

Time reference frames and gravitating quantum clocks

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Causality in the Quantum World
Anacapri, September 2019



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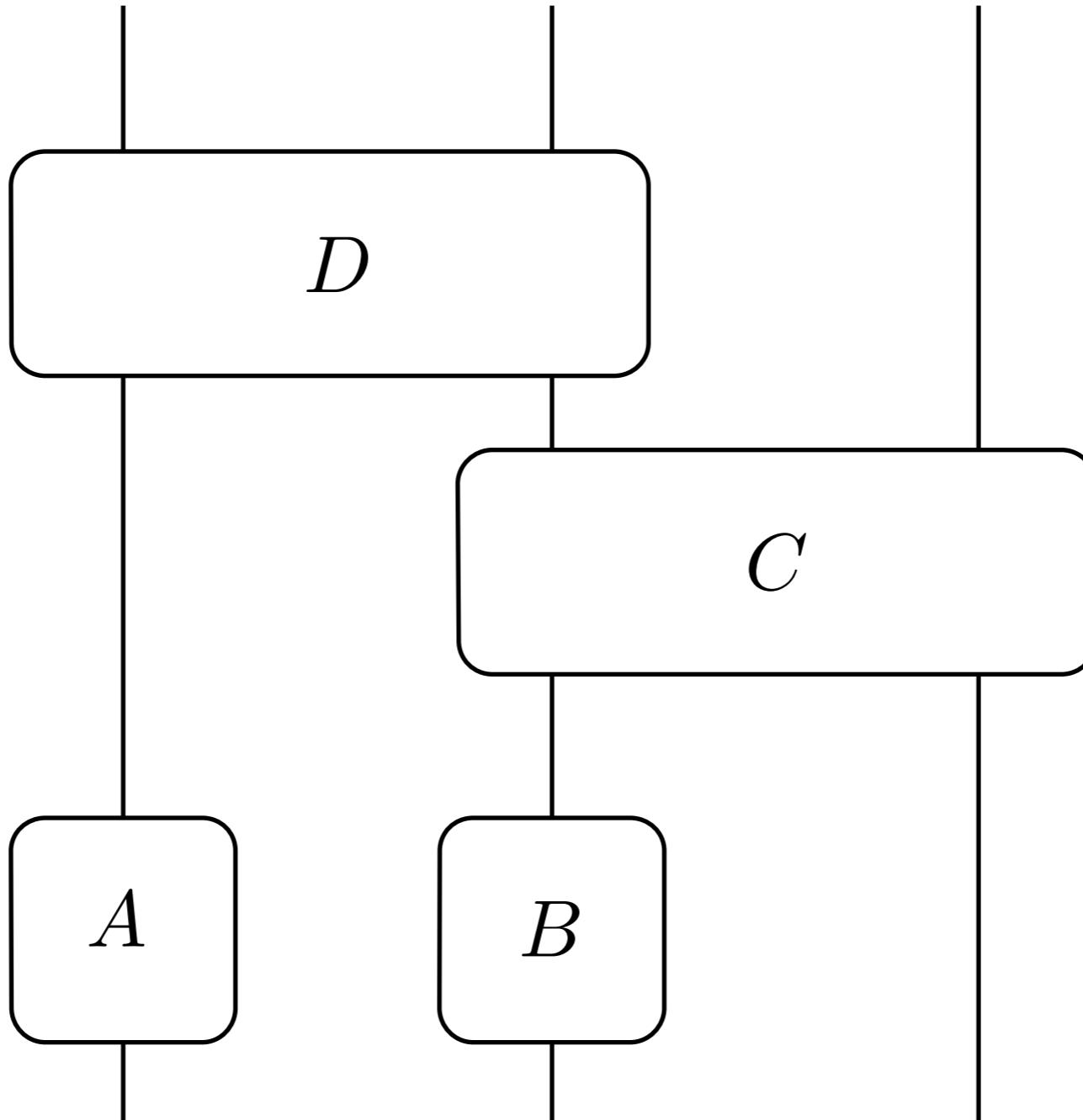


Outline

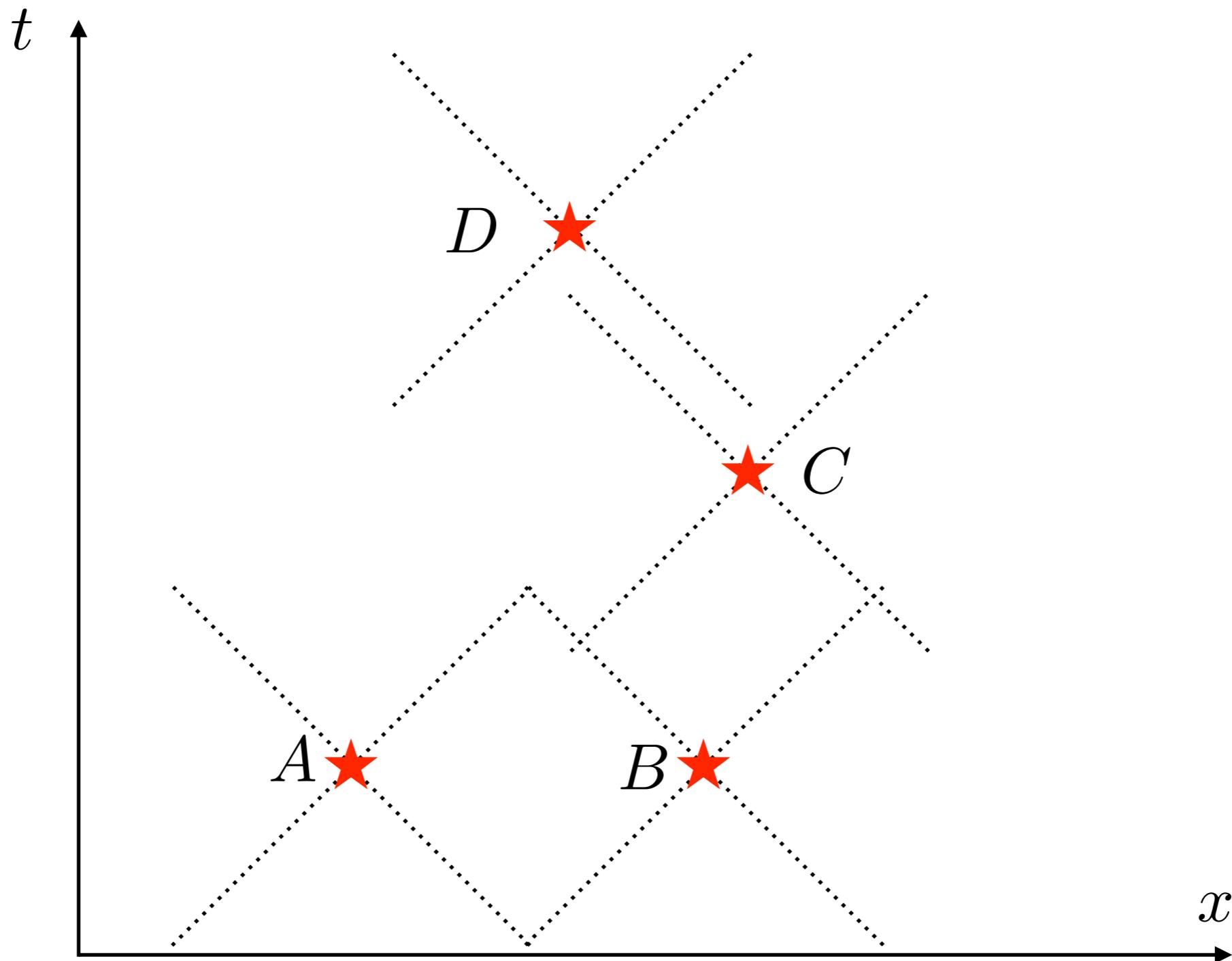
1. Events (introduction)
2. Gravitating quantum clocks (motivation)
3. Quantum clocks as temporal reference frames
4. Outlook

1 . Events (introduction)

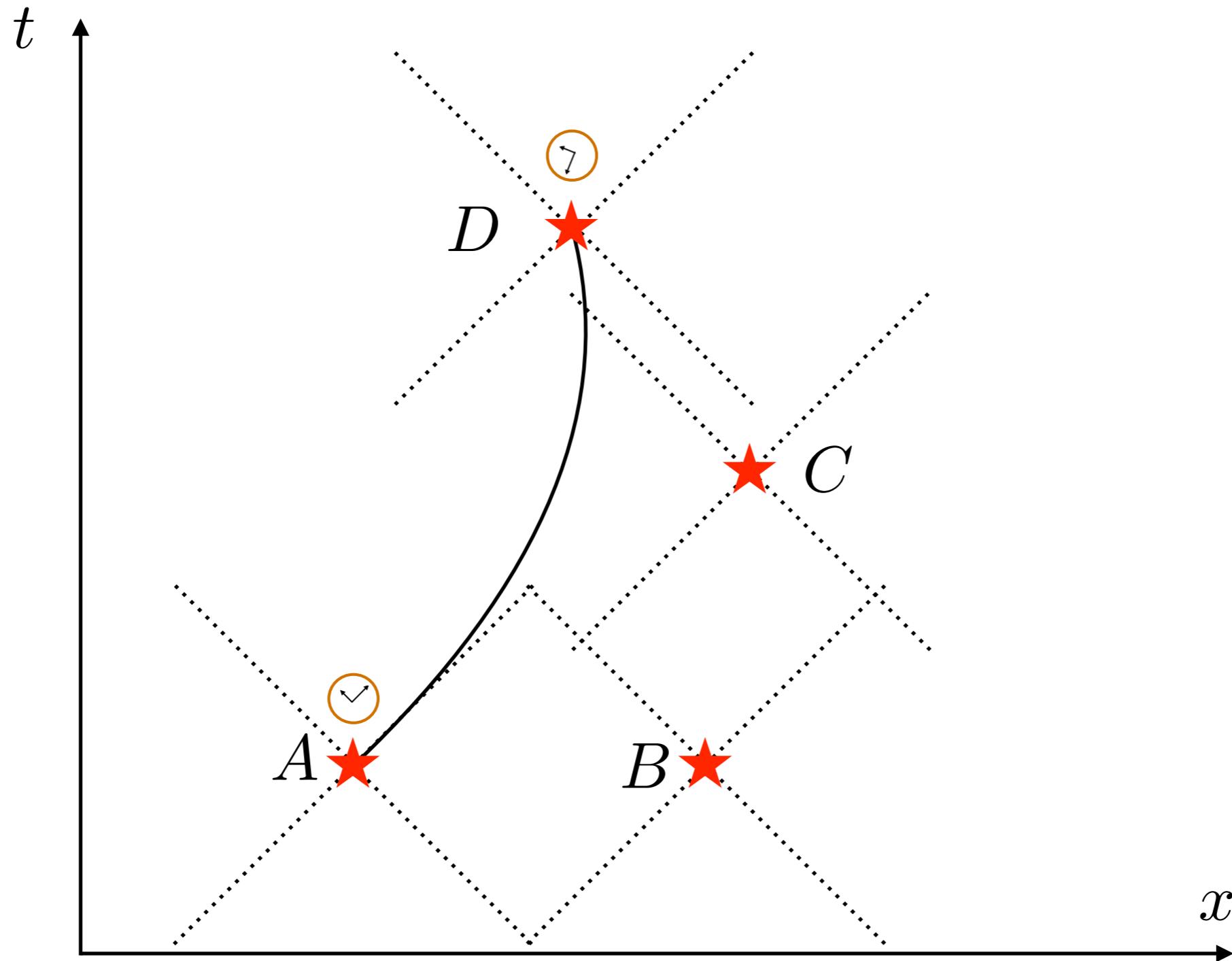
1.1 Relative localisation and causal order of events



