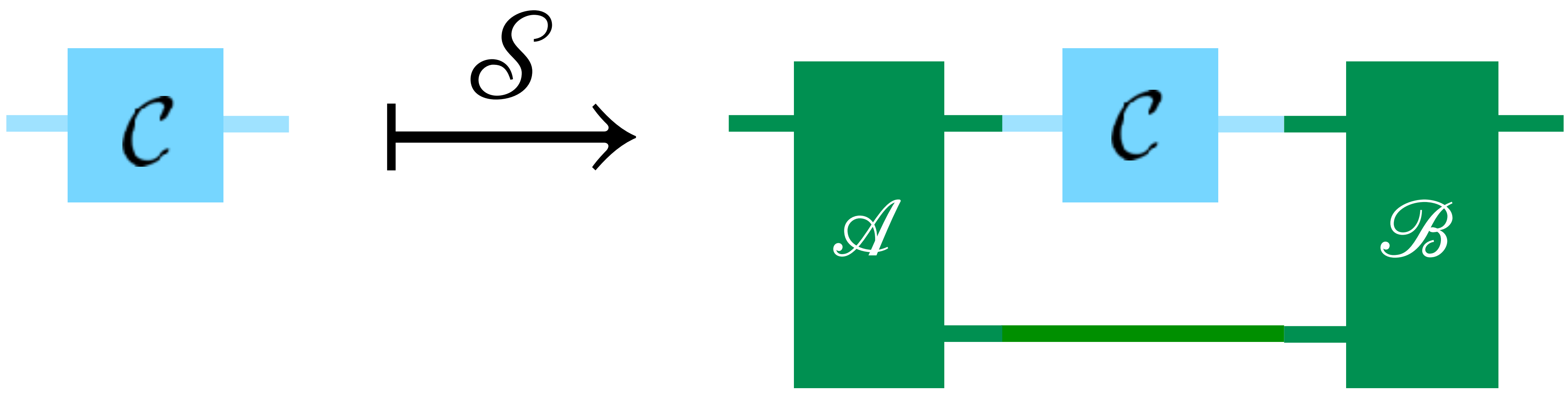
Supermap:
higher order
transformation

Supermap = linear, completely positive,
and normalization-preserving map

EXAMPLE

Supermaps from a *single channel to a single channel* are easy to characterize.

They have all the form
“sandwitch the input channel between two fixed channels”



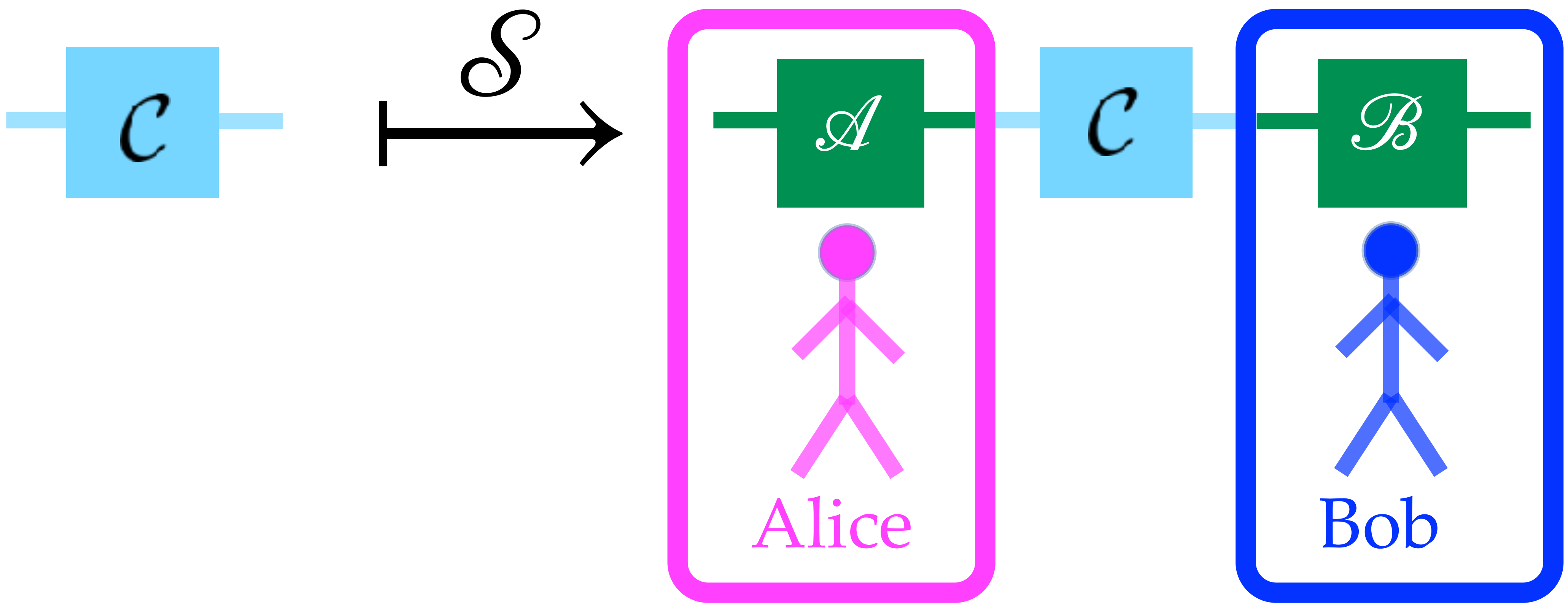
Chiribella, D’Ariano, Perinotti, EPL 83, 30004 (2008);
Phys. Rev. A 80, 022339 (2009)

