Quantum computation with indefinite causal structures

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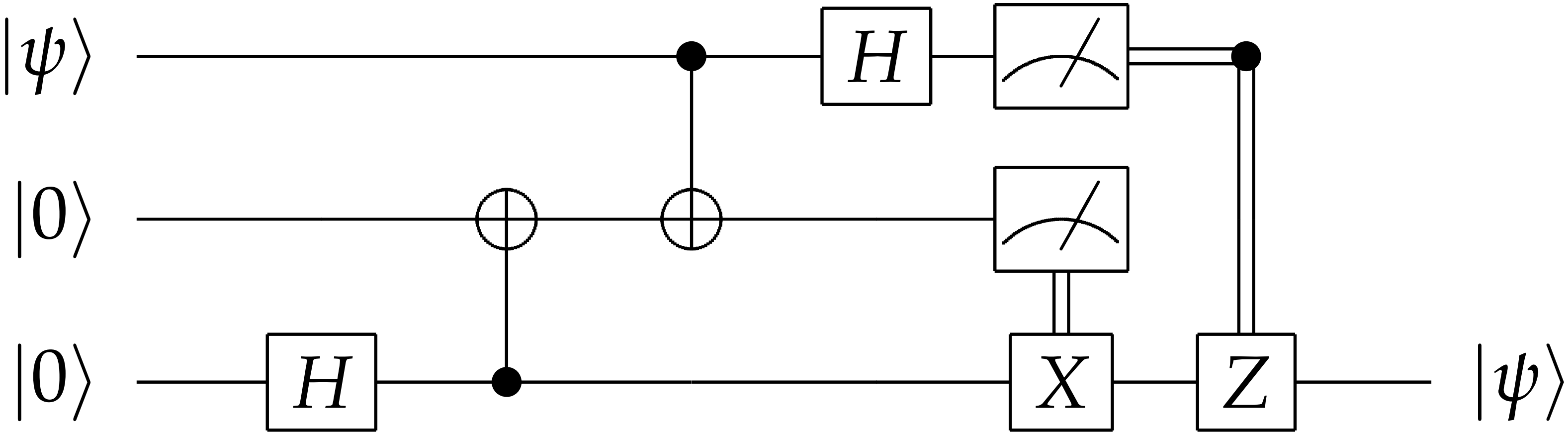
Universität zu Köln

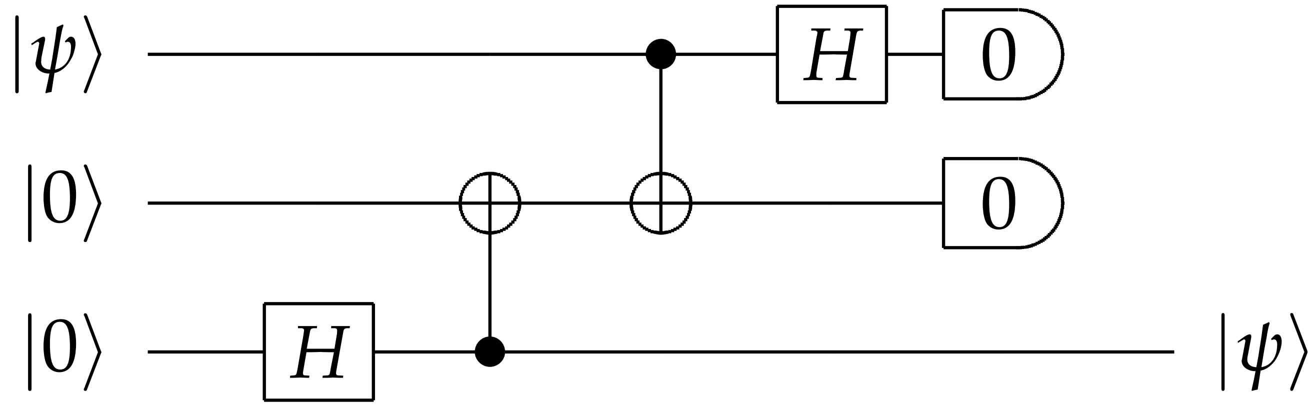
ÖAW ÖSTERREICHISCHE AKADEMIE DER WISSENSCHAFTEN