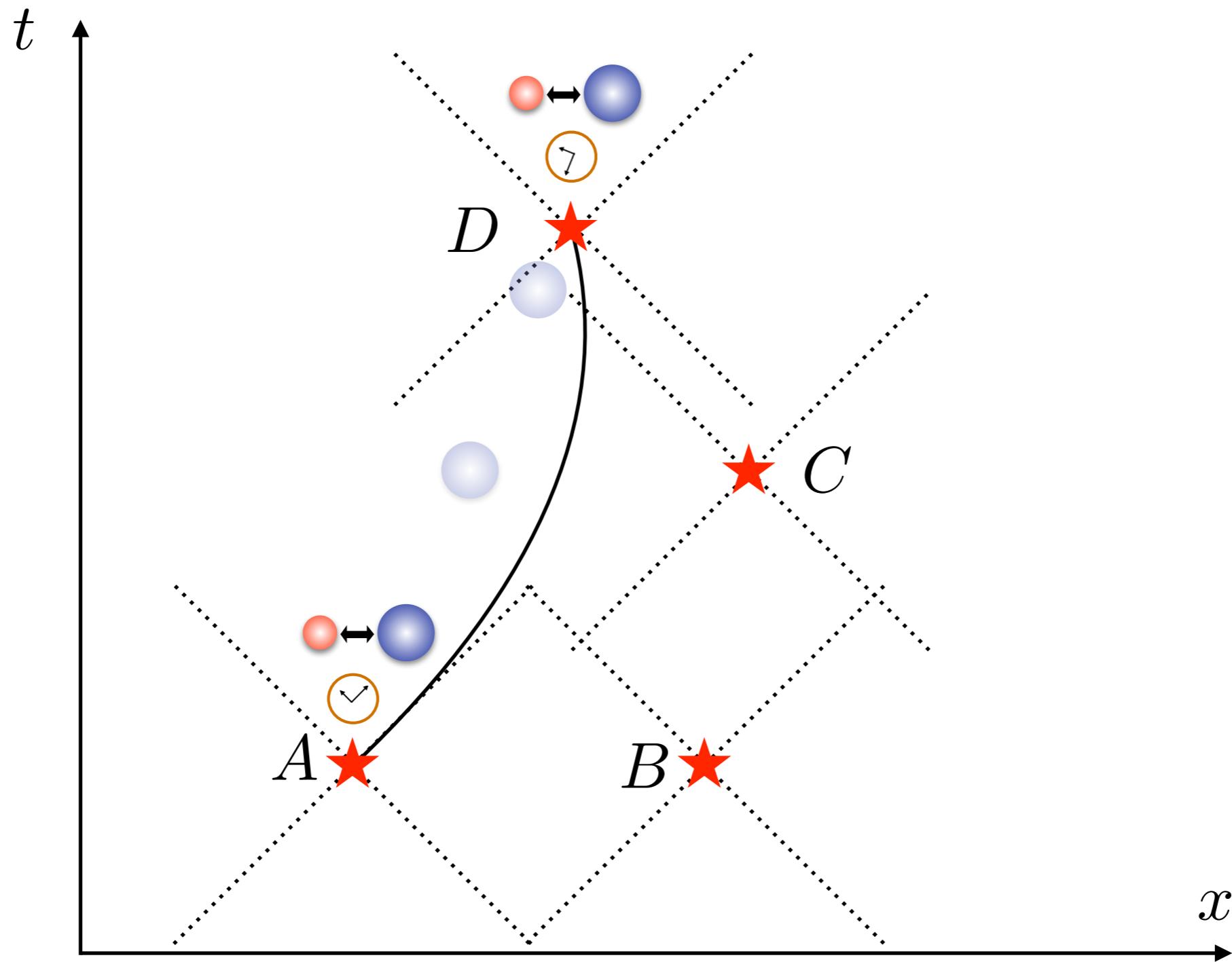
1.1 Localisation and causal order of events