FREE SUPERMAPS

Supermaps define *all* possible operations on channels, (cf. the set **Op** in the Coecke-Fritz-Spekkens scheme).

What about the *free* operations?

In the case of a single channel, the standard choice is to take **FreeOp** to be supermaps without internal memory.

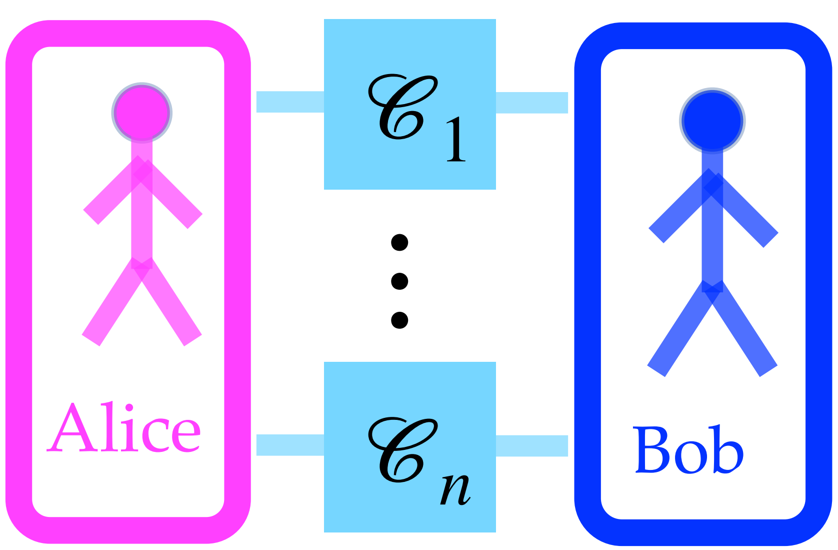


MULTIPLE CHANNELS

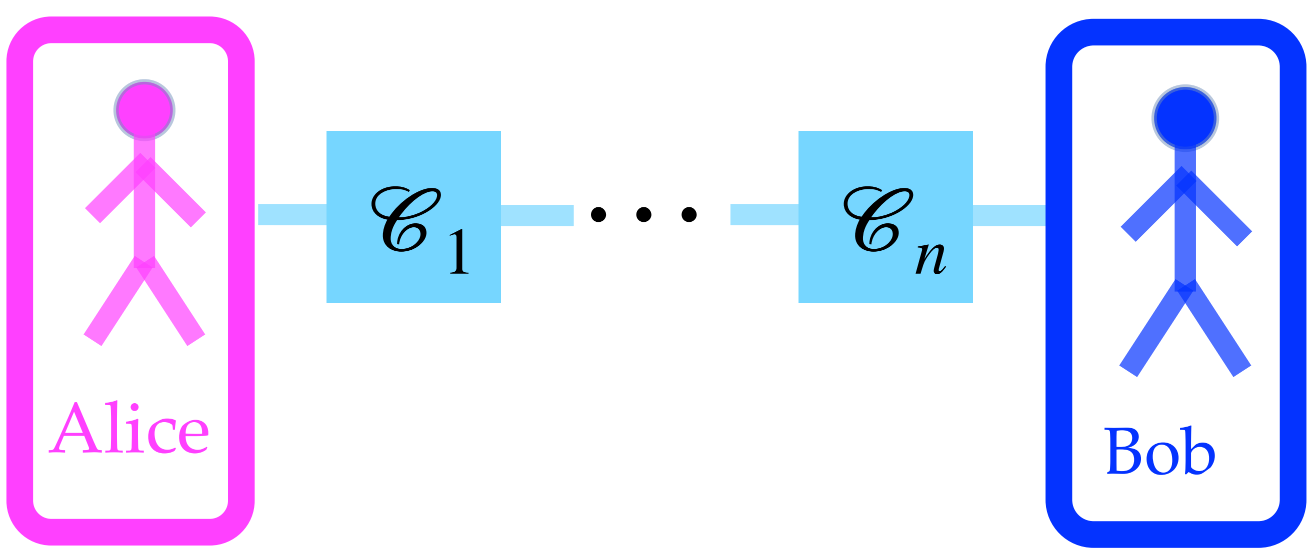
Suppose that the communication between Alice and Bob uses n communication devices, corresponding to channels $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n$

To specify a communication protocol, we need first to know *how the devices are placed between Alice and Bob.*

e.g. parallel placement



vs sequential placement



PLACEMENTS

A *placement* is a supermap transforming a list of channels $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n$ into a communication channel between Alice and Bob

e.g. parallel placement:

$$(\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n) \mapsto \mathscr{C}_1 \otimes \mathscr{C}_2 \otimes \cdots \otimes \mathscr{C}_n$$

e.g. sequential placement with repeaters:

$$(\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n) \mapsto \mathscr{C}_n \circ \mathscr{R}_{n-1} \circ \mathscr{C}_{n-1} \circ \cdots \mathscr{C}_2 \circ \mathscr{R}_1 \circ \mathscr{C}_1$$

