

Computational advantage from quantum control of multiple causal orders: a demonstration of the N-switch

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INCT-IQ



serrapilheira



In collaboration with the group of the University of Concepción, Chile:



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J. Cariñe, M. M. Tadei, T. García, N. Guerrero, E. S. Gómez, L. Aolita, and G. Lima,
in preparation (2019).

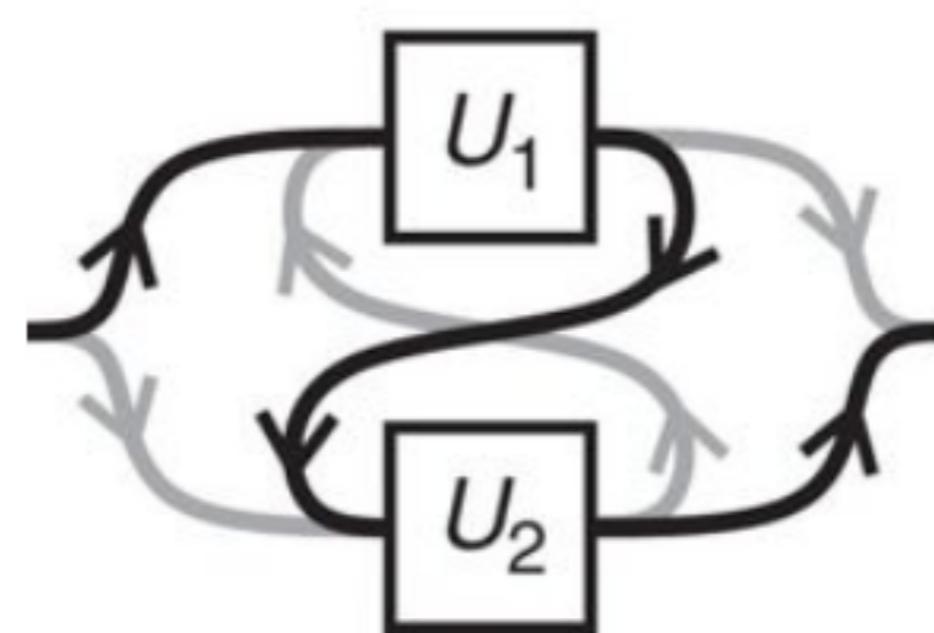
A new computational paradigm: quantum control of the connections between gates

- The **quantum switch**: implements **quantum control of gate orders with fewer queries to the gates** than any circuit with fixed gate order.

Chiribella, D'Ariano, Perinotti, and Valiron, Phys. Rev. A **88**, 022318 (13); Colnaghi, G. M. D'Ariano, S. Facchini, and P. Perinotti, Phys. Lett. A **376**, 2940 (2012).

- Optical-table implementations are based on **conceptually simple interferometers**.

Araújo, Costa, and Brukner, PRL (14); Procopio et al., Nat. Comms. (15); Rubino et al., Sci. Adv. (17); Goswami et al., PRL **121**, 090503 (2018), Wei et al., PRL (18).



*One degree of a photon coherent controls
the gate order on another degree of freedom*

Quantum control of causal orders as resource for information-theoretic tasks

- Allows for **perfect discrimination of no-signalling channels** not perfectly distinguishable by circuits with a fixed gate order.
Chiribella, PRA (2012).
- Solves **a decision problem about black-box unitary permutations** efficiently.
Araújo, Costa, and Brukner, PRL (14); Procopio et al., Nat. Comms. (15).
- This translates into an **exponential advantage in (quantum) communication complexity** for a multi-partite communication-complexity problem.
Guerin et al., PRL (16); Wei et al., PRL (18).
- Activates the (classical and quantum) **capacities of zero-capacity communication channels**.
Ebler, Salek, and Chiribella, PRL 120, 120502 (2018); Goswami, Romero, and White, arXiv:1807.07383; Chiribella et al., arXiv:1810.10457.

So far, quantum control of only 2 causal orders experimentally demonstrated

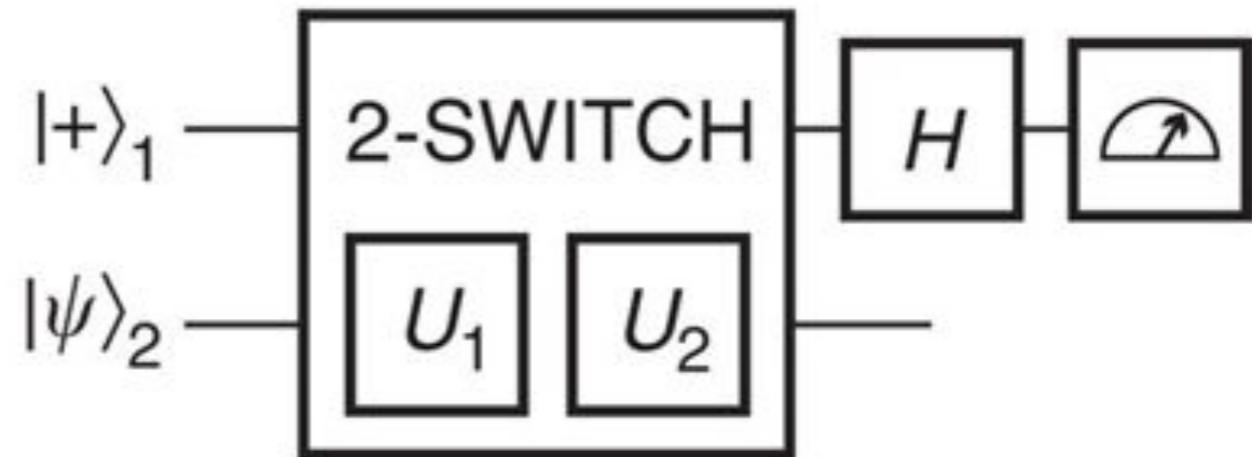
How about the N-switch for $N > 2$?