1.3 Clocks tick according to the spacetime metric

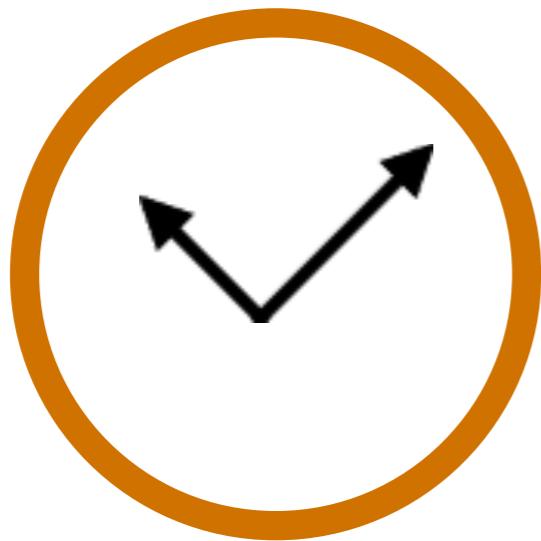


1.3 Clocks track time evolution and events

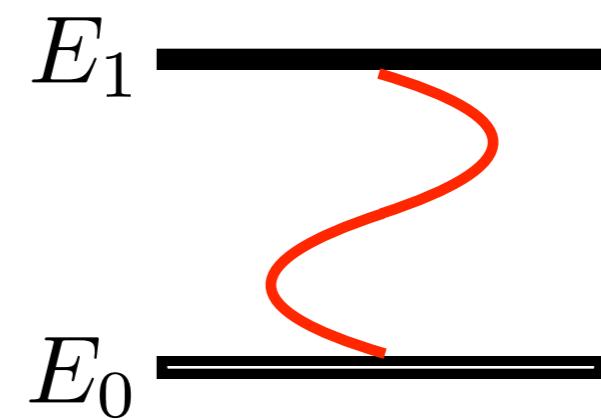


2. Gravitating quantum clocks (motivation)

2.1 A simple clock model



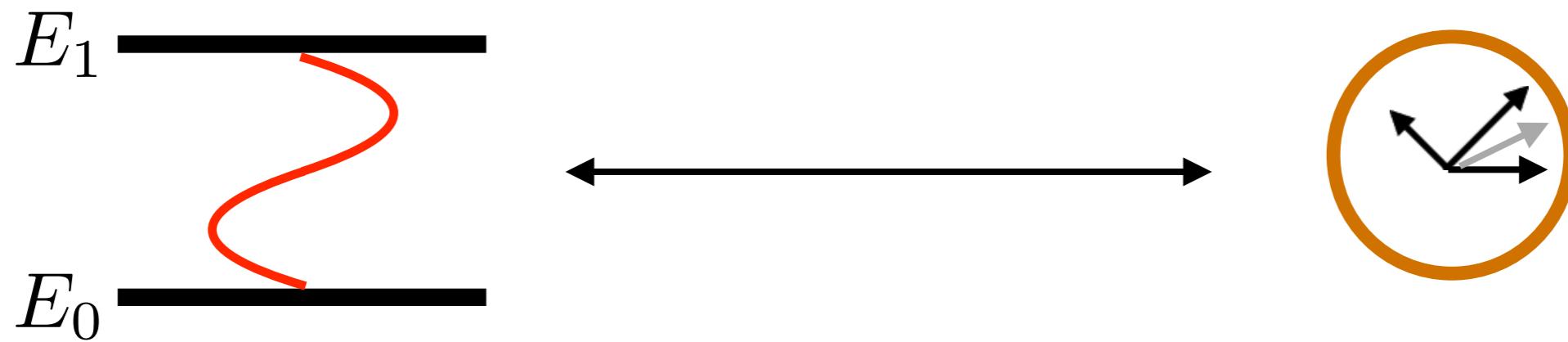
=



$$|\psi_0\rangle = \frac{|E_0\rangle + |E_1\rangle}{\sqrt{2}}$$

$$t_{\perp} = \frac{\hbar\pi}{E_1 - E_0}$$

2.2 Gravitating clocks lead to a non-fixed metric background

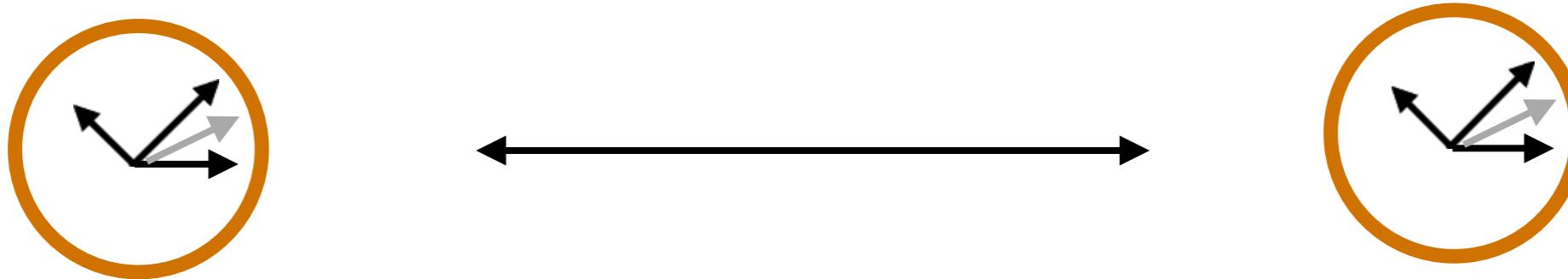


$$\Delta\Phi(x) = \frac{G(E_1 - E_0)}{c^4 x}$$

$$t_{\perp} \Delta t = \frac{\pi \hbar G t}{c^4 x}$$

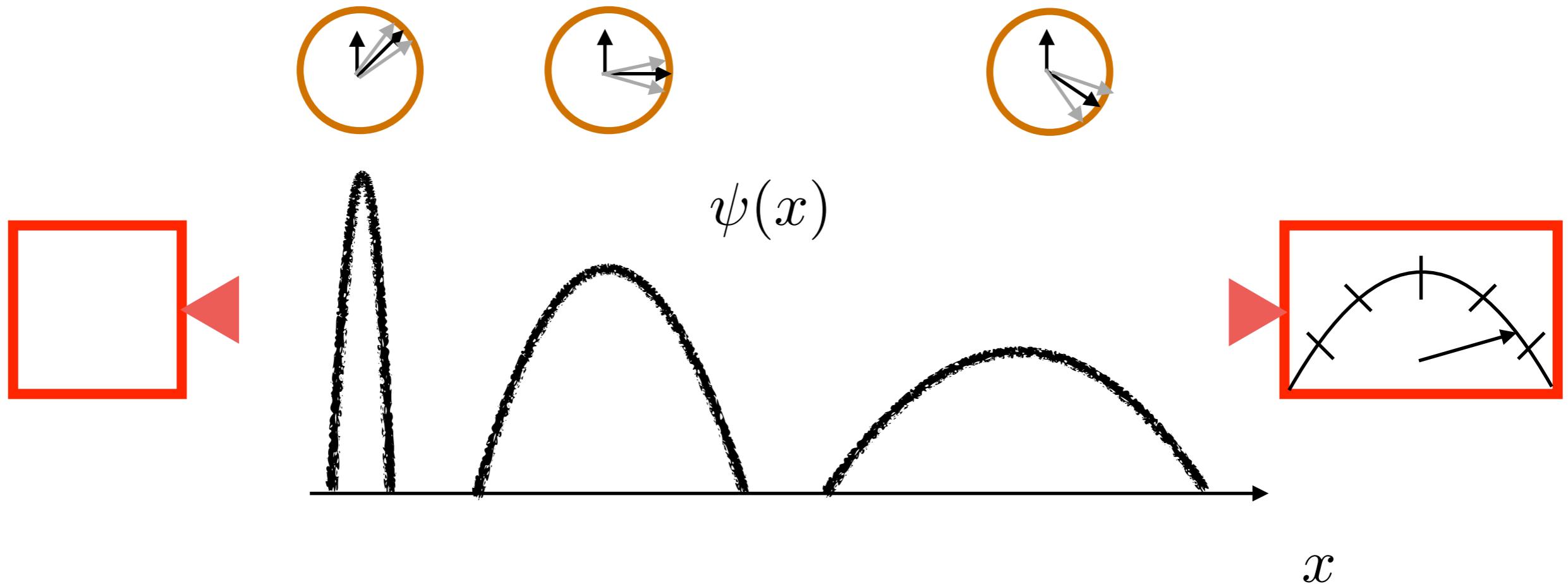
2.3 Entanglement of Quantum clocks through gravity

$$H = H_A + H_B - \frac{G}{c^4 x} H_A H_B$$



$$\begin{aligned} |\psi\rangle &= \frac{1}{\sqrt{2}}|0\rangle \left[\frac{1}{\sqrt{2}} \left(|0\rangle + e^{-\frac{it}{\hbar}\Delta E} |1\rangle \right) \right] \\ &+ \frac{1}{\sqrt{2}}|1\rangle \left[\frac{1}{\sqrt{2}} \left(|0\rangle + e^{-\frac{it}{\hbar}\Delta E \left(1 - \frac{G\Delta E}{c^4 x} \right)} |1\rangle \right) \right] \end{aligned}$$

2.4 The general problem: gravitating quantum systems make the metric “fuzzy”



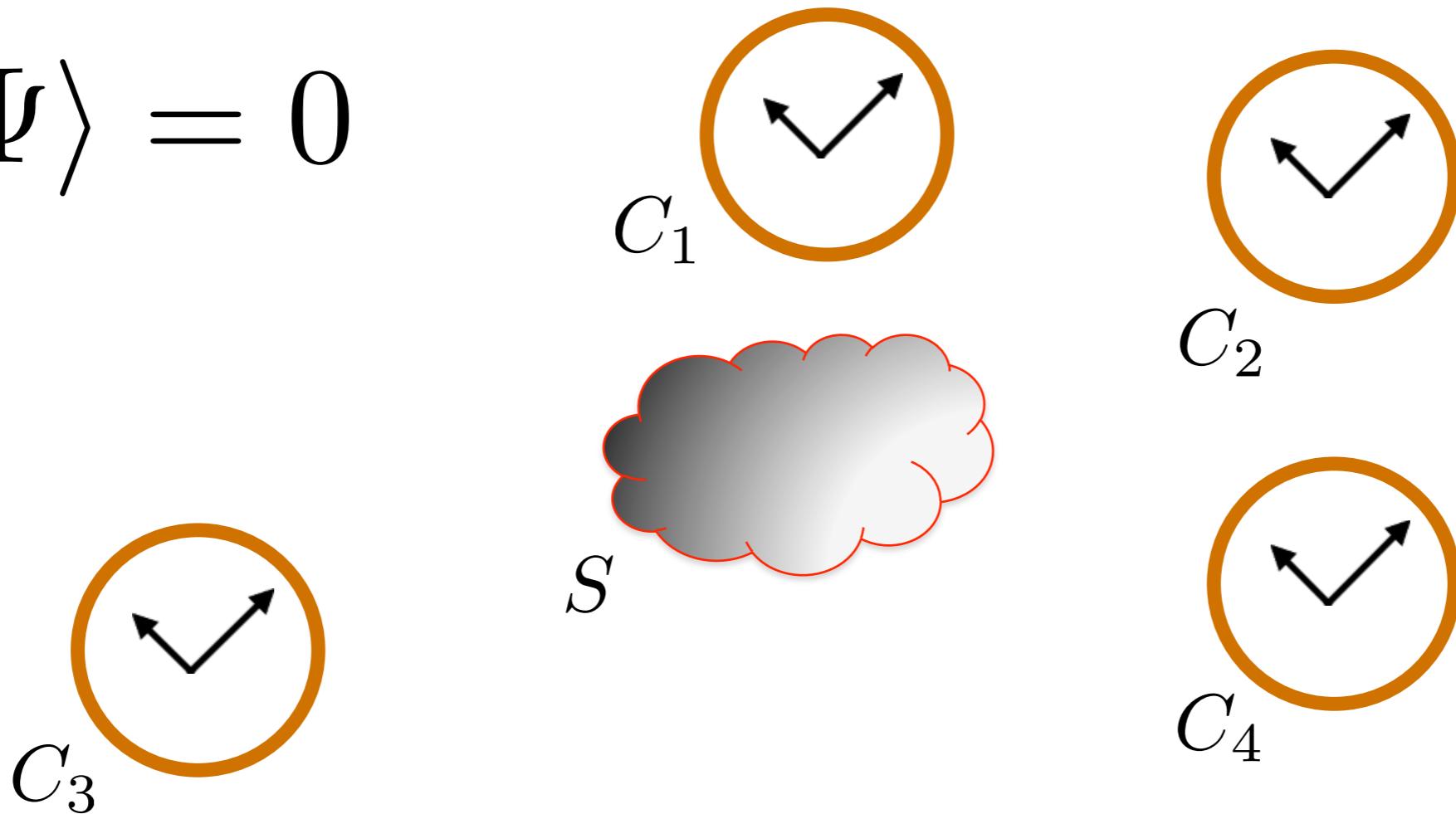
$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle$$

What is $\frac{d}{dt}$?

3. Quantum clocks as temporal reference frames

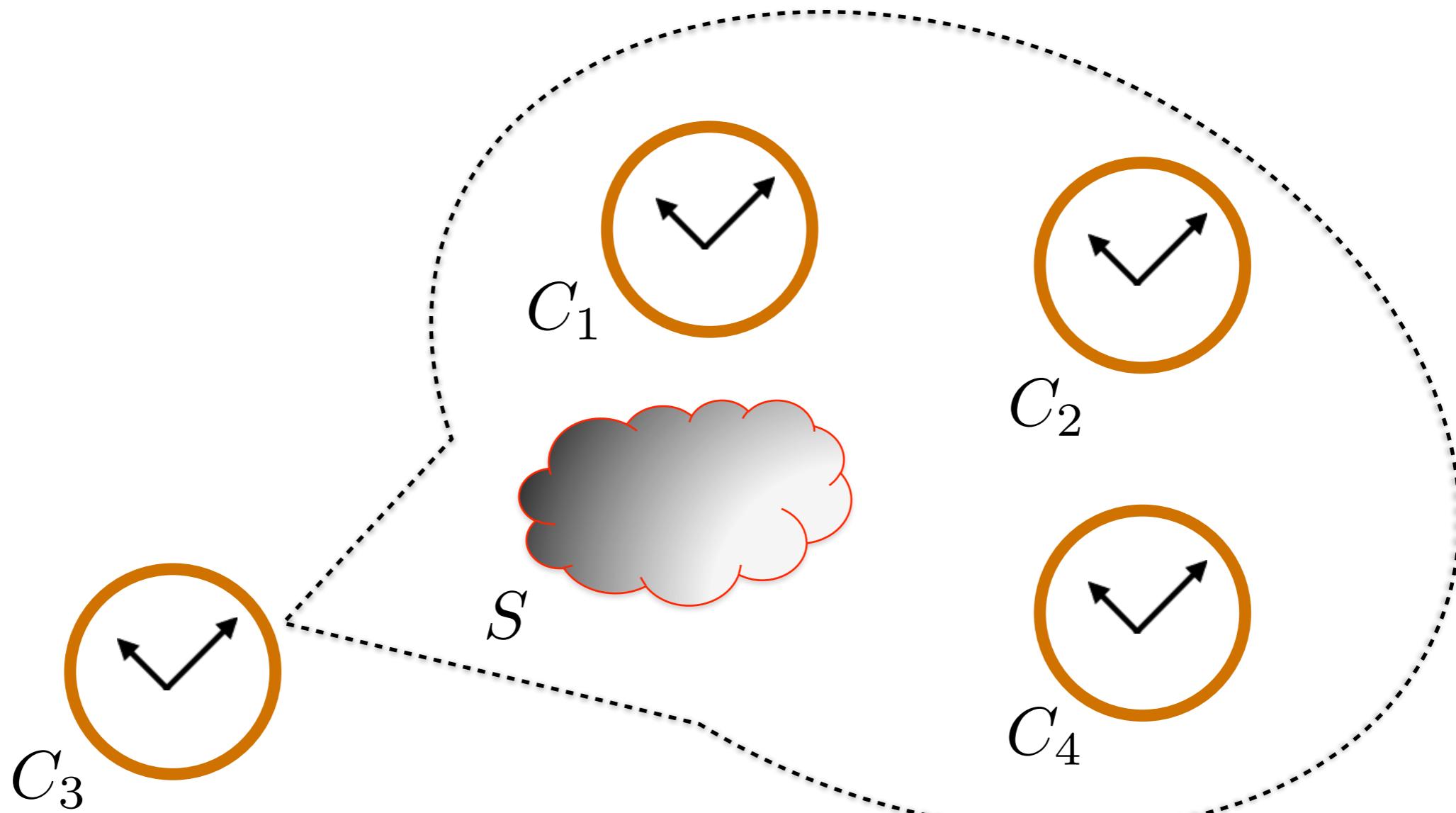
3.1 “Jumping” onto a (quantum) clock’s reference frame?