Operationally, the placement is implemented by a *communication provider.*

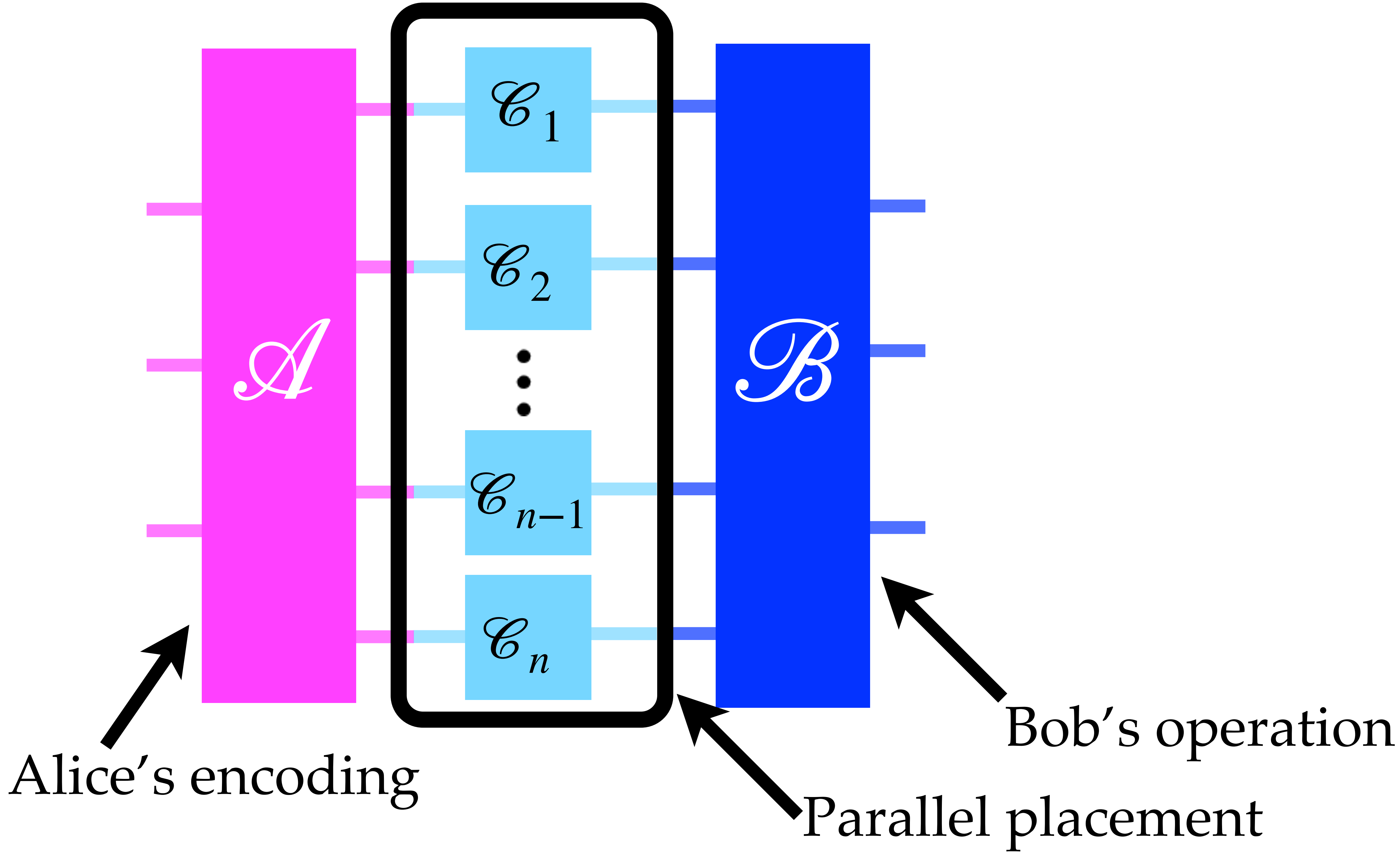
RESOURCE THEORIES OF COMMUNICATION

- Possible operations **Op** = set of all supermaps on lists of channels $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n$
- Free operations **FreeOp** = given set of placements + encoding/decoding

Goal: use the n channels "in the best possible way"
i.e. find the free operation $\mathscr{S} \in$ **FreeOp**
such that the channel $\mathscr{S}(\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n)$
transmits the maximum amount of information.