Time travel

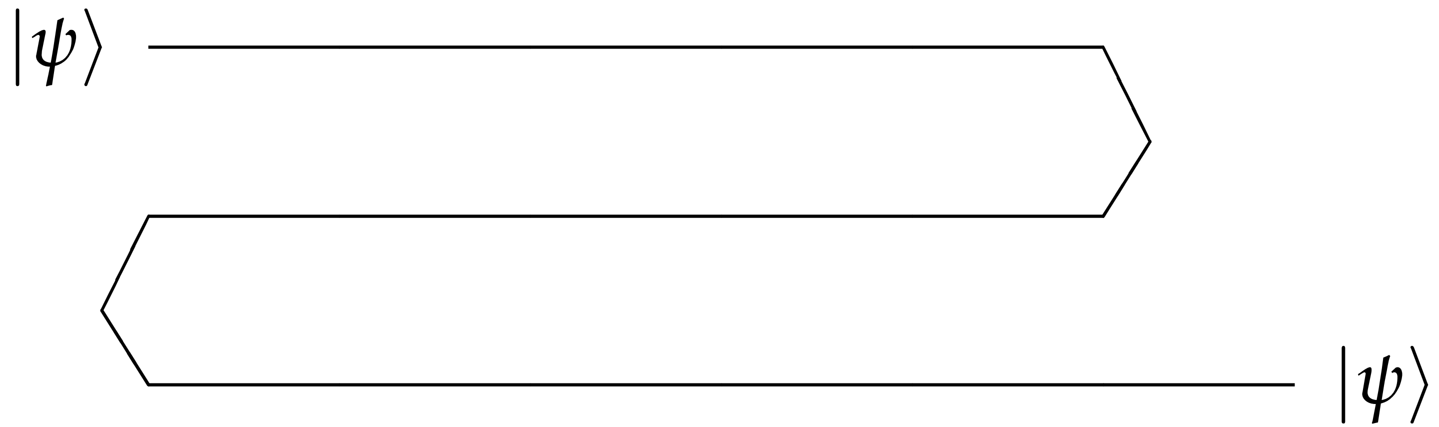
Teleportation



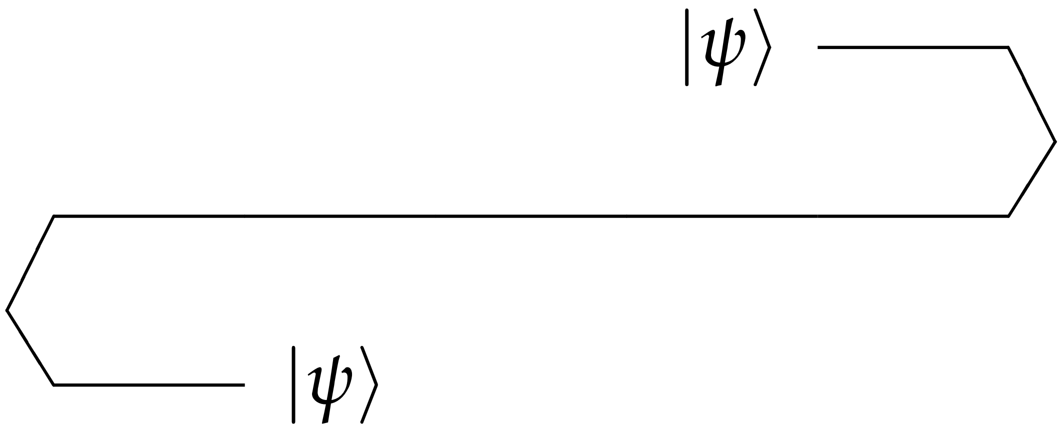
Teleportation



Post-selected teleportation

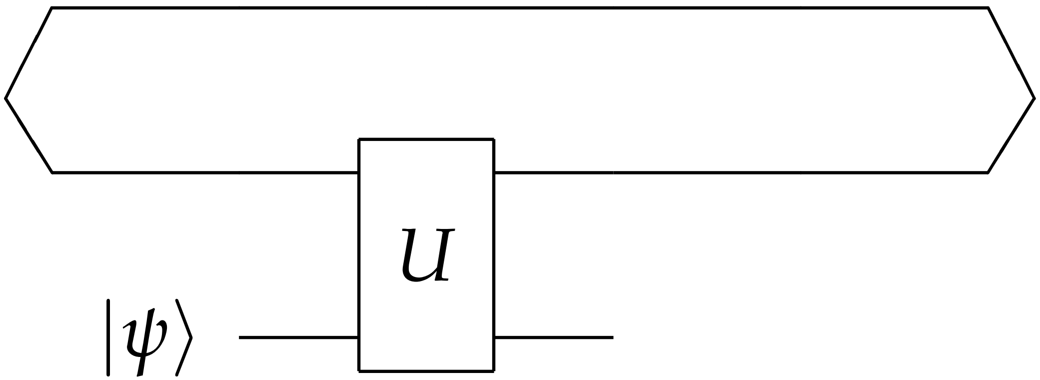


Post-selected teleportation



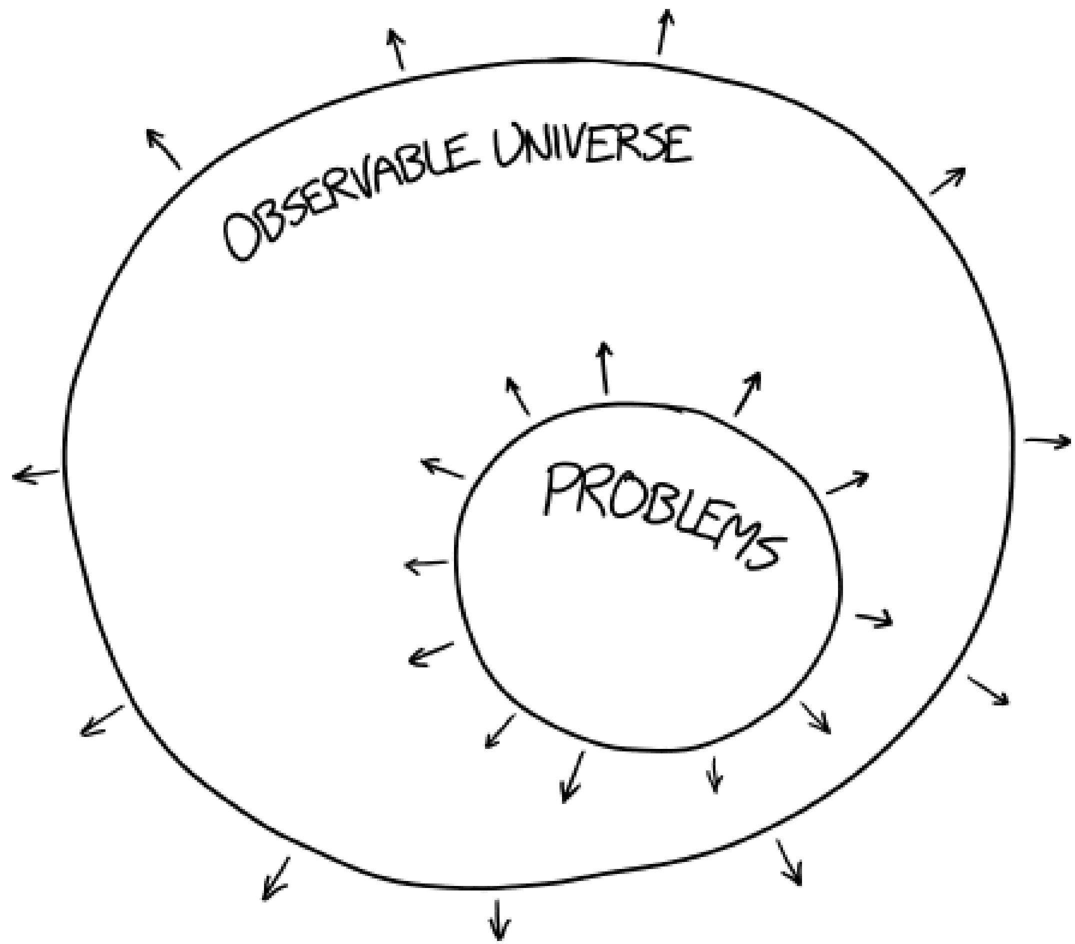
Post-selected teleportation

Post-selected closed timelike curve

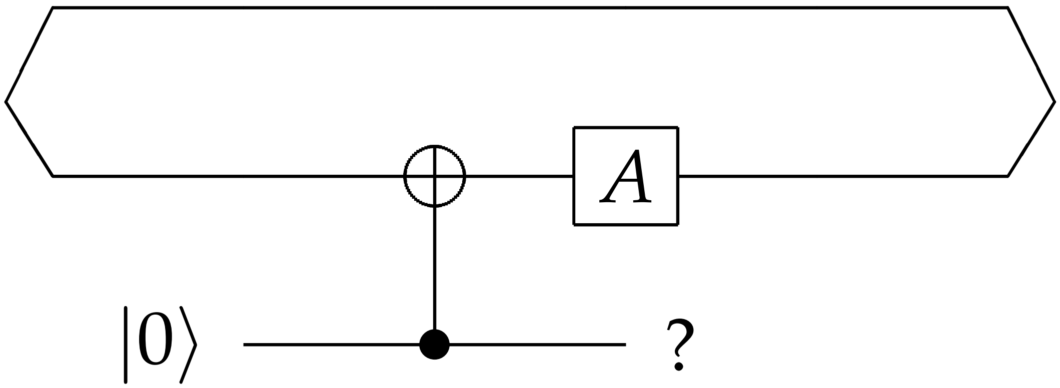


$$K = \mathrm{tr}_{\mathrm{CTC}}\, U$$

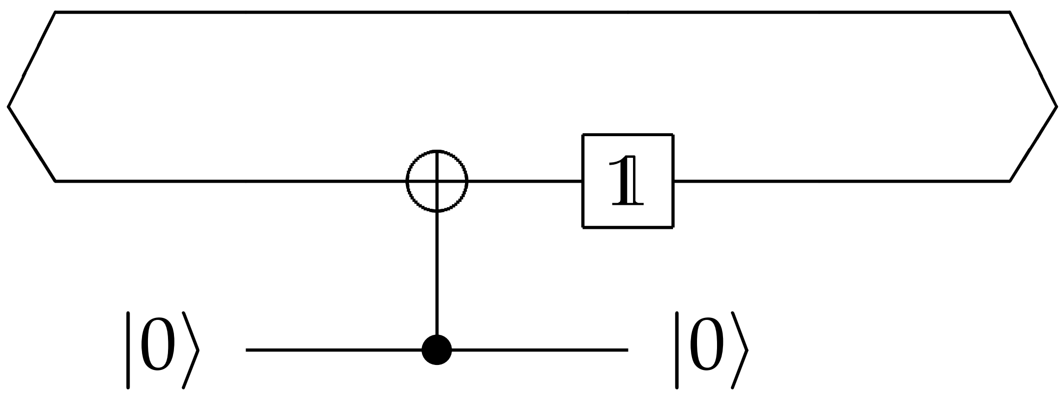
$$|\psi\rangle \mapsto \frac{1}{\| K|\psi\rangle \|} K|\psi\rangle$$