$$\hat{C}|\Psi\rangle = 0$$



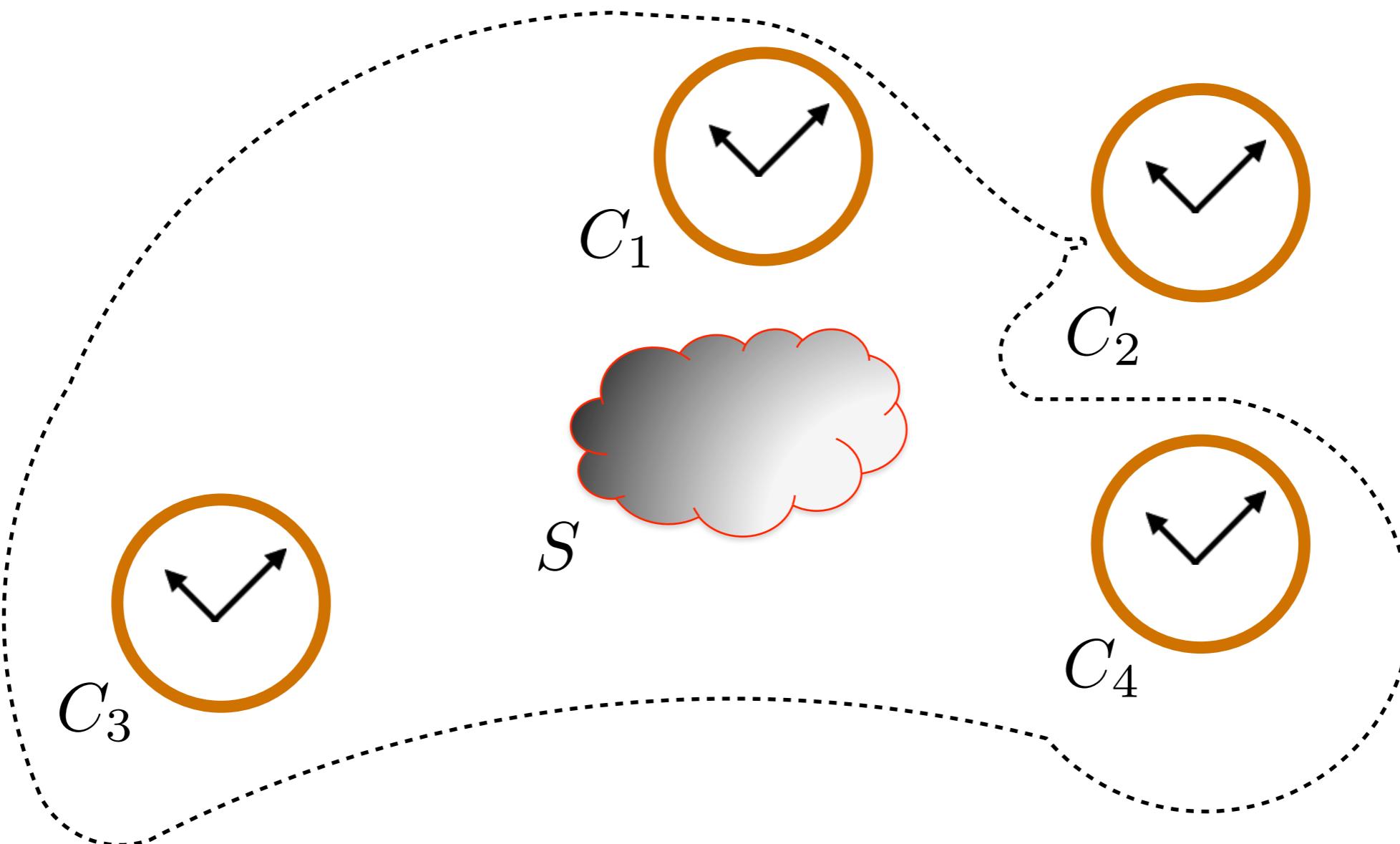
$$|\Psi\rangle = \int d\alpha e^{-i\alpha \hat{C}} |\varphi\rangle$$

3.3 Timeless approach for multiple clocks



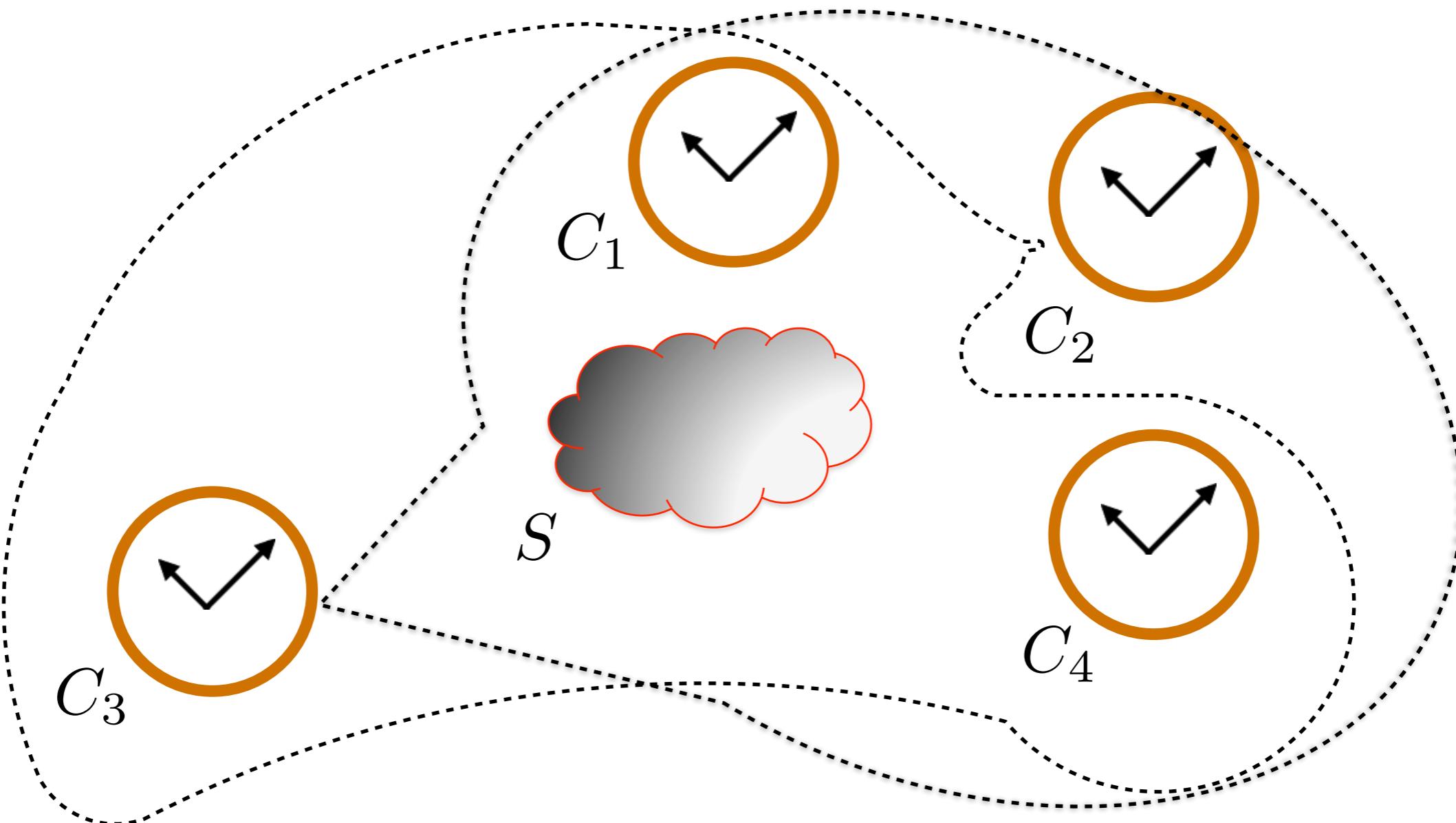
$$|\Psi\rangle = \int dt |t\rangle_3 \otimes \hat{U}_{\bar{3}} |\psi_3(0)\rangle_{\bar{3}}$$