EXAMPLE 1: STANDARD QUANTUM SHANNON THEORY OF DIRECT COMMUNICATION

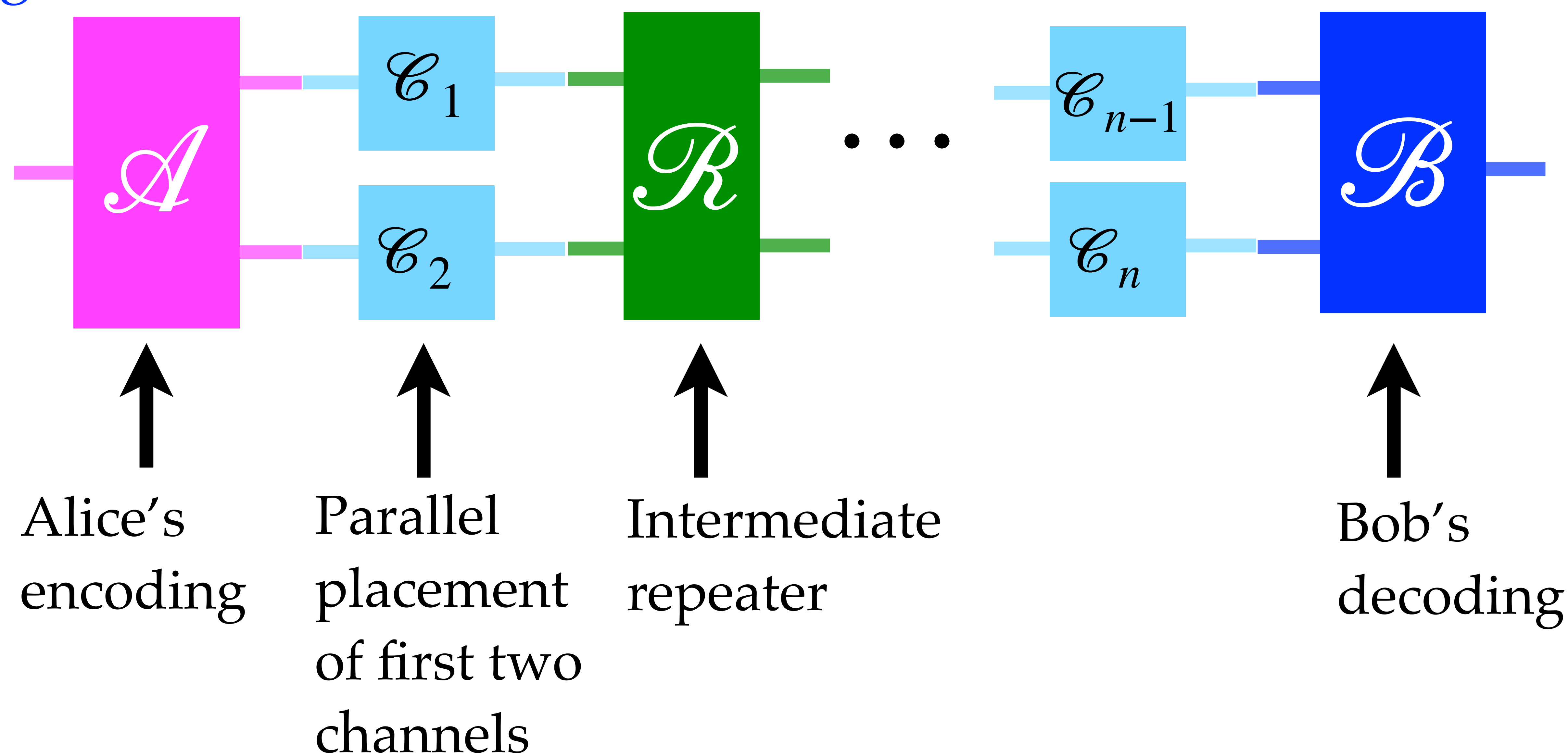
Allowed placements: **only parallel**



EXAMPLE 2: STANDARD QUANTUM SHANNON THEORY (NETWORK COMMUNICATION)

Allowed placements: **all combinations of parallel and sequential placements**

e.g.



AN ADDITIONAL RESOURCE:

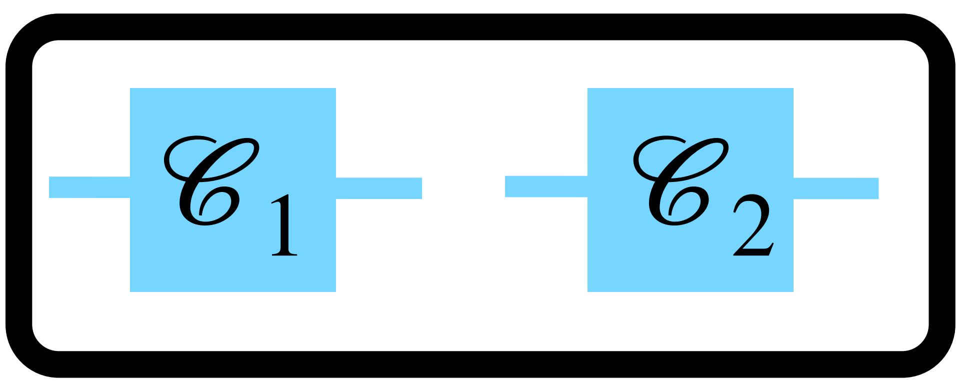
SUPERPOSITION
OF
CAUSAL ORDERS

THE QUANTUM SWITCH

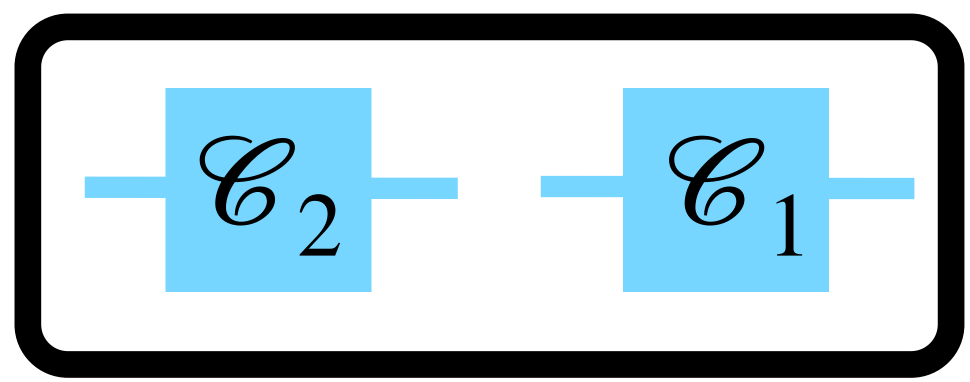
The quantum SWITCH is the supermap that

takes as input the two channels $\mathcal{C}_1$ and $\mathcal{C}_2$
(with the same input and output)

and places them in a coherent superposition of the two configurations



and



The two configurations are flagged by two orthogonal states of an "order qubit", which controls the causal order.

Chiribella, D'Ariano, Perinotti, Valiron, arXiv:0912.0195; Phys. Rev. A 88, 022318 (2013),

THE QUANTUM SWITCH, EXPLICITLY

Input channels:

$$\mathscr{C}_1(\rho) = \sum_i C_{1i} \rho C_{1i}^\dagger \quad \text{and} \quad \mathscr{C}_2(\rho) = \sum_j C_{2j} \rho C_{2j}^\dagger$$

NB: definition of q-switch ***independent of Kraus representations*** for $\mathscr{C}_1$ and $\mathscr{C}_2$

Output channel:

$$\mathscr{S}(\mathscr{C}_1, \mathscr{C}_2)(\rho) = \sum_{i,j} S_{ij} \rho S_{ij}^\dagger$$

$$S_{ij} := \alpha\, C_{1i} \otimes C_{2j} \otimes |0\rangle + \beta\, C_{2j} \otimes C_{1i} \otimes |1\rangle$$

acting on system in the first time slot

acting on system in the second time slot

$$|\alpha|^2 + |\beta|^2 = 1$$

QUANTUM SHANNON THEORY WITH THE ASSISTANCE OF THE QUANTUM SWITCH

Idea: add the quantum SWITCH
to the set of allowed placements.