The Araujo-Costa-Brukner (ACB) algorithm

- The N -switch solves a **decision problem on the commutation relations between permutations of black-box unitaries** with query complexity $O(N)$ instead of $O(N^2)$.

Araújo, Costa, and Brukner, PRL (14).

If U_1 and U_2 commute, the control-qubit measurement outputs 0; if they anti-commute, it outputs 1!



- Not only an **interesting computational primitive** but also a **potentially good tool to benchmark experimental realisations** of the N -switch.
- However, the **target-system dimension must be at least N** , making the protocol **non-optimal for experimental applications**.



Outline of the talk:

Modify the protocol to relax the dimension constraint



Achieve experimental demonstration for $N=4$



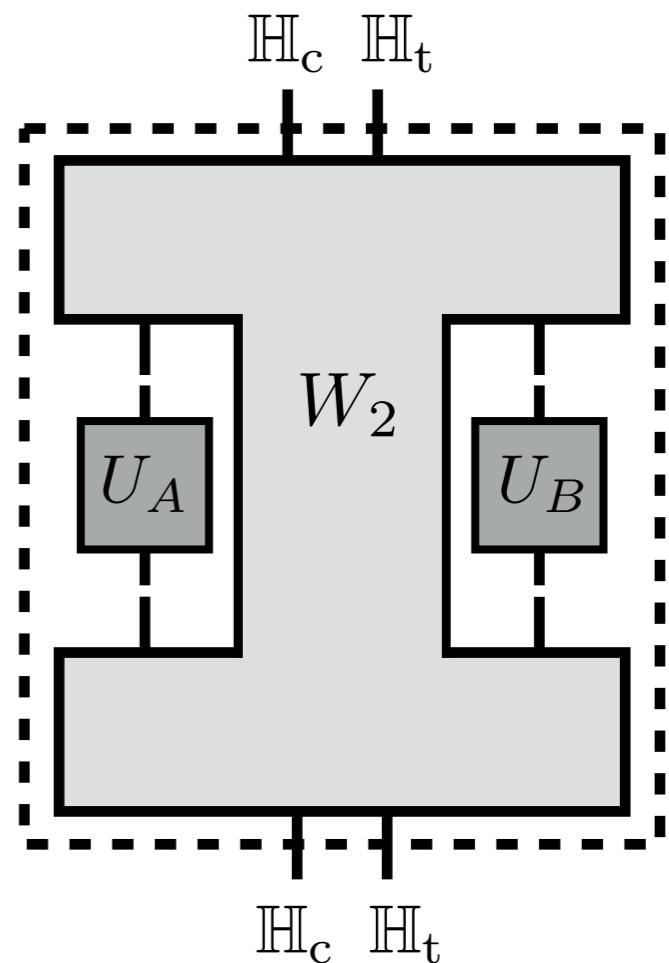
The N-switch gate

The N -switch: process versus gate

- The 2-switch unitary gate:

$$S_2 := |0\rangle\langle 0| \otimes U_B U_A + |1\rangle\langle 1| \otimes U_A U_B$$

Concatenation of the target-system unitaries U_A and U_B with the process matrix W_2



- More generally, the N -switch unitary gate:

$$d_c := \dim(\mathbb{H}_c) \geq N! \quad d_t := \dim(\mathbb{H}_t)$$

$$S_{N!} := \sum_{x \in [N!]^N} |x\rangle\langle x| \otimes \Pi_x$$

$$[N!] := \{0, 1, \dots, N! - 1\}$$

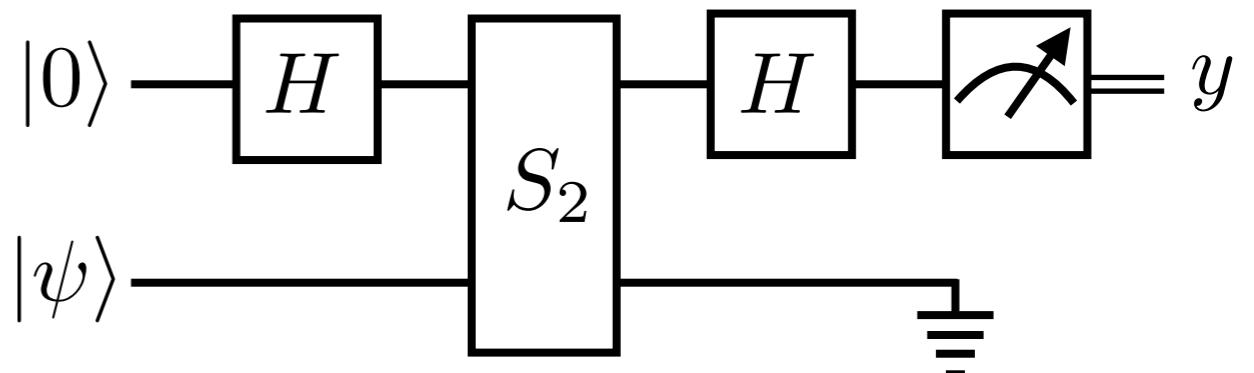
$$\Pi_x := U_{\sigma_x(N)} \dots U_{\sigma_x(2)} U_{\sigma_x(1)}$$

x -th permutation of $\{1, 2, \dots, N\}$

The Unitary Permutation problem as a decision problem

Problem statement

- **Promise:** Given an N -tuple $\mathcal{U} := \{U_i\}_{i=1,2,\dots,N}$ of black-box unitaries U_i on \mathbb{H}_t , satisfying
 $P_y : \forall x \Pi_x = \omega^{x \cdot y} \Pi_0$, with $\omega := e^{i \frac{2\pi}{N!}}$, for some $y \in [N!]$.
- **Task:** Decide which one of the $N!$ properties \mathcal{U} satisfies, i.e. **find** y .
- **Algorithm:** based on the Hadamard test.