3.3 Timeless approach for multiple clocks



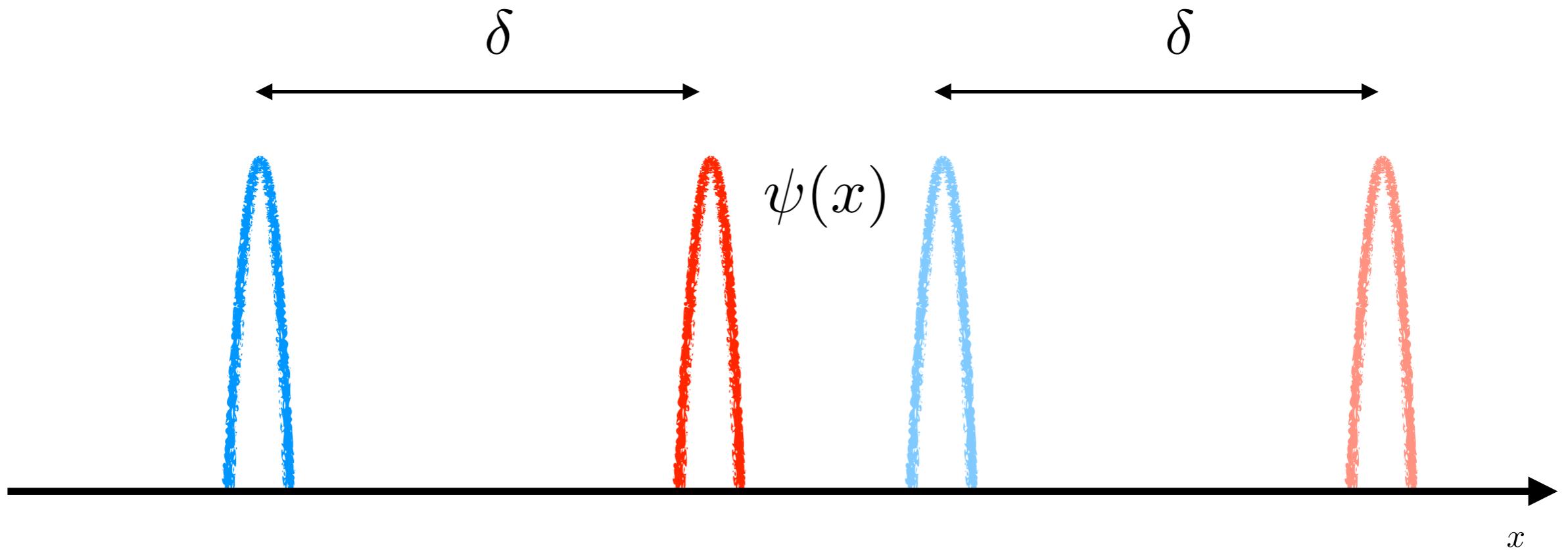
$$|\Psi\rangle = \int dt |t\rangle_2 \otimes \hat{U}_{\bar{2}} |\psi_2(0)\rangle_{\bar{2}}$$

3.3 Timeless approach for multiple clocks



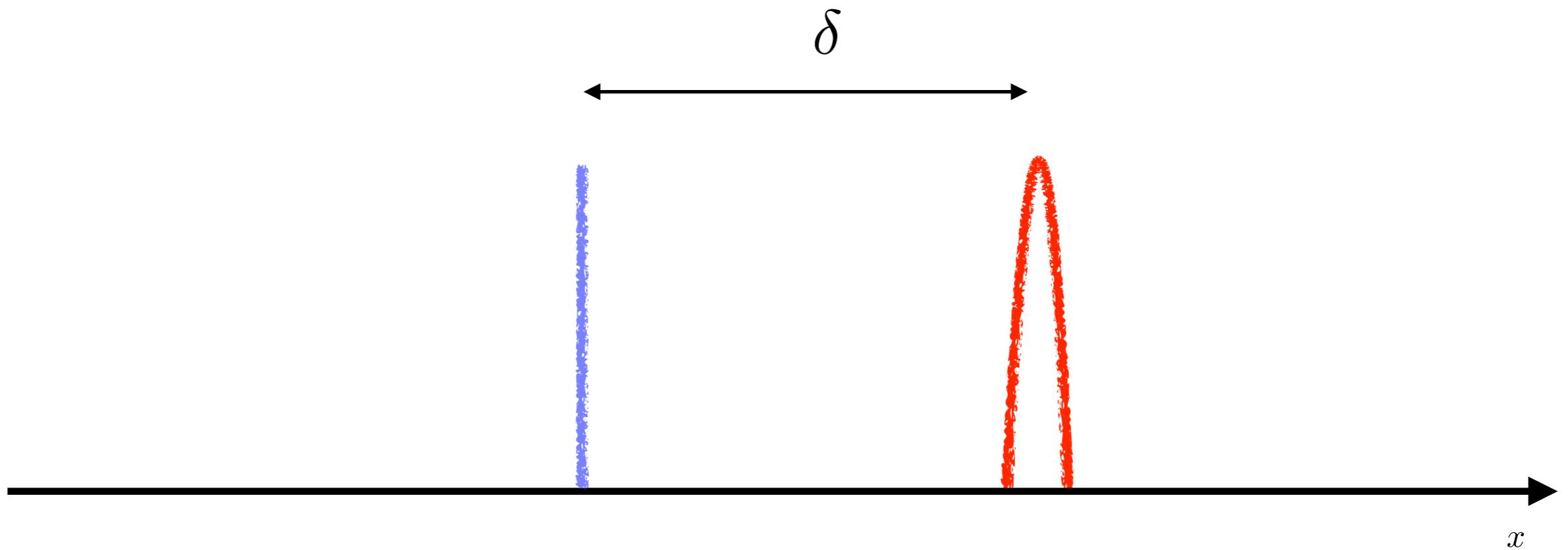
What if the clocks get entangled?

3.2 Analogy: quantum reference frames for space



$$|\Psi\rangle = \frac{1}{\sqrt{2}} |0\rangle |\delta\rangle + \frac{1}{\sqrt{2}} |1 - \delta\rangle |1\rangle$$

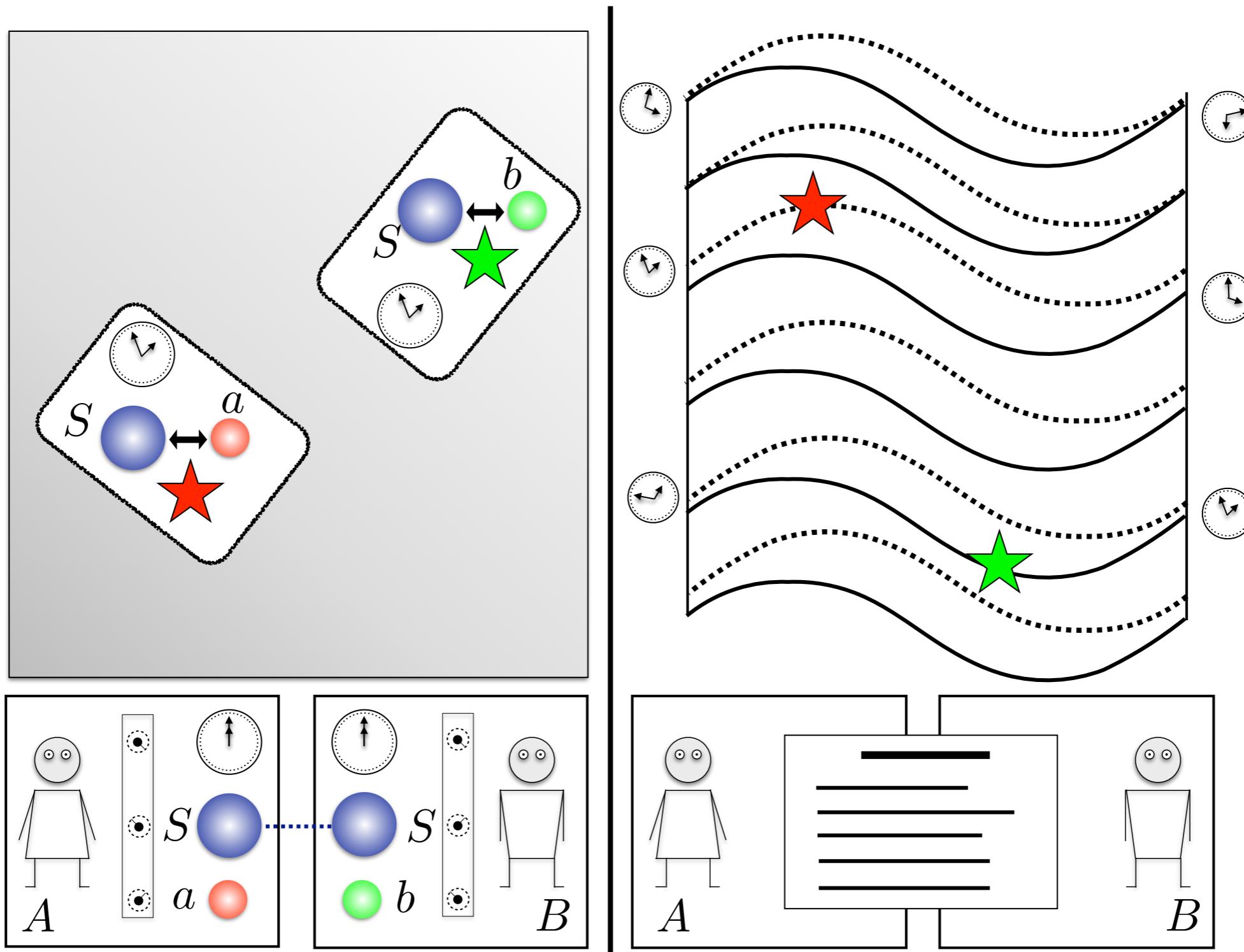
3.2 Analogy: quantum reference frames for space



$$|\Psi\rangle = |\delta\rangle$$

3.3 Operational definition of events

$$(\hat{H}_A + \hat{H}_B + \hat{f}_A(\hat{T}_A) + \hat{f}_B(\hat{T}_B))|\Psi\rangle = 0$$



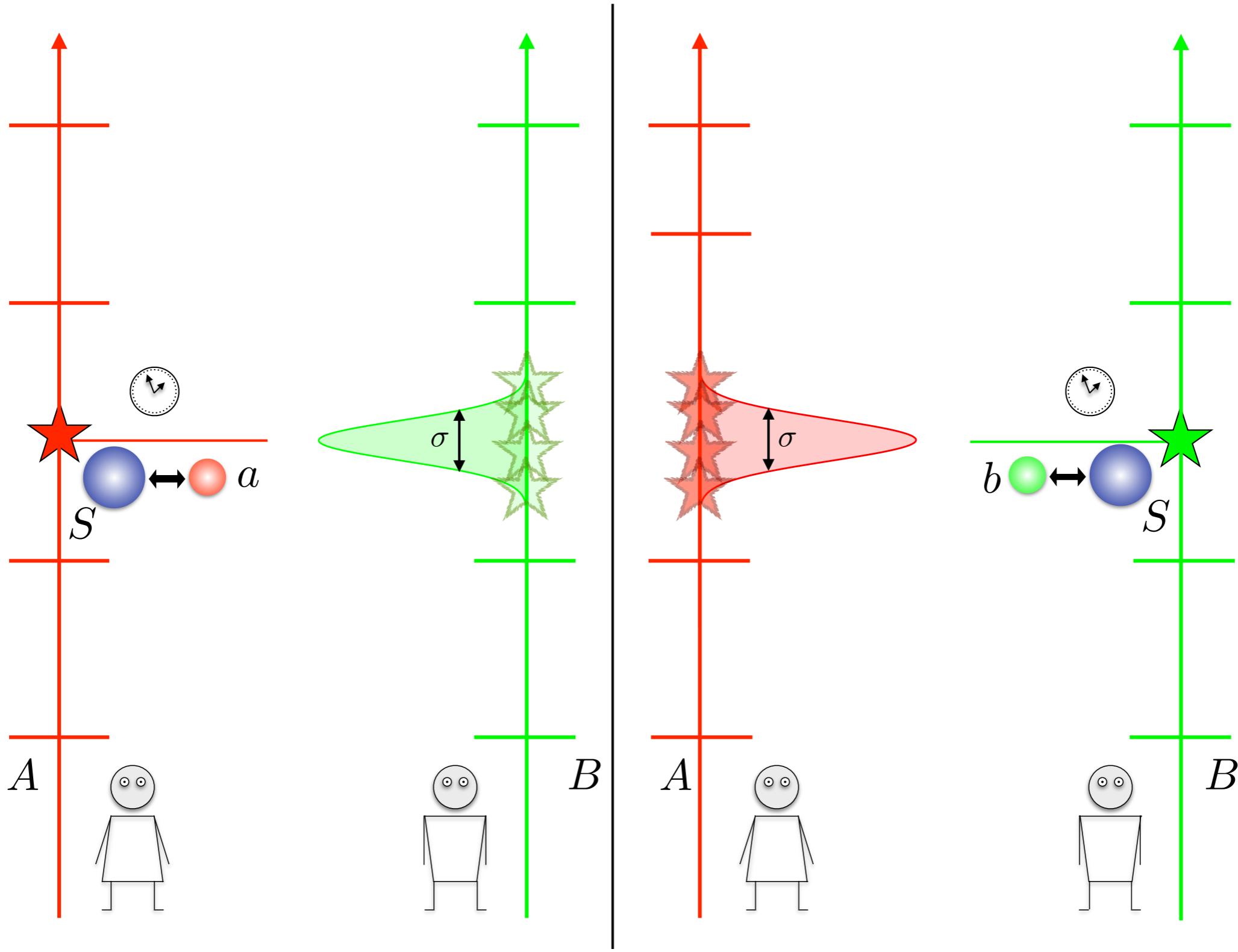
3.4 The simplest case: non-interacting clocks

$$(\hat{H}_A + \hat{H}_B + \delta(\hat{T}_A - t_A^*)\hat{K}^{(A)} + \delta(\hat{T}_B - t_B^*)\hat{K}^{(B)})|\Psi\rangle = 0$$

3.4 The simplest case: non-interacting clocks

$$|\Psi\rangle = \int dt_A |t_A\rangle_A e^{-it_A \hat{H}_B} T e^{-i \int_0^{t_A} ds (\hat{f}_A(s) + \hat{f}_B(s + \hat{T}_B))} |\psi_A(0)\rangle_{\bar{A}}$$

3.4 The simplest case: non-interacting clocks



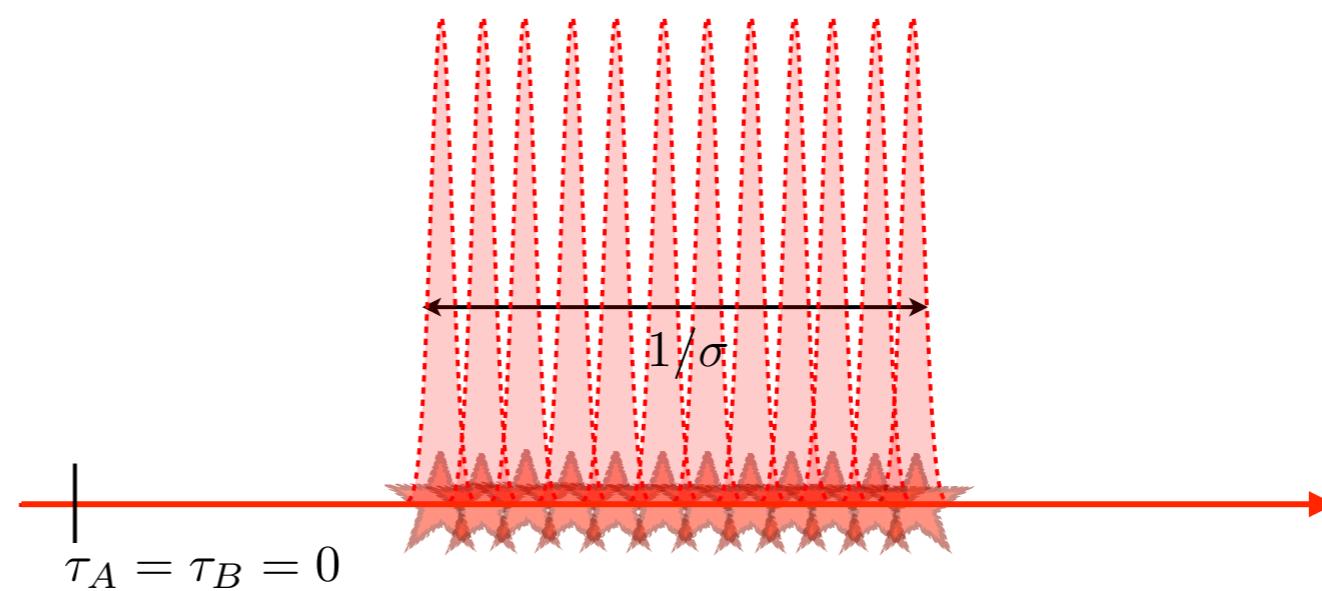
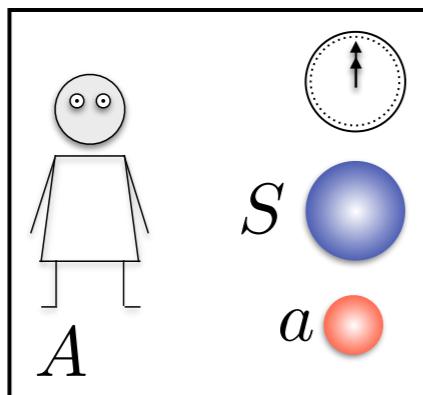
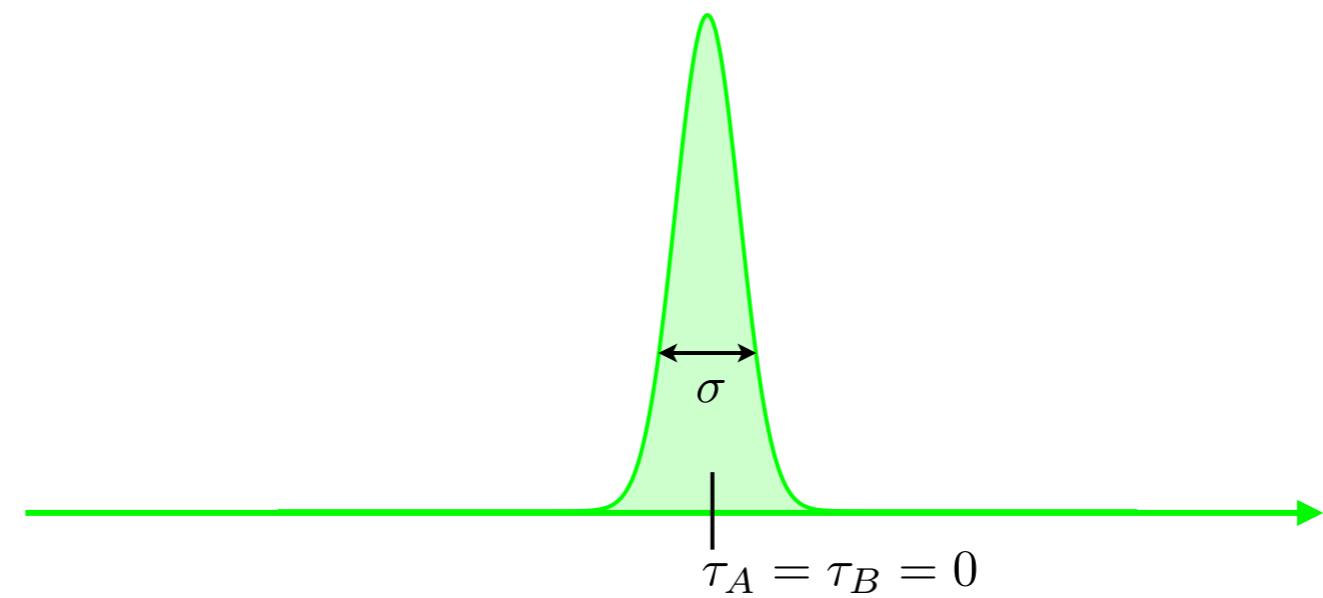
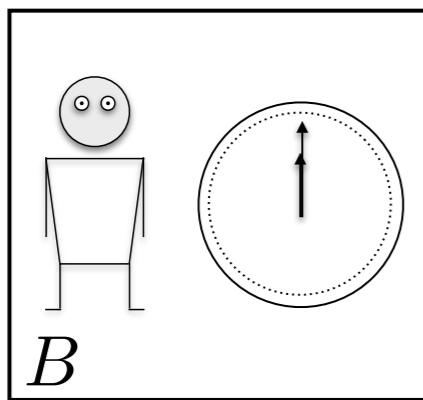
3.5 Entanglement of quantum clocks through gravity (once more)

$$\left(\hat{H}_A + \hat{H}_B + \hat{H}_C - \frac{G}{c^4 x} \hat{H}_A \hat{H}_B + \hat{f}(\hat{T}_A) \left(1 - \frac{G}{c^4 x} \hat{H}_B \right) \right) |\Psi\rangle = 0$$

3.5 Entanglement of quantum clocks through gravity (once more)

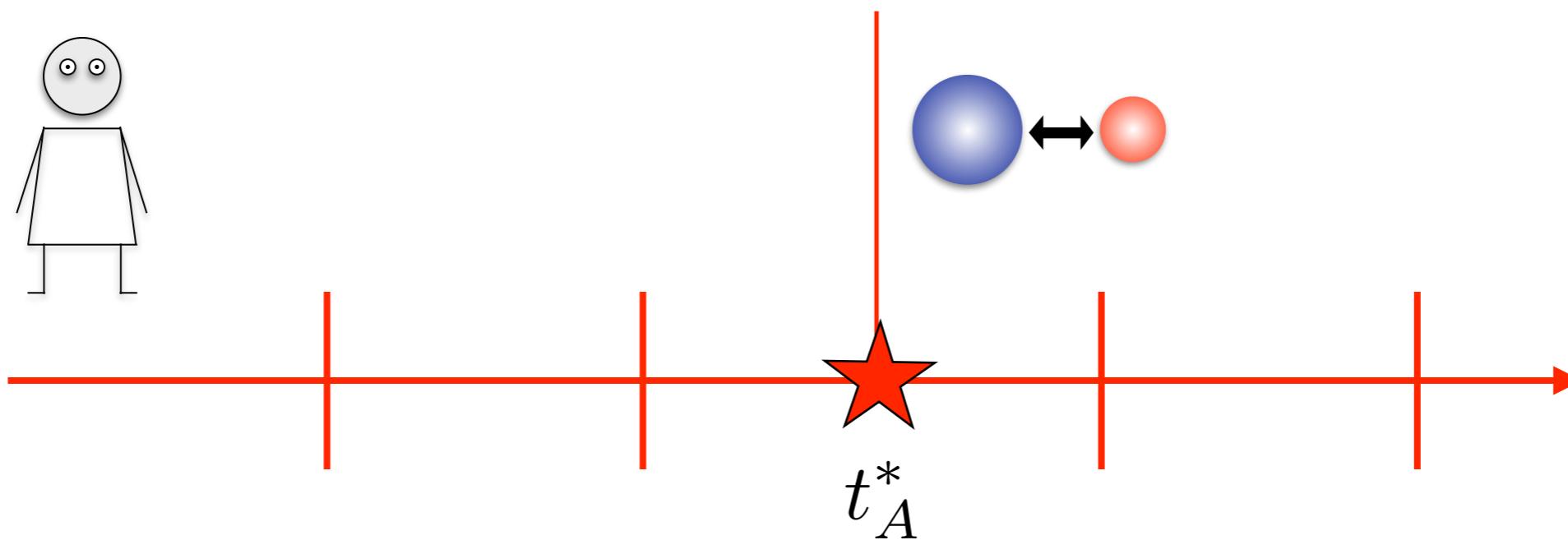
$$\hat{U}_{\bar{C}} = e^{-i\tau_C(\hat{H}_A + \hat{H}_B + \lambda \hat{H}_A \hat{H}_B)} T e^{-i \int_0^{\tau_C} ds (1 + \lambda \hat{H}_B)} \hat{f}(s(1 + \lambda \hat{H}_B) + \hat{T}_A)$$

3.5 Entanglement of quantum clocks through gravity (once more)



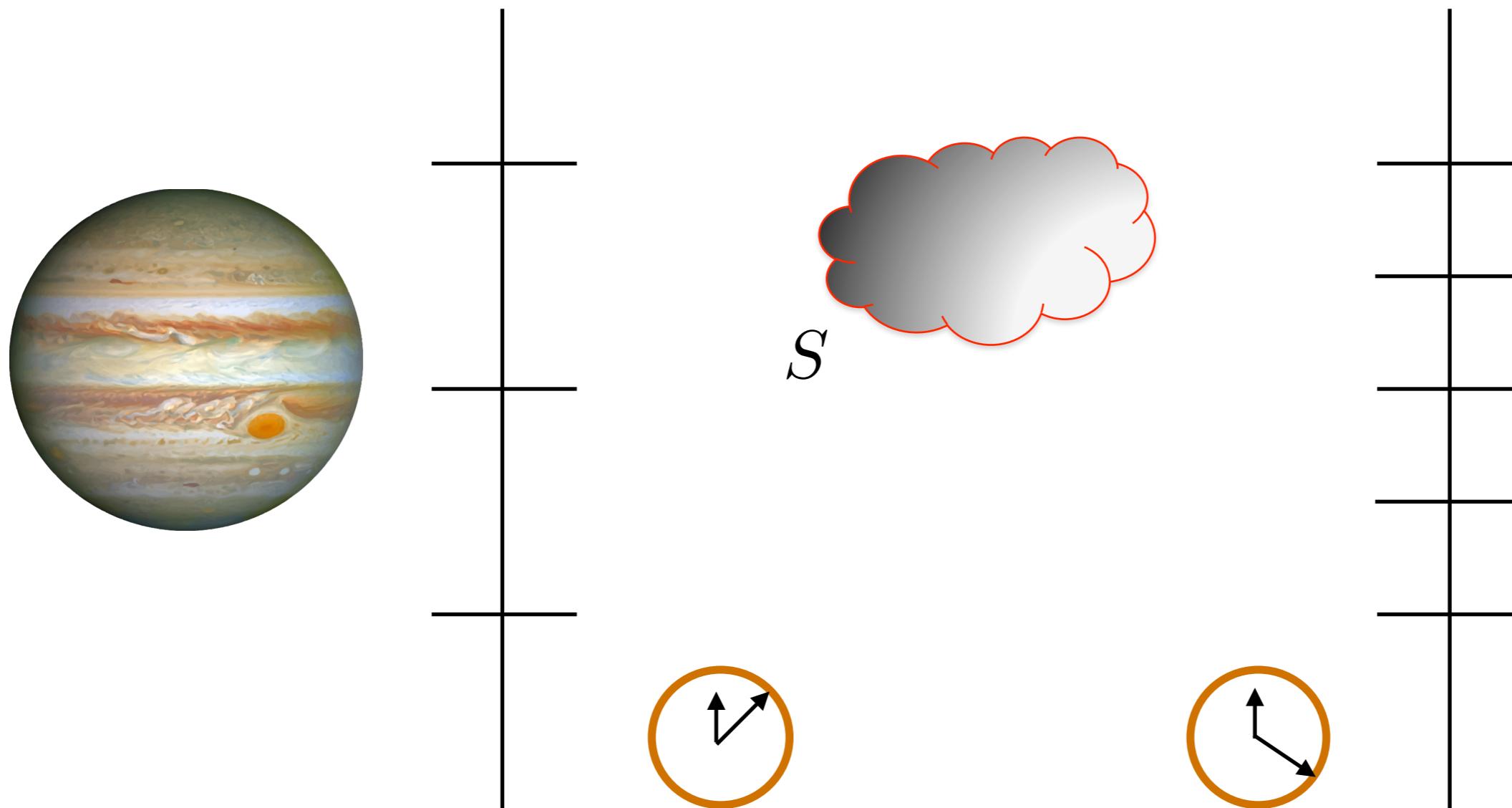
3.5 Entanglement of quantum clocks through gravity (once more)

$$\hat{U}_{\bar{A}} = T e^{-i \int_0^{\tau_A} ds \left(\frac{\hat{H}_B + \hat{H}_C}{1 + \lambda \hat{H}_B} + \hat{f}(s) \right)}$$



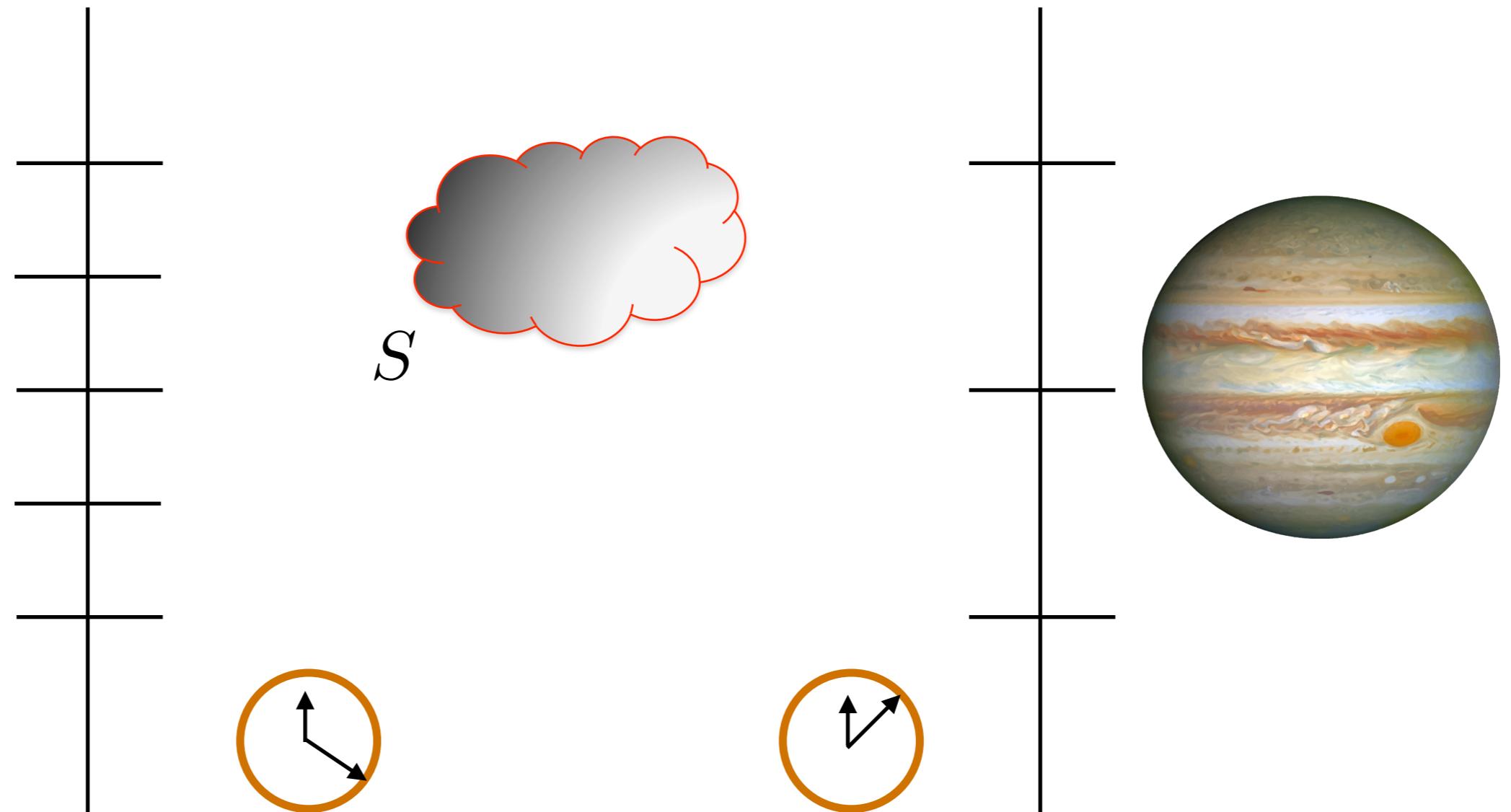
3.6 Gravitational quantum switch

$$\left(\sum_I \hat{H}_I (1 + \hat{\Phi}_I) + \sum_I \hat{f}_I (\hat{T}_I) (1 + \hat{\Phi}_I) \right) |\Psi\rangle = 0$$



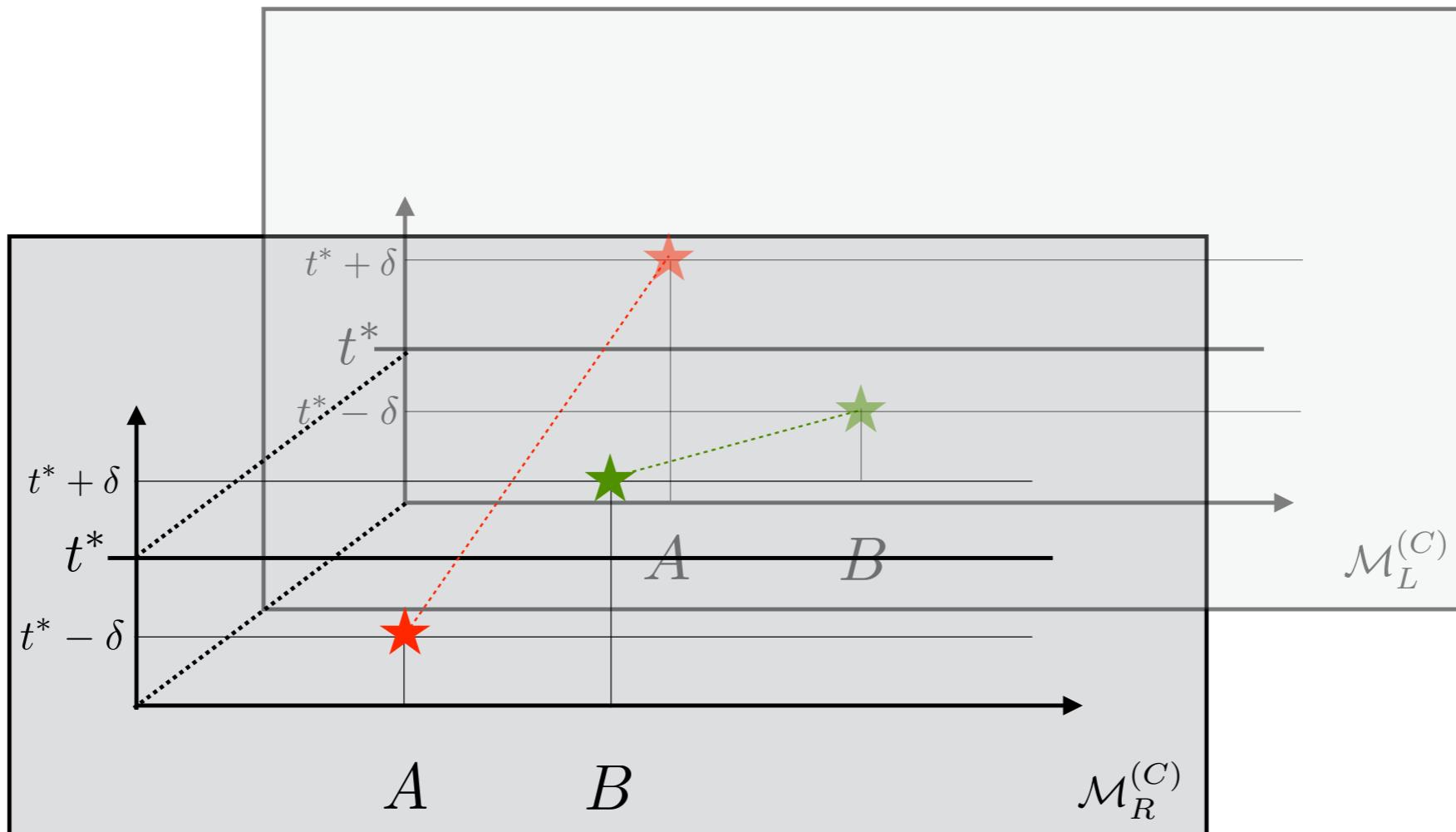
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$$\left(\sum_I \hat{H}_I (1 + \hat{\Phi}_I) + \sum_I \hat{f}_I (\hat{T}_I) (1 + \hat{\Phi}_I) \right) |\Psi\rangle = 0$$



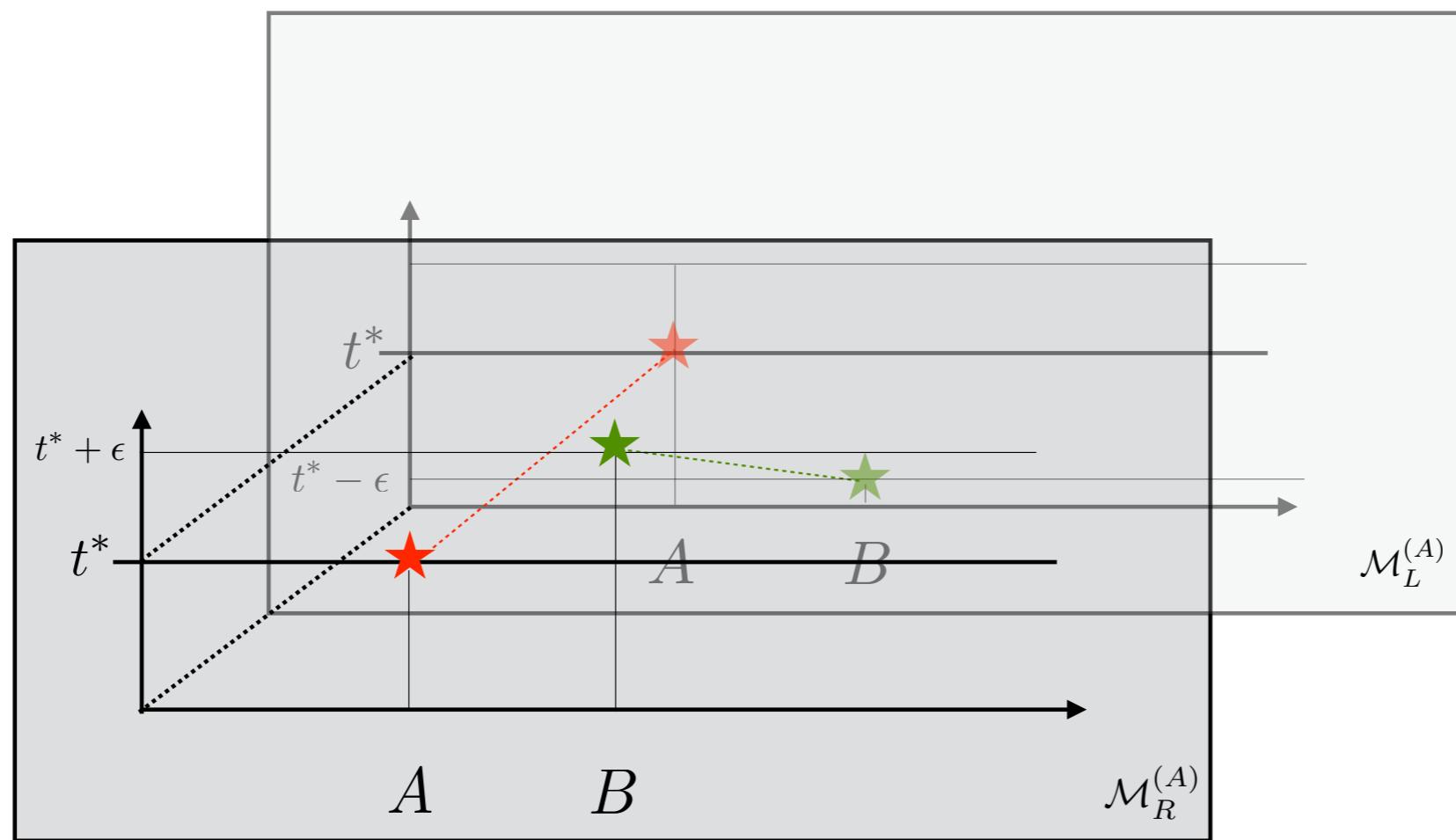
3.6 Gravitational quantum switch

$$|\Psi\rangle = \int dt |t\rangle_C \otimes (U_A^{aS} U_B^{bS} |\phi\rangle |L\rangle_M + U_B^{bS} U_A^{aS} |\psi\rangle |R\rangle_M)$$



3.6 Gravitational quantum switch

$$|\Psi\rangle = \int dt |t\rangle_A \otimes (U_A^{aS}(t-t^*) U_B^{bS} |\phi\rangle |L\rangle_M + U_B^{bS} U_A^{aS}(t-t^*)_A |\psi\rangle |R\rangle_M)$$



4. Outlook

- Beyond superpositions of semiclassical states?
- Inclusion of spatial quantum reference frames?
- Connection with indefinite causal structures via causal reference frames?

F Giacomini, ECR, C Brukner, *Nat. Commun.* (2019)

O Oreshkov, *arXiv:1801.07594* (2018)

P Allard Guérin, C Brukner *NJP* (2018)

Thank you