This gives us a new resource theory of comunication:

Input resource: list of channels $\mathscr{C}_1, \mathscr{C}_2, \ldots, \mathscr{C}_n$

Free operations, generated from
(1) parallel placement
(2) sequential placement
(3) quantum SWITCH
(4) encoding/decoding

NO SIDE-CHANNEL GENERATION

Isn't adding the quantum SWITCH a way of cheating?

Yes, it is, but an interesting way.

The switch ***does not generate side-channels,***
in the following sense:

A supermap $\mathcal{S}$ *generates a side-channel* if there exists encoding/decoding operations such that the channel $\mathscr{B} \circ \mathcal{S}(\mathscr{C}_1, \ldots, \mathscr{C}_n) \circ \mathscr{A}$ is independent of the input channels $\mathscr{C}_1, \ldots, \mathscr{C}_n$ and can transmit information.

CAUSAL ACTIVATION
OF
THE CLASSICAL CAPACITY

Ebler, Salek, Chiribella, Phys. Rev. Lett. 120, 120502 (2018)
with hindsights from
Chiribella and Kristjánsson, Proc. Royal Soc. A 475, 20180903 (2019)

TWO ERASURE CHANNELS

Suppose we are given **two erasure channels**

$$\mathscr{C}_1 = \mathscr{C}_2 \quad \text{with} \quad \mathscr{C}_1(\rho) = \mathscr{C}_2(\rho) = \mathrm{Tr}[\rho]\,\rho_0$$

In the standard quantum Shannon theory,
these two channels are useless,
no matter if we combine them in parallel, in sequence, or sandwiched between encoding and decoding operations.