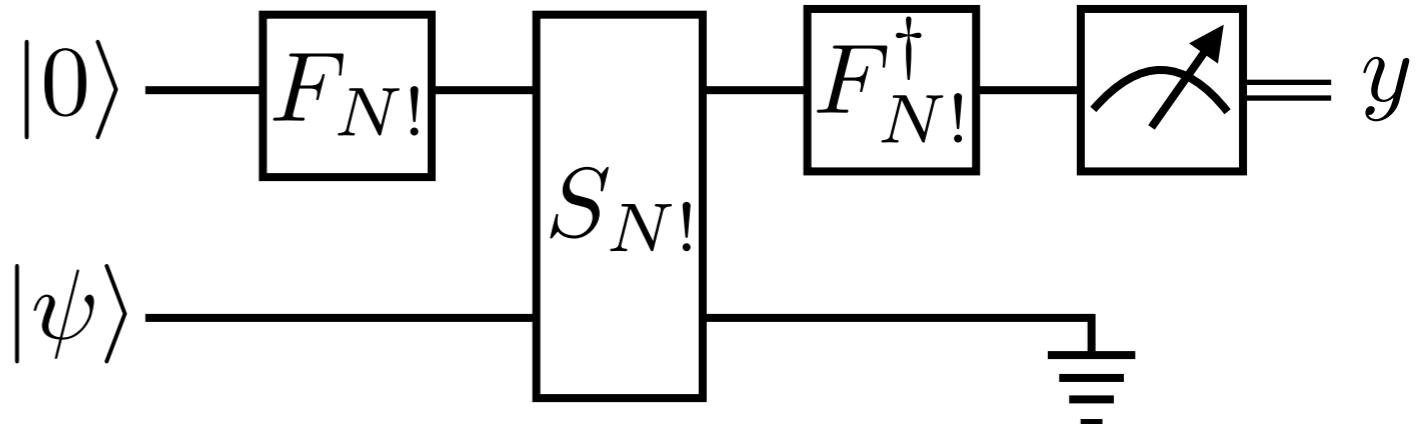
*For $N=2$, consumes only 1 use
of U_A and U_B while all other
known algorithms consume 2!!!*



$$P(y') = \left\| \frac{U_A U_B + (-1)^{y'} U_B U_A}{2} |\psi\rangle \right\|_2^2 \\ = \delta_{y,y'} \quad \forall |\psi\rangle$$

The Araujo-Costa-Brukner (ACB) algorithm

- More generally, for arbitrary N :



$$|0\rangle_c |\psi\rangle_t \rightarrow \frac{1}{\sqrt{N!}} \sum_{x \in [N!]} |x\rangle_c |\psi\rangle_t$$

$$\rightarrow \frac{1}{\sqrt{N!}} \sum_{x \in [N!]} |x\rangle_c \Pi_x |\psi\rangle_t$$

Only $O(1)$ uses of \mathcal{U} [query complexity $O(N)$] while all other known algorithms use $O(N)$ [query complexity $O(N^2)$]!!!



$$\stackrel{\textcolor{red}{\rightarrow}}{(because\ of\ P_y)} = \left(\frac{1}{\sqrt{N!}} \sum_{x \in [N!]} e^{i \frac{2\pi x y}{N!}} |x\rangle_c \right) \Pi_0 |\psi\rangle_t$$

$$\rightarrow |y\rangle_c \Pi_0 |\psi\rangle_t$$

The target-register dimension caveat

- The type of promise imposes **non-trivial constraint on the target system dimension d_t** :

$$P_y : \forall x \ \Pi_x = \omega^{x \cdot y} \Pi_0, \text{ with } \omega := e^{i \frac{2\pi}{N!}}, \text{ for } y \in [N!].$$

$$\Rightarrow \Pi_1 = e^{i \frac{2\pi}{N!}} \Pi_0 \Rightarrow \text{Det}[\Pi_1] = e^{i \frac{2\pi d_t}{N!}} \text{Det}[\Pi_0] \Rightarrow d_t \geq N!$$



$$\text{Det}[\Pi_x] = \text{Det}[\Pi_0]$$

(Determinant of a product equals
product of determinants)

*The target register must be at least
as large as the control register*



Idea: modify the type of promise to relax the constraint on d_t .