OBSERVABLE UNIVERSE
PROBLEMS

Fragility



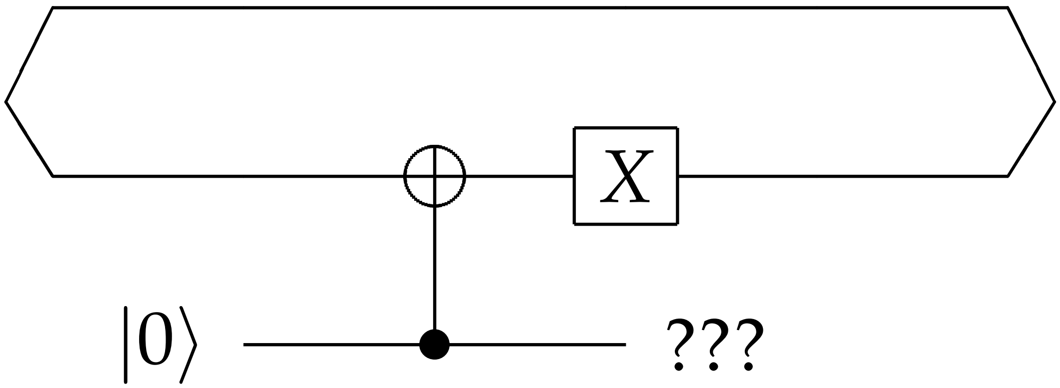
Fragility



$$K = \mathrm{tr}_{\mathrm{CTC}}\left(|0\rangle\langle 0| \otimes \mathbb{1} + |1\rangle\langle 1| \otimes X\right) = |0\rangle\langle 0|$$

$$|0\rangle \mapsto \frac{1}{\| |0\rangle\langle 0| |0\rangle \|} |0\rangle\langle 0| |0\rangle = |0\rangle$$

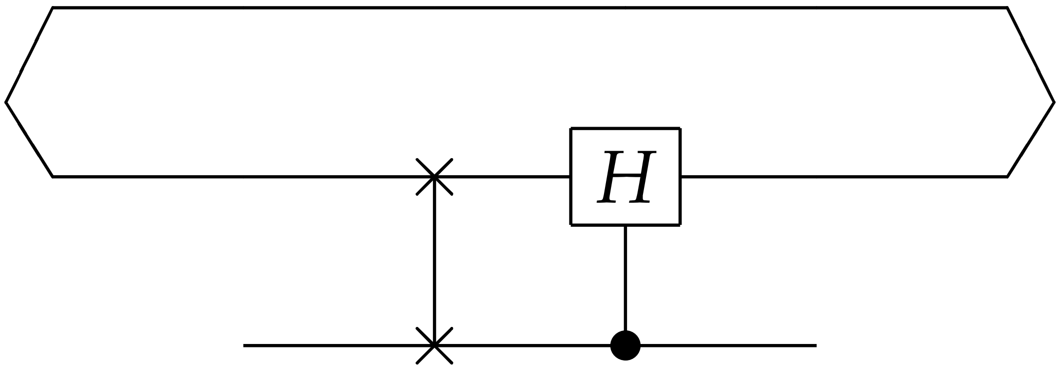
Fragility



$$K = \mathrm{tr}_{\mathrm{CTC}}\left(|0\rangle\langle 0| \otimes X + |1\rangle\langle 1| \otimes \mathbb{1}\right) = |1\rangle\langle 1|$$

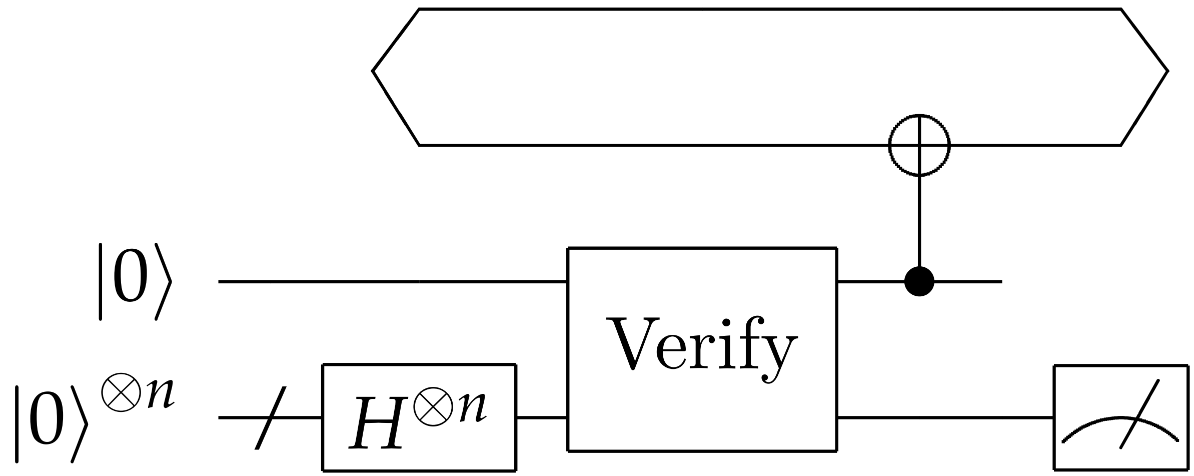
$$|0\rangle \mapsto \frac{1}{\| |1\rangle\langle 1||0\rangle \|}|1\rangle\langle 1||0\rangle = \frac{0}{0}$$