In particular, the **classical capacity is 0 bits.**

EXPLICITLY

Input channels: $\mathscr{C}_1(\rho) = \mathscr{C}_2(\rho) = \mathrm{Tr}[\rho]\,\rho_0$

Output channel:
assuming control qubit in the state $|\psi\rangle = \sqrt{p}\,|0\rangle + \sqrt{1-p}\,|1\rangle$

$$
\begin{aligned}
\mathscr{S}(\mathscr{C}_1, \mathscr{C}_2)(\rho) = {} & p\,\rho_0 \otimes |0\rangle\langle 0| \\
& + (1-p)\,\rho_0 \otimes |1\rangle\langle 1| \\
& \boxed{\begin{aligned} & + \sqrt{p(1-p)}\,\rho_0\rho\rho_0 \otimes |0\rangle\langle 1| \\ & + \sqrt{p(1-p)}\,\rho_0\rho\rho_0 \otimes |1\rangle\langle 0| \end{aligned}}
\end{aligned}
$$

CAUSAL ACTIVATION

Two useless channels, combined by the quantum SWITCH, become **useful for classical communication.**

When placed in a superposition of time configurations, the two erasure channels *are* a resource. *Causal activation:* activation by means of causal order

(although similar phenomena may arise in other scenarios
cf. P. A. Guérin, G. Rubino, and Č. Brukner, Phys. Rev. A 99, 062317 (2019).)

Seemingly paradoxical: the two channels are **identical.**
Classically, randomizing the order of two identical channels cannot give any advantage.
Quantumly, it does.

HERALDED
QUANTUM COMMUNICATION
IN
THE QUANTUM SWITCH

Salek, Ebler, Chiribella, arXiv:1809.06655

TWO DEPHASING CHANNELS

Consider the two dephasing channels

$$\mathscr{C}_1(\rho) = (1-p)\,\rho + p\,X\rho X$$

$$\mathscr{C}_2(\rho) = (1-q)\,\rho + q\,Z\rho Z$$

When placed in a superposition of causal orders, the two channels $\mathscr{C}_1$ and $\mathscr{C}_2$ yield the new channel

$$\mathcal{S}(\mathscr{C}_1, \mathscr{C}_2)(\rho) =$$

$$\Big[(1-p)(1-q)\,\rho + p(1-q)\,X\rho X + q(1-p)\,Z\rho Z\Big] \otimes |+\rangle\langle+|$$

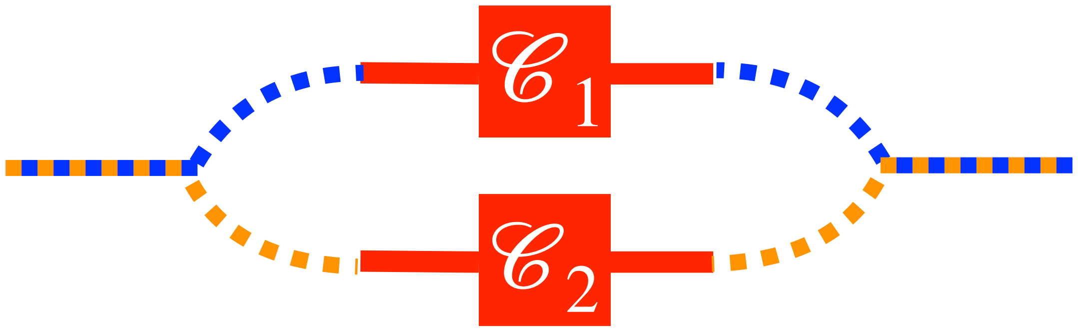
$$\boxed{+pq\,Y\rho Y \otimes |-\rangle\langle-|}$$