J. Cariñe, M. M. Tadei, T. García, N. Guerrero, E. Sepulveda, L. Aolita, and G. Lima,
in preparation (2019).

A slightly different decision problem

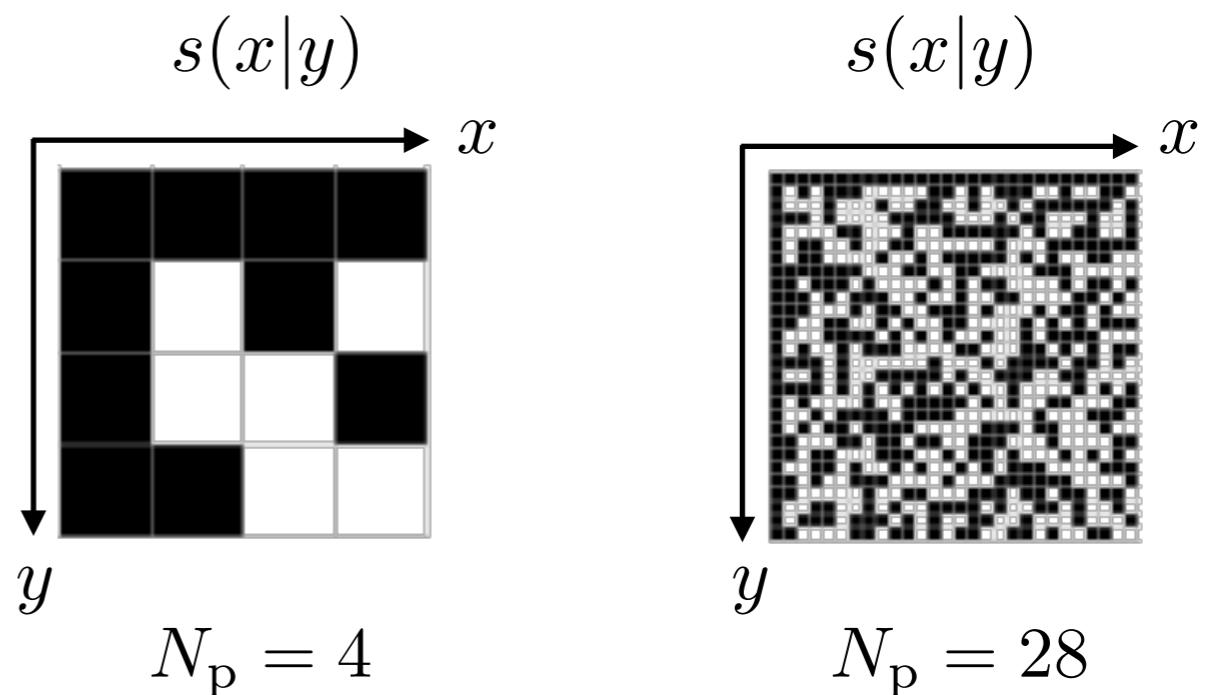
- **Promise:** Given an N -tuple $\mathcal{U} := \{U_i\}_{i=1,2,\dots,N}$ of black-box unitaries U_i on \mathbb{H}_t , satisfying

$P_y : \forall x \in [N_p], \Pi_x = s(x|y) \Pi_0$, with $s(x|y) \pm 1$, for some $y \in [N_p]$.

$$N \leq N_p \leq N!$$

(Number of permutations)

- **Task:** Decide which one of the N_p properties \mathcal{U} satisfies, i.e. **find** y .



- **Determinant condition:** much less stringent constraint on d_t .

$$\text{Det}[\Pi_x] = \text{Det}[\Pi_0]$$

↓

$$\text{Det}[\Pi_x] = s(x|y)^{d_t} \text{Det}[\Pi_0] \Rightarrow d_t \text{ must just be even!!!}$$

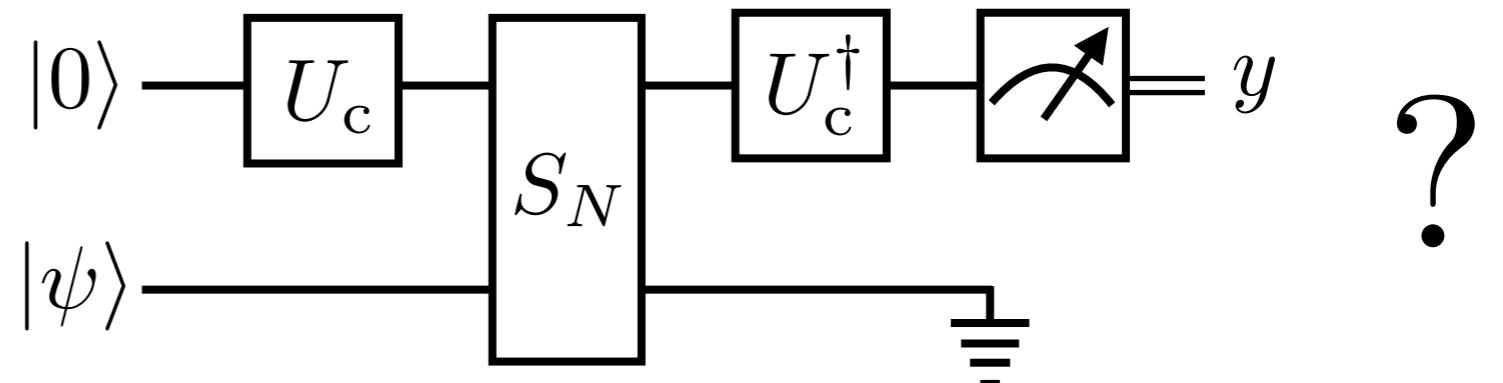
Qubits IN!!!



(instead of equal to N_p)

Can one do something similar to the ACB algorithm?

Is there U_c on \mathbb{H}_c such that



$$|0\rangle_c |\psi\rangle_t \rightarrow \sum_{x \in [N_p]} \phi_x |x\rangle_c |\psi\rangle_t \rightarrow \sum_{x \in [N_p]} \phi_x |x\rangle_c \Pi_x |\psi\rangle_t$$

$$= \left(\sum_{x \in [N_p]} \phi_x s(x|y) |x\rangle_c \right) \Pi_0 |\psi\rangle_t \rightarrow |y\rangle_c \Pi_0 |\psi\rangle_t$$

?

(because of P_y)

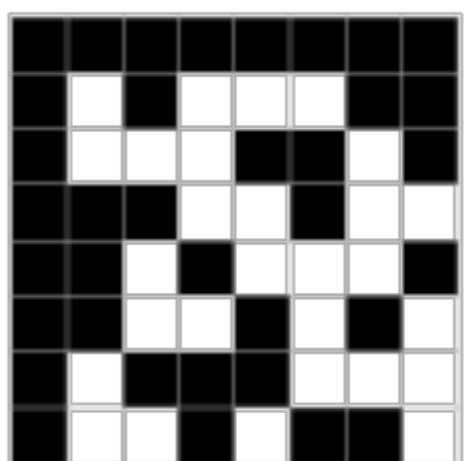
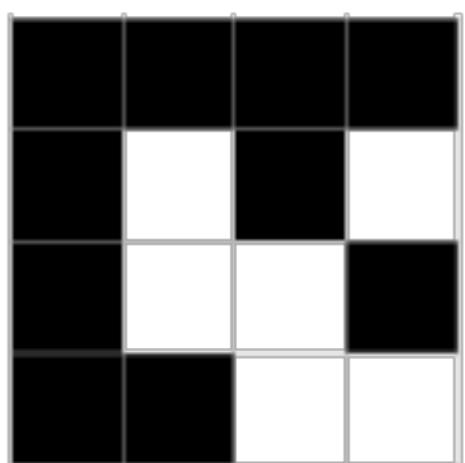
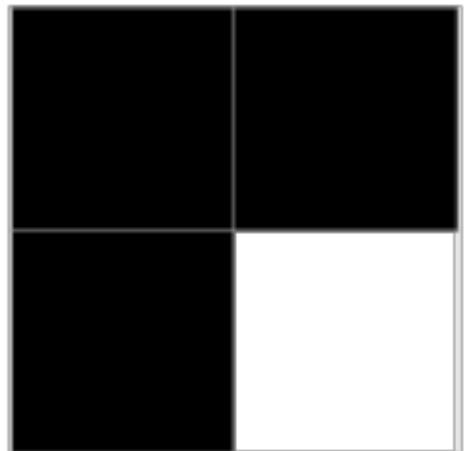
$$\Leftrightarrow U_c |y\rangle_c = \sum_{x \in [N_p]} \phi_x s(x|y) |x\rangle_c, \quad \forall y \in [N_p], \text{ and } s(x|0) = 1, \quad \forall x \in [N_p].$$

The answer was in front of our nose...

Take $\phi_x = \frac{1}{\sqrt{N_p}}$, $\forall x \in [N_p]$, and $s(x|y)$ a Hadamard matrix!!!

Hadamard matrices:

- Square matrices, entries ± 1 , orthogonal columns and rows;
- Any two rows (columns) have same entries in half the columns (rows) and opposite ones in the other half;
- Can only exist for dimensions 1, 2, or multiples of 4;
- Conjectured to exist for all multiples of 4.



$$\Rightarrow U_c = H_{N_p} !!!$$

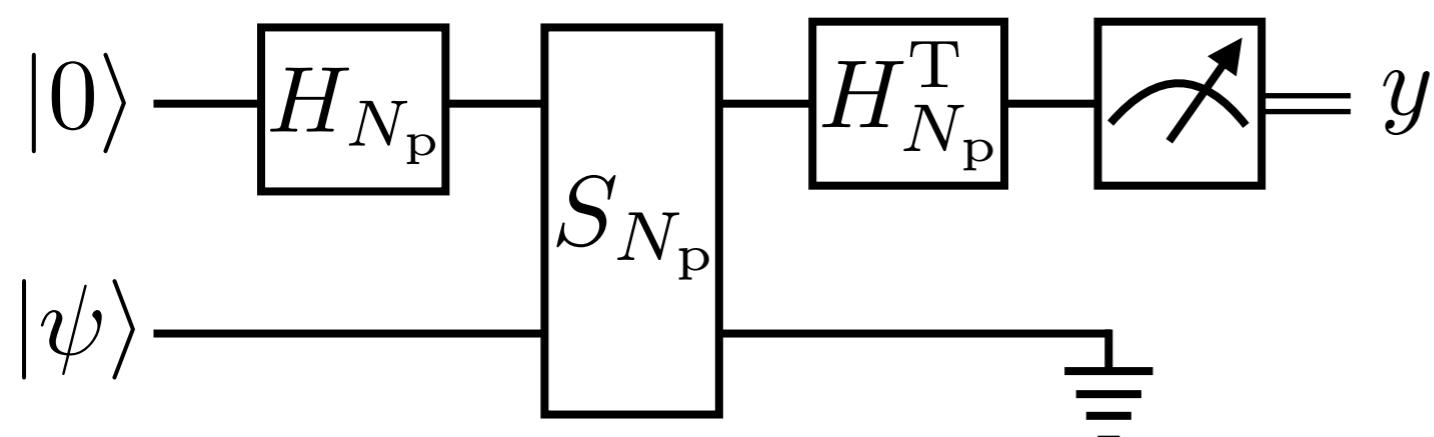


↑
Quantum Hadamard gate of dimension N_p

(# of permutations must equal a dimension for which Hadamard matrices exist)

Final form of the problem statement

- **Promise:** Given N -tuples $\mathcal{U} := \{U_i\}_{i=1,2,\dots,N}$ of black-box unitaries U_i on \mathbb{H}_t , satisfying
$$\mathcal{P}_y : \forall x \in [N_p], \Pi_x = s(x|y) \Pi_0, \text{ with } s(\cdot|y) \text{ the } y\text{-th row of a Hadamard matrix } H_{N_p}, \text{ for some } y \in [N_p].$$
- **Task:** Decide which one of the N_p properties \mathcal{U} satisfies, i.e. **find which row y is.**



Query complexity $O(N)$ against $O(N^2)!!$

$$d_c = N_p, d_t = 1, 2, 4, 8, \dots$$



Can the algorithm be experimentally implemented in a practical way?

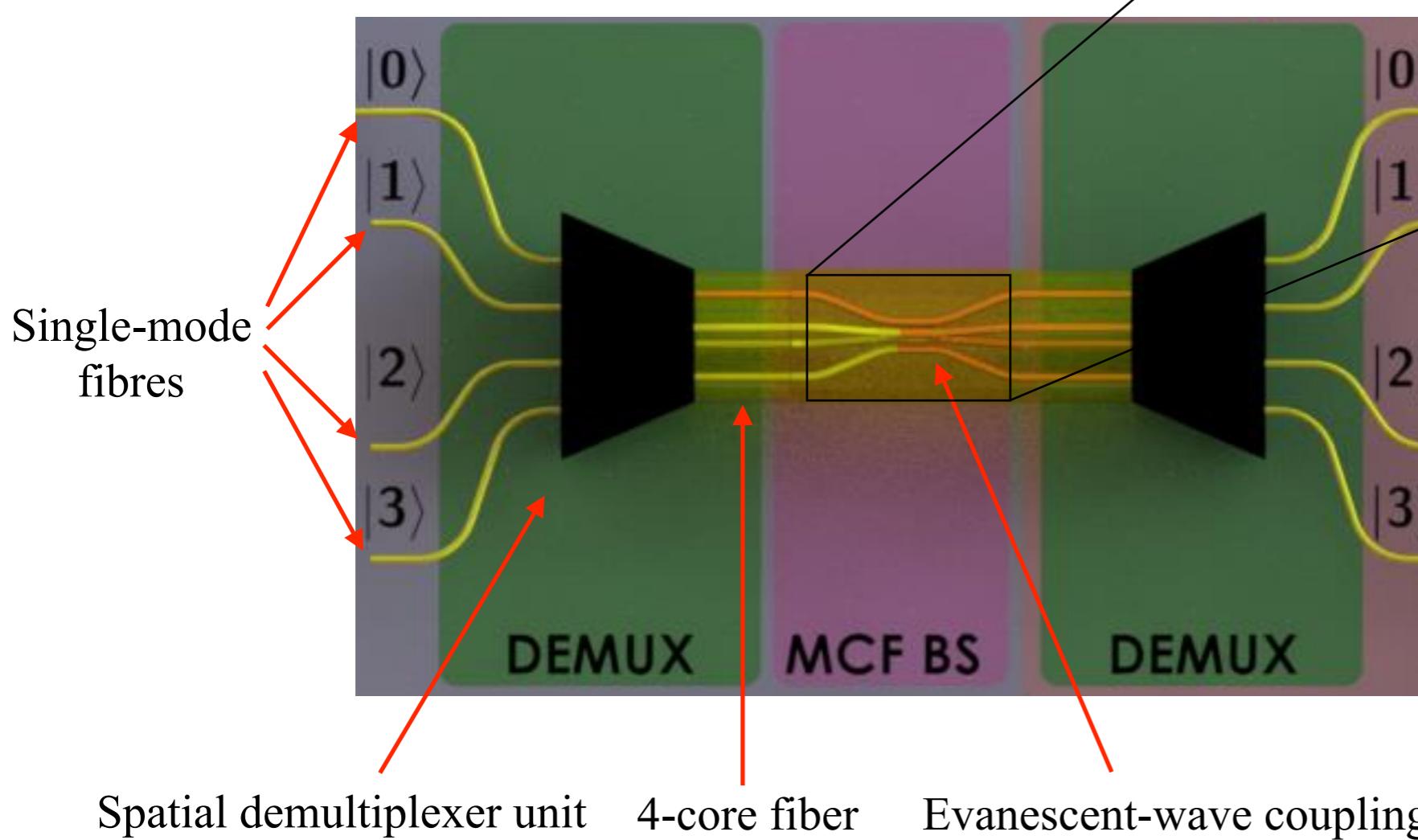
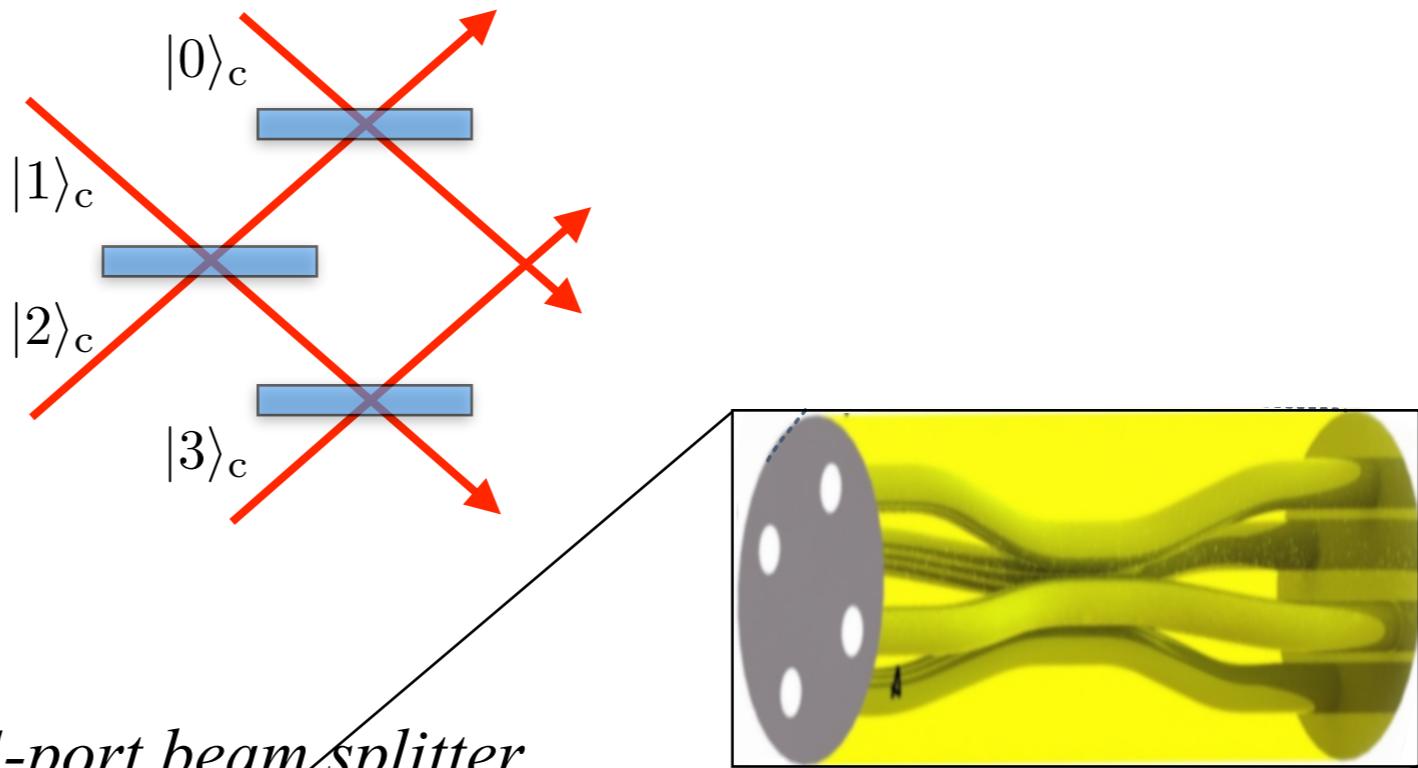


Experimental demonstration of the N-switch for N=4

J. Cariñe, M. M. Tadei, T. García, N. Guerrero, E. Sepulveda, L. Aolita, and G. Lima, in preparation (2019).

Experimental high-dimensional quantum Hadamard gates

- A sequence of 2^k balanced beams splitters gives a Hadamard matrix:

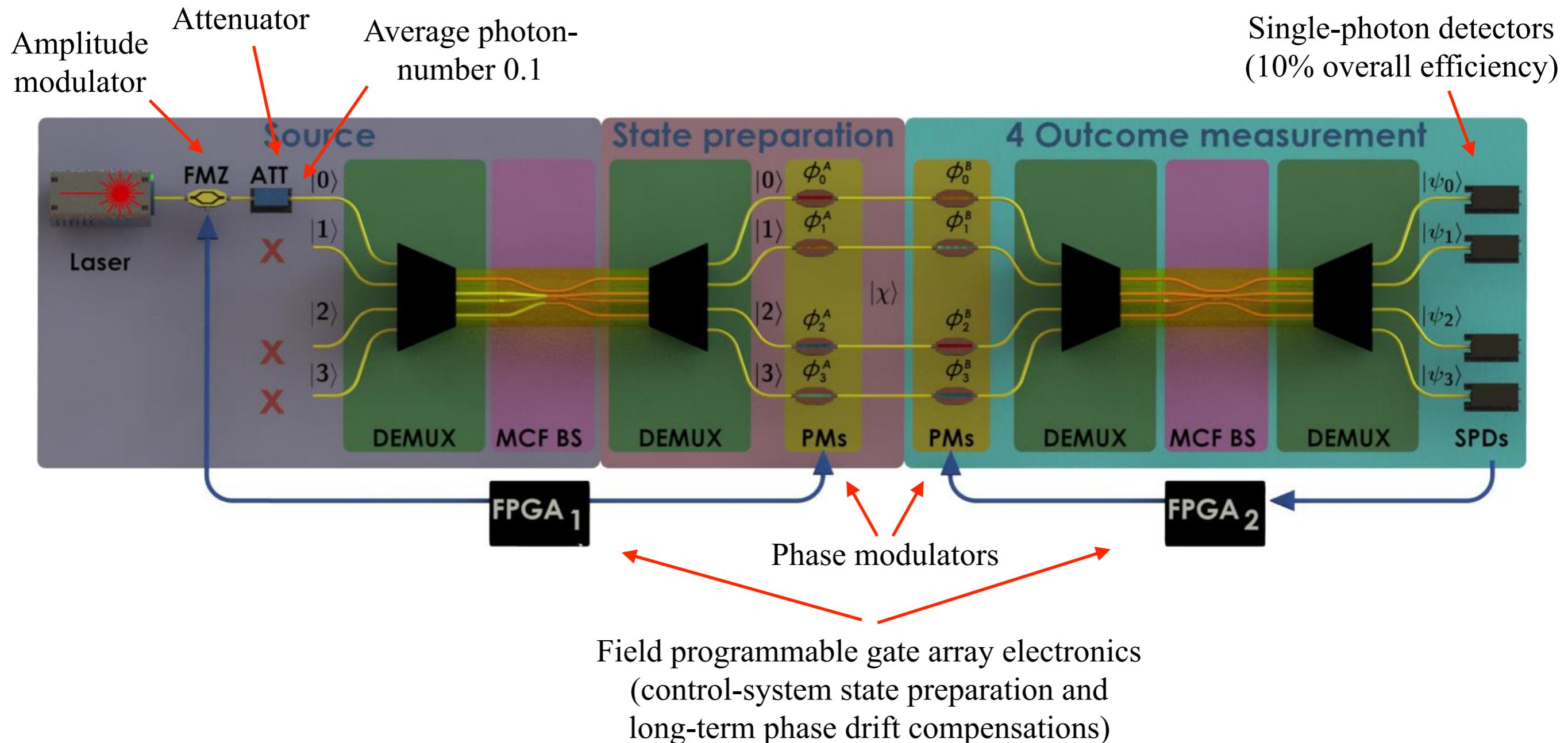


Implements the gate

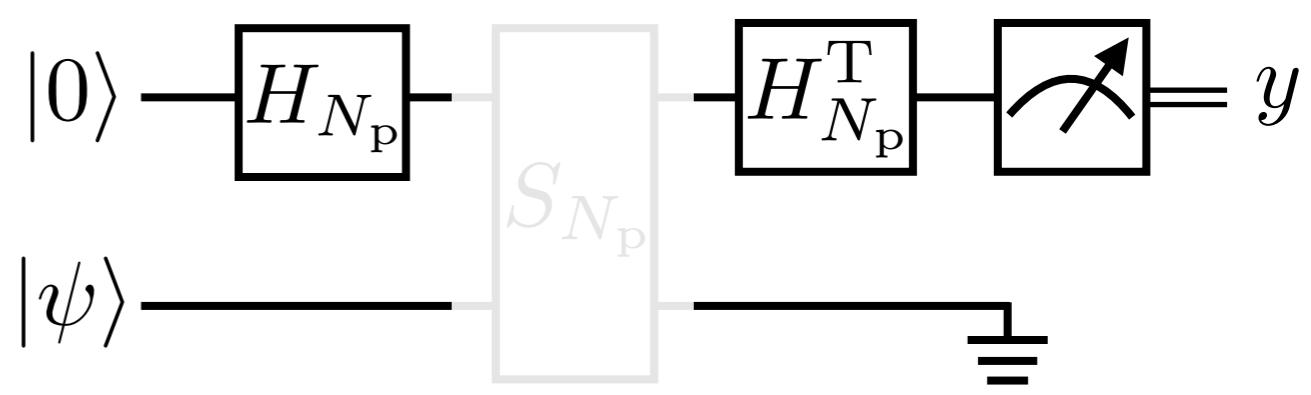
$$H_4 = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$



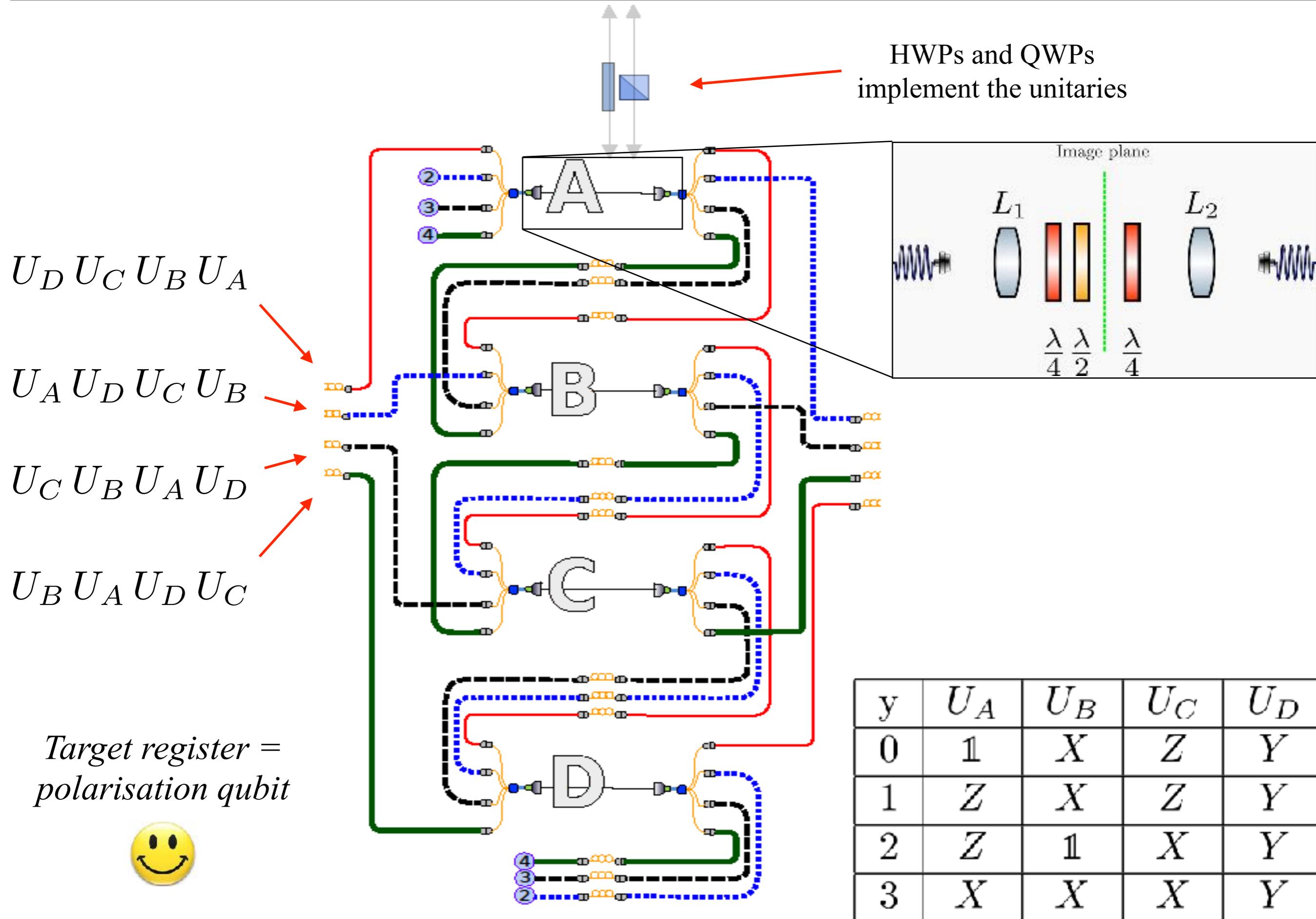
A compact, self-realigning 4-port Mach-Zehnder interferometer:



Control register = path ququart



The experimental 4-router and the unitaries