Non-orthogonal states

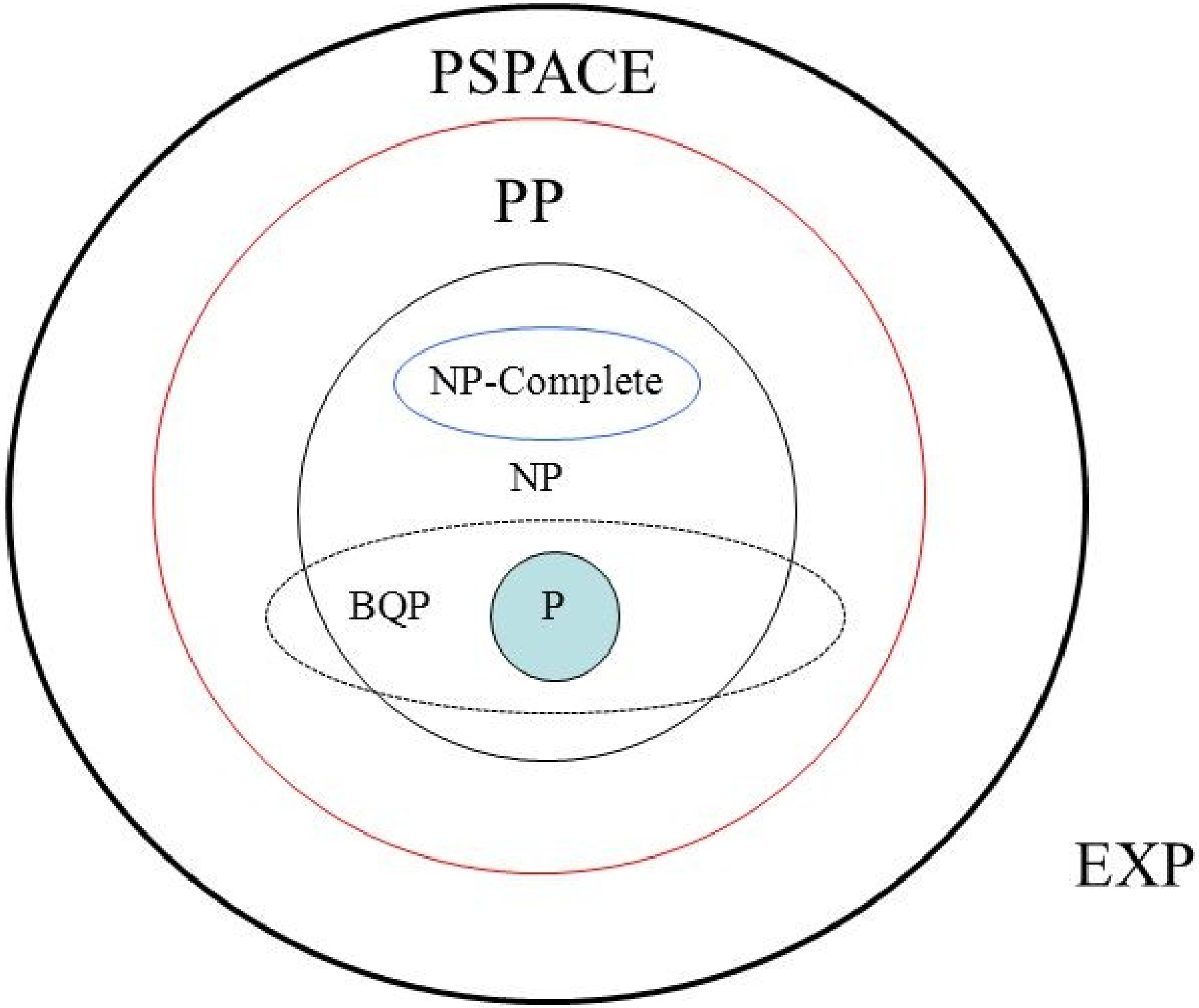


$$K = |0\rangle\langle 0| + |1\rangle\langle -|$$

NP-complete problems



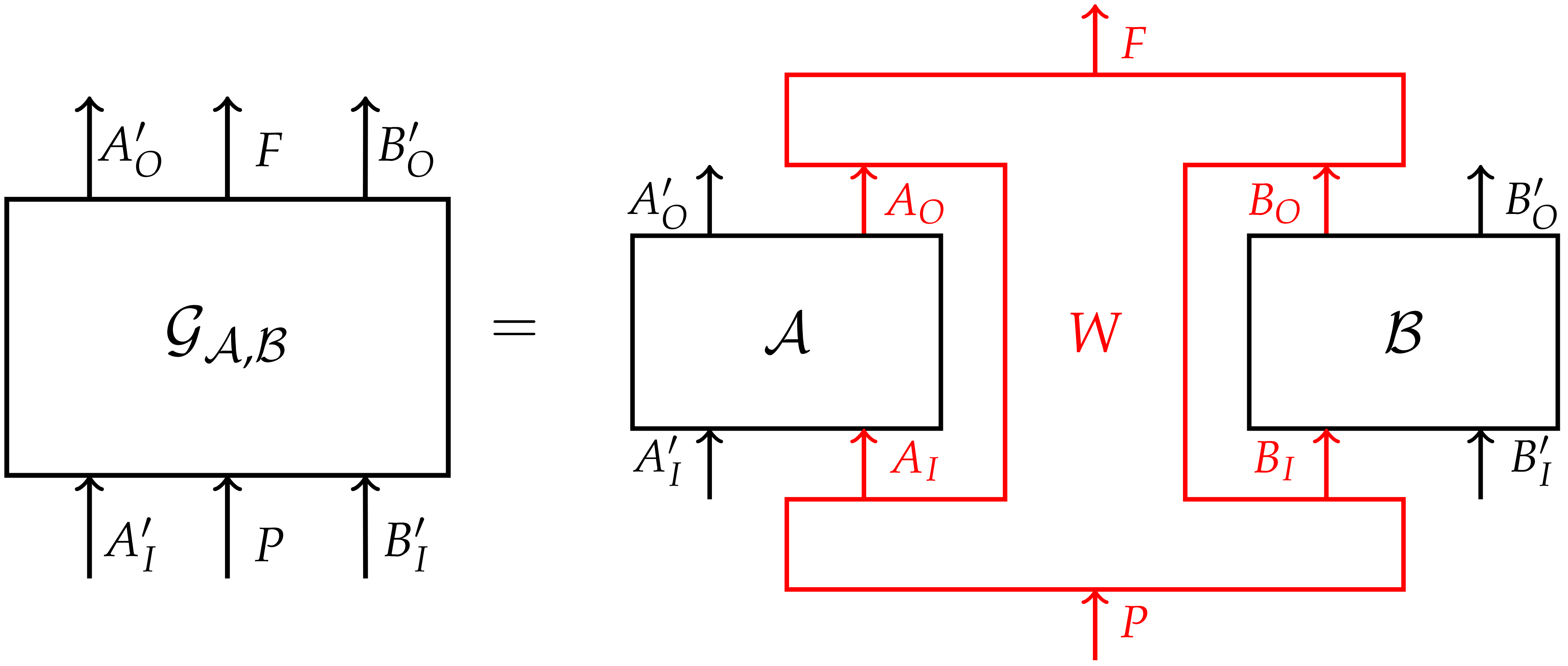
The Complexity Zoo



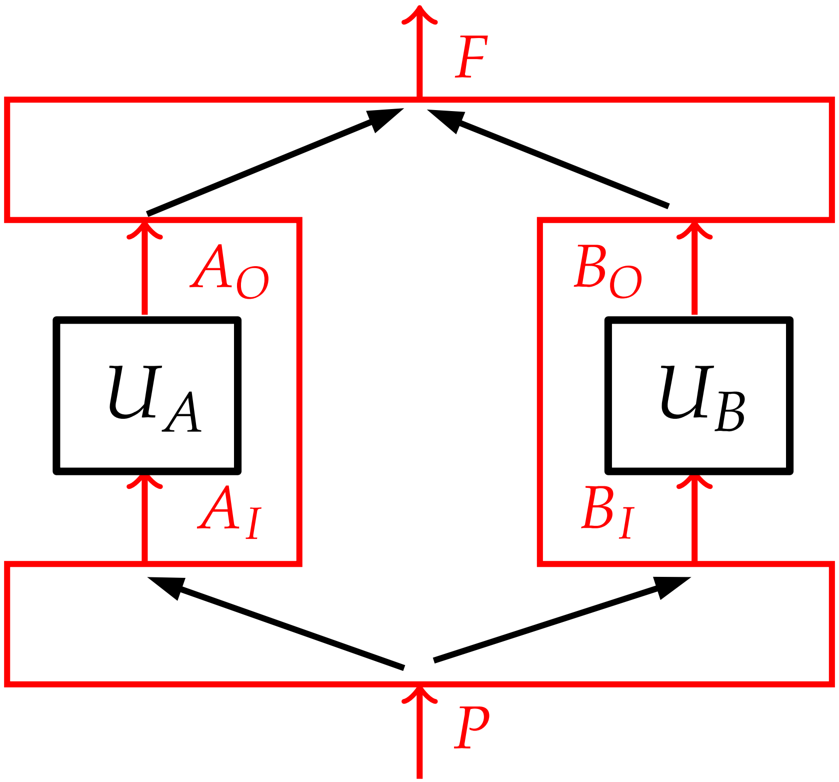
$$\mathbf{BQP}_{\mathrm{CTC}} = \mathrm{Post}\mathbf{BQP} = \mathbf{PP}$$

And now for something completely different

Definition: a process is the most general linear function that maps all CPTP maps to CPTP maps

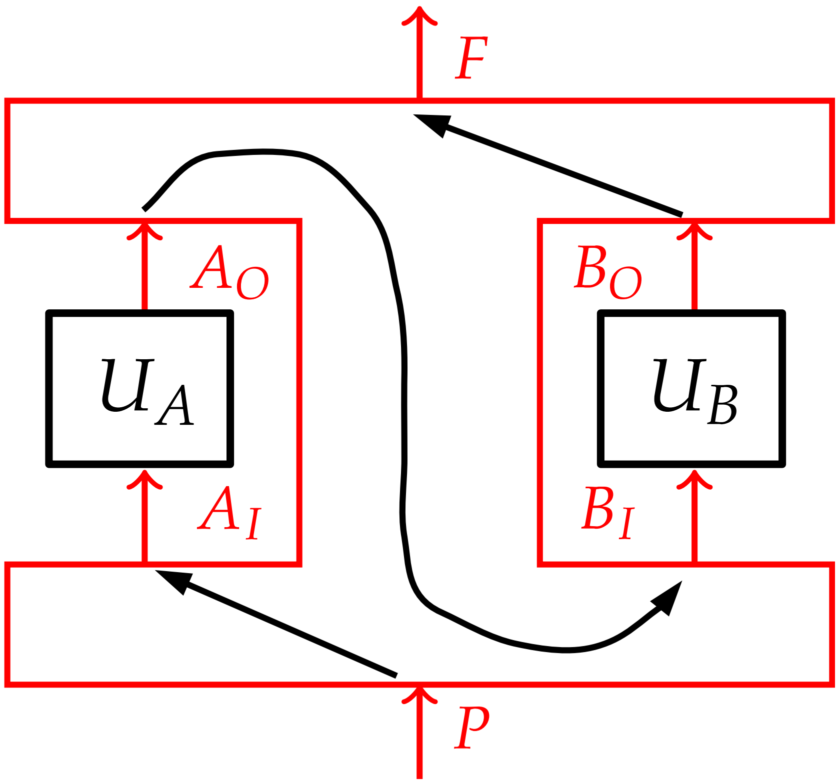


A nonsignalling process



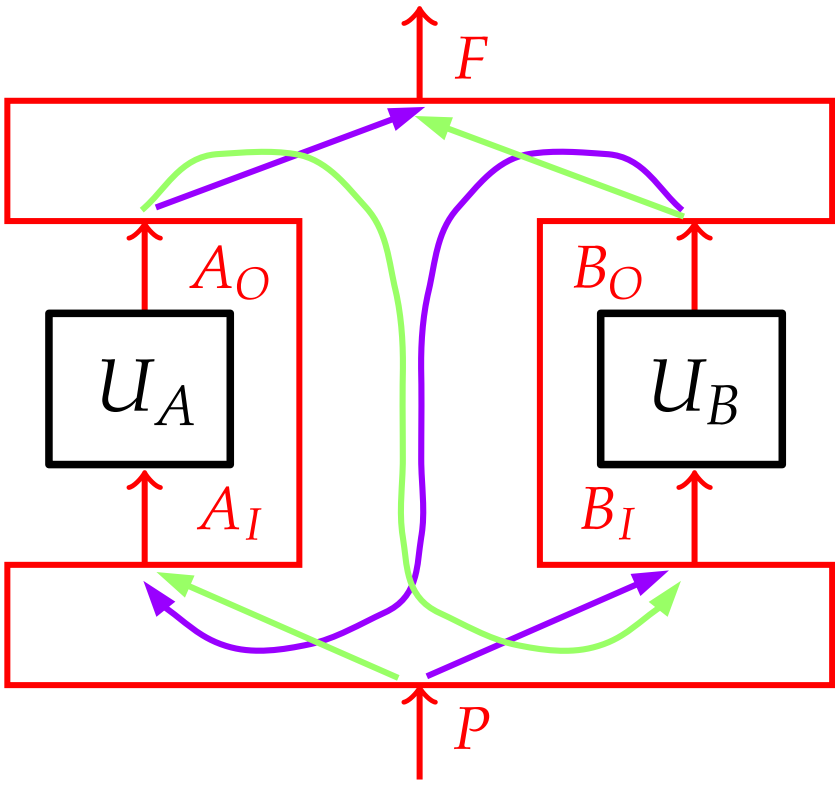
$$U_A, U_B \mapsto U_A \otimes U_B$$

A channel from Alice to Bob



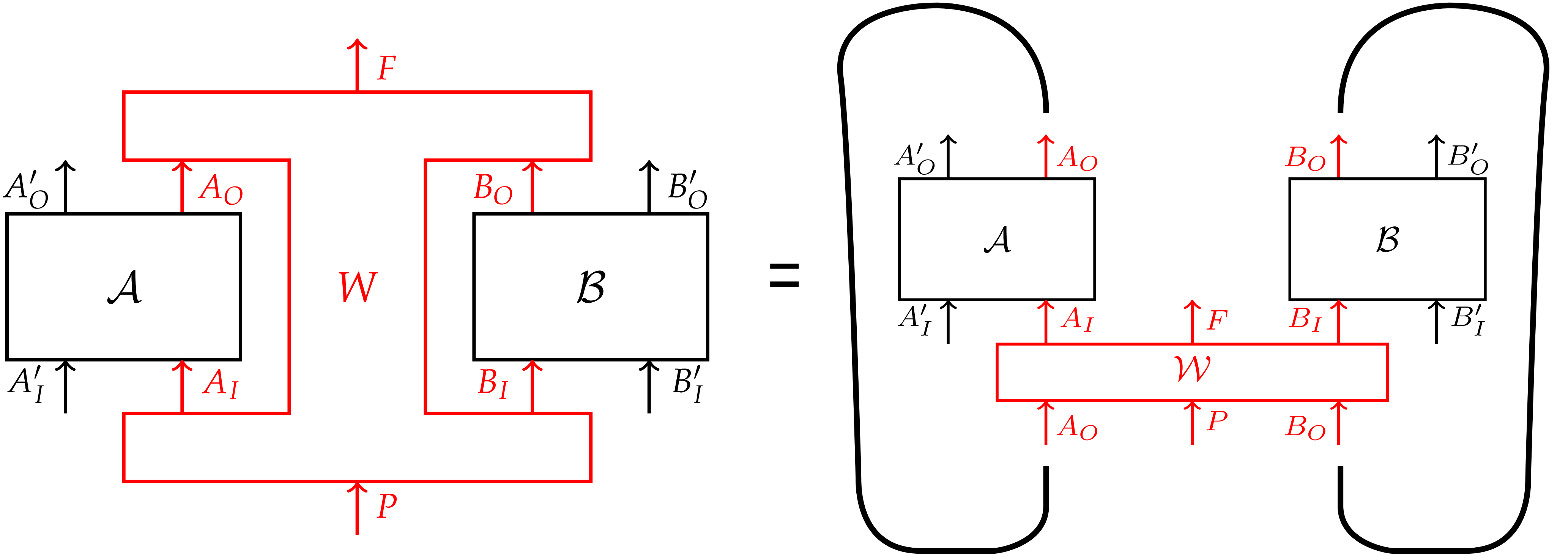
$$U_A, U_B \mapsto U_B U_A$$

The quantum switch

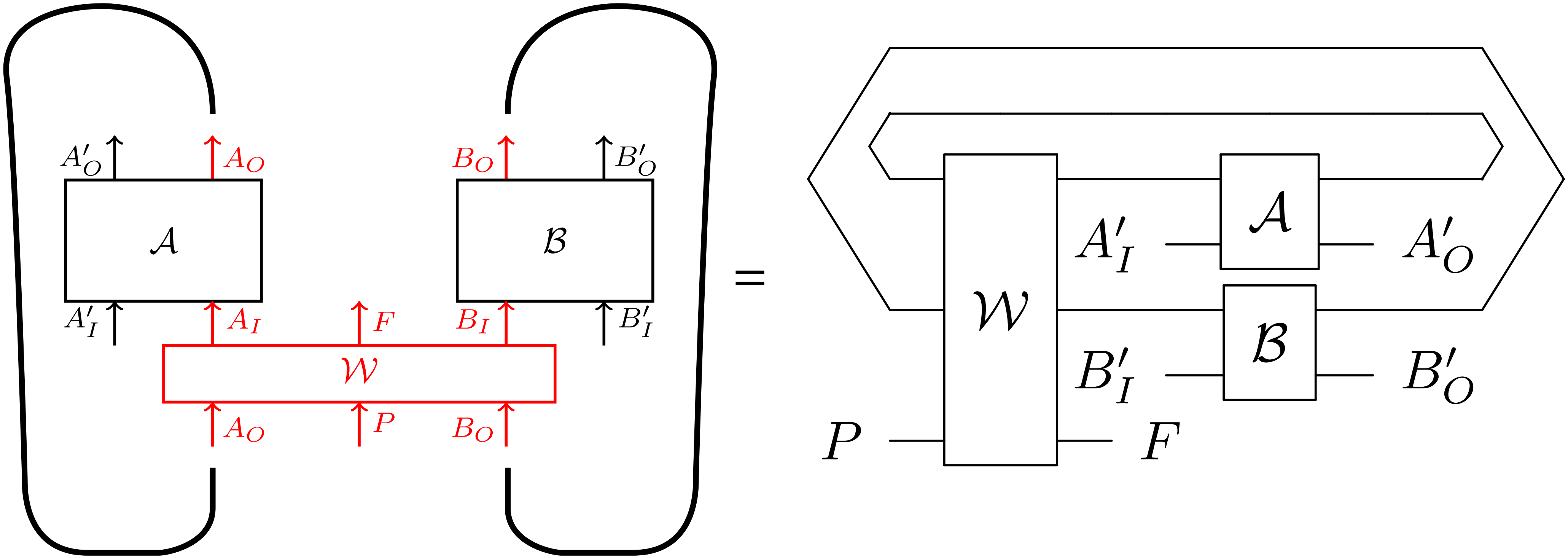


$$U_A, U_B \mapsto |0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B$$

What is a process?



A process is a bunch of CTCs

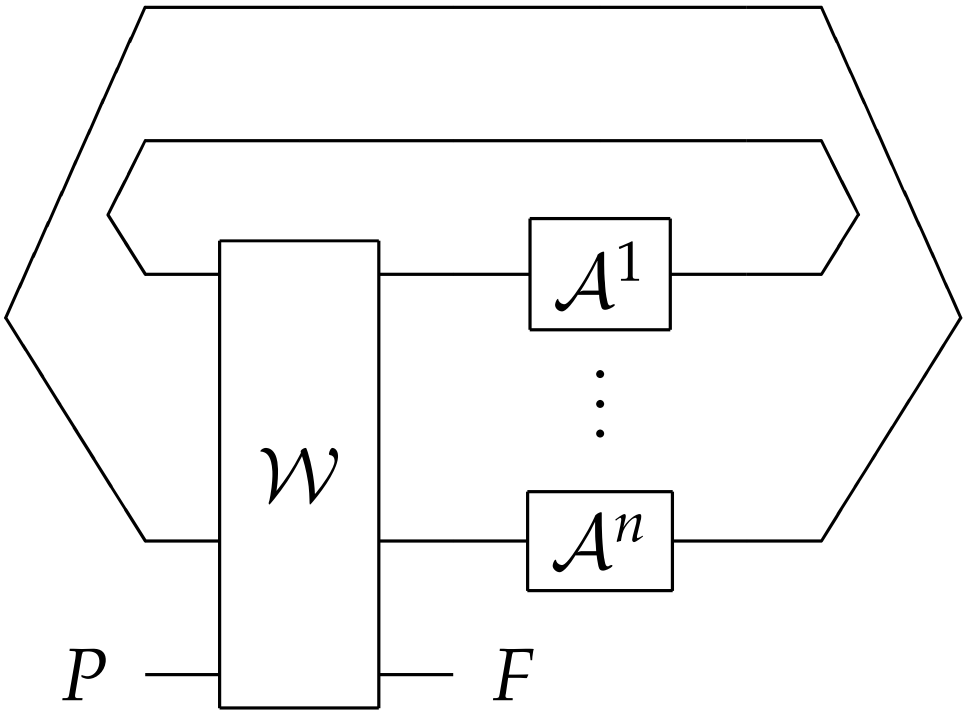


Processes are linear CTCs

Theorem: W is a valid process iff the probability of post-selection for all A and B is $\frac{1}{d_A^2 d_B^2}$

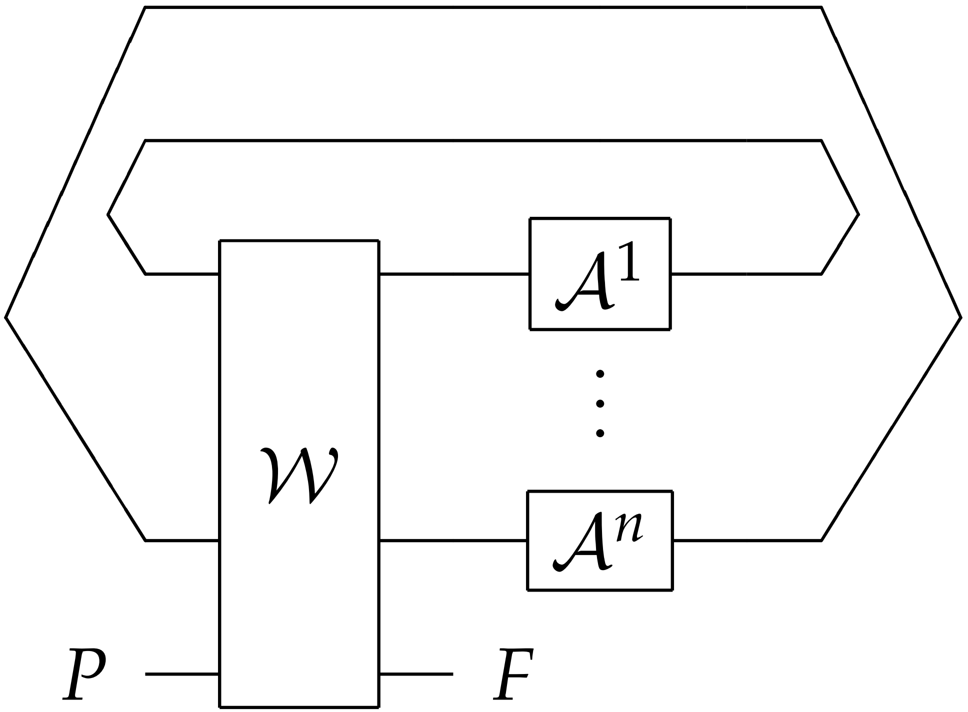
Complexity

Definition: $\mathbf{BQP}_{\ell\mathrm{CTC}}$ is the complexity class of problems that can be efficiently solved by a process



Complexity

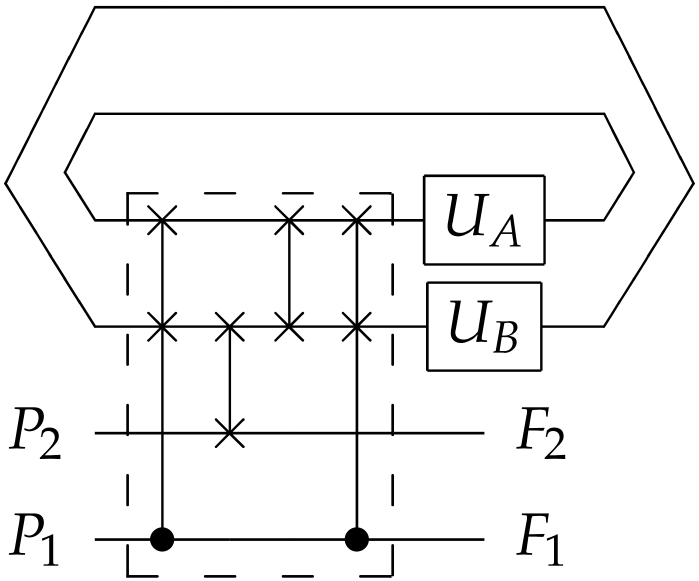
Definition: $\mathbf{BQP}_{\ell\mathrm{CTC}}$ is the complexity class of problems that can be efficiently solved by linear CTCs



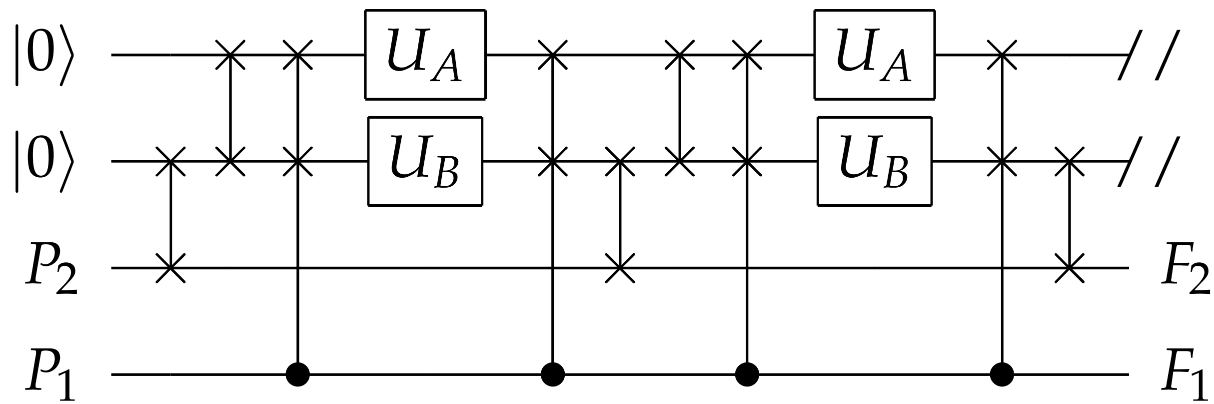
A loose upper bound

$$\mathbf{BQP}_{\ell\mathrm{CTC}} \subseteq \mathbf{BQP}_{\mathrm{CTC}} = \mathbf{PP}$$

The quantum switch

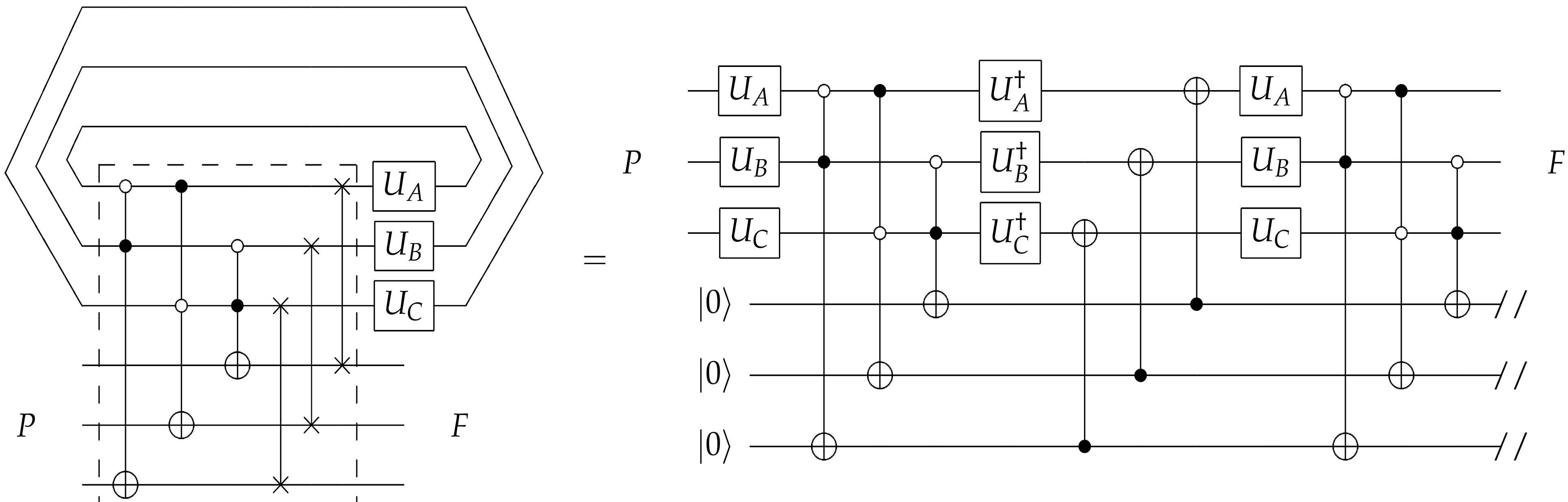


$=$



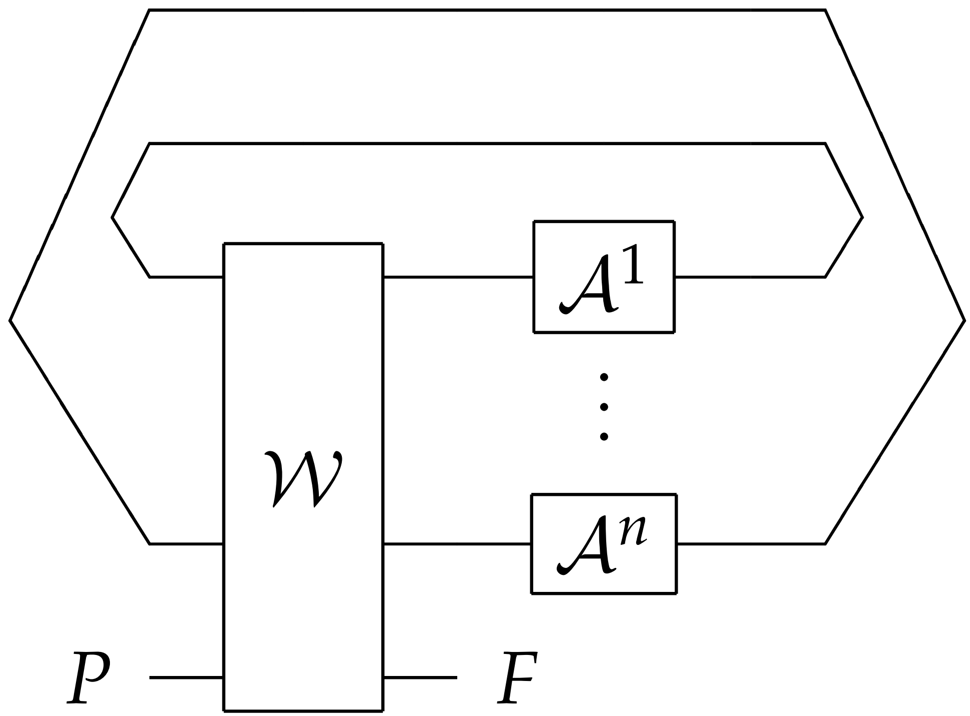
$O(n)$ $\Omega(n^2)$

The Swiss process



$O(n)$ $\Omega(n)$

A general process

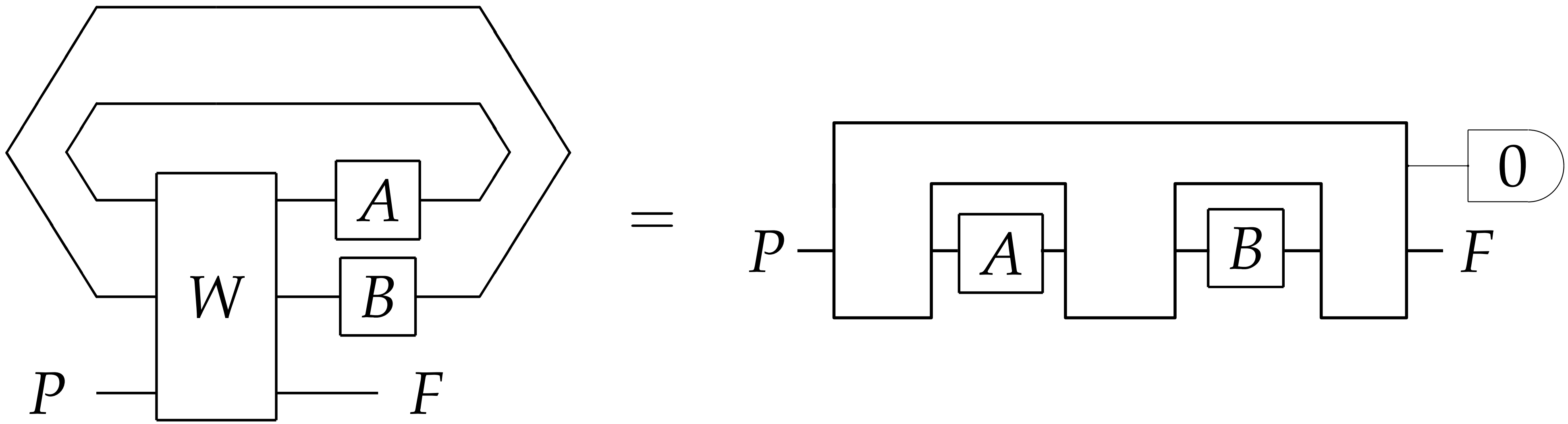


$p_{\text{success}} = \frac{1}{d^{2n}}$, so repeat it $O(d^{2n})$ times

There is room for improvement

Process	Complexity
Switch	$\Omega(n^2)$
Swiss	$\Omega(n)$
General	$O(d^{2n})$

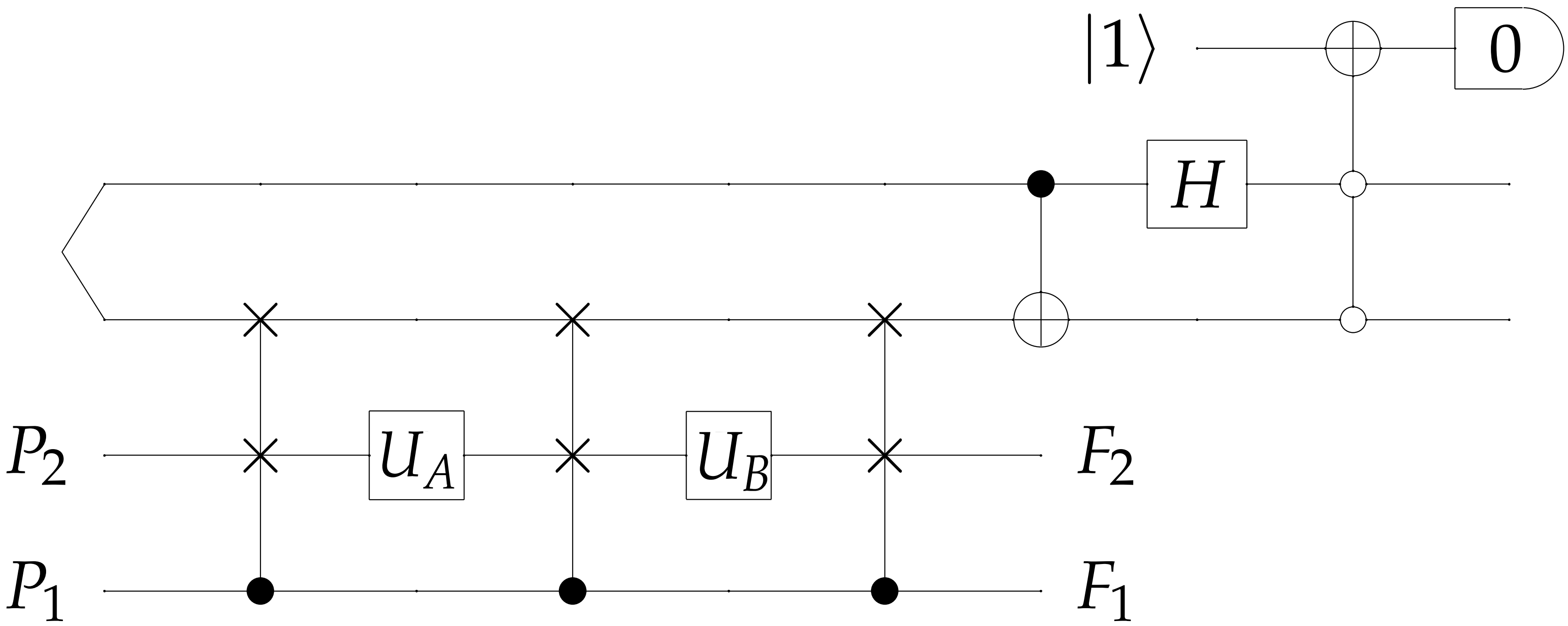
How to post-select optimally?



This is a SDP

$$
\begin{aligned}
\max p \\
\text{s.t.} \quad L \geq 0 \\
{}_{F}(pW + L) = {}_{B_O F}(pW + L) \\
{}_{B_I B_O F}(pW + L) = {}_{A_O B_I B_O F}(pW + L) \\
{}_{A_I A_O B_I B_O F}(pW + L) = {}_{P A_I A_O B_I B_O F}(pW + L) \\
\mathrm{tr}(pW + L) = d_P d_{A_O} d_{B_O}
\end{aligned}
$$

The quantum switch



$$p_{\text{success}} = \frac{1}{d^{2(n-1)}}$$

Challenge

Find a general way to simulate processes without postselection