HERALDED COMMUNICATION

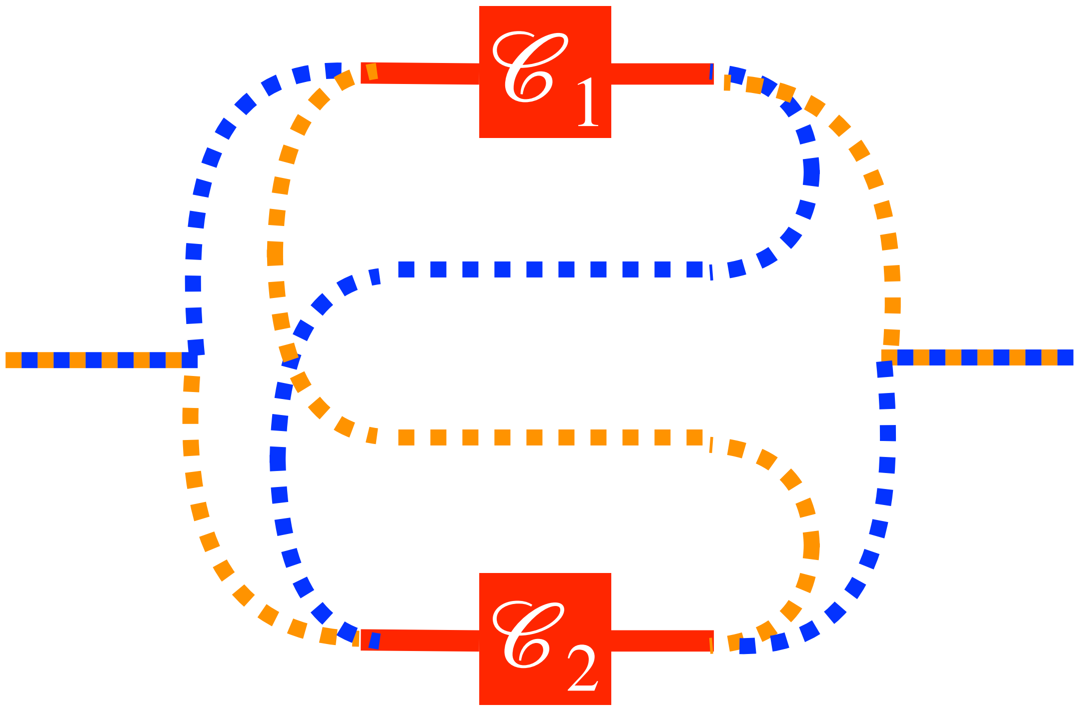
The quantum SWITCH offers a
heralded noiseless channel with probability *pq*

- can be used to implement BB84 or other key distribution protocols.

- this heralded removal of the noise does not take place when independent channels are used in alternative



VS



PERFECT ACTIVATION
OF
THE QUANTUM CAPACITY

Chiribella, Banik, Bhattacharya, Guha, Alimuddin, Roy, Saha, Agrawal, Kar

arXiv:1810.10457

TWO ENTANGLEMENT-BREAKING CHANNELS

Consider the two entanglement-breaking channels

$$\mathscr{C}_1(\rho) = \mathscr{C}_2(\rho) = \frac{1}{2}\, X\rho X + \frac{1}{2}\, Y\rho Y$$

These two channels cannot transmit any quantum information.

When placed in a superposition of causal orders,
the two channels $\mathscr{C}_1$ and $\mathscr{C}_2$ yield the new channel

$$\mathcal{S}(\mathscr{C}_1, \mathscr{C}_2)(\rho) = \frac{1}{2}\, \rho \otimes |+\rangle\langle+| + \frac{1}{2} Y\rho Y \otimes |-\rangle\langle-|$$

which can transmit a qubit ***perfectly and deterministically.***

Agan, perfect removal of the noise *not possible for superposition of independent channels.*

CONCLUSIONS

TAKE HOME MESSAGES

- The properties of quantum channels depend **not only on the way the channels are defined**, but also on t**he way they are placed** between the sender and the receiver.

- Quantum Shannon Theory With Assistance of Quantum Switch: including placement of channels in superposition of orders

- Examples of advantages over standard quantum Shannon theory.

POSTDOC POSITION AT HKU CS/OXFORD CS

A 2-years postdoc position in quantum information is about to be advertised jointly at the Computer Science Department of The University of Hong Kong.

Contact me if interested!
giulio@cs.hku.hk or giuliochiribella@cs.ox.ac.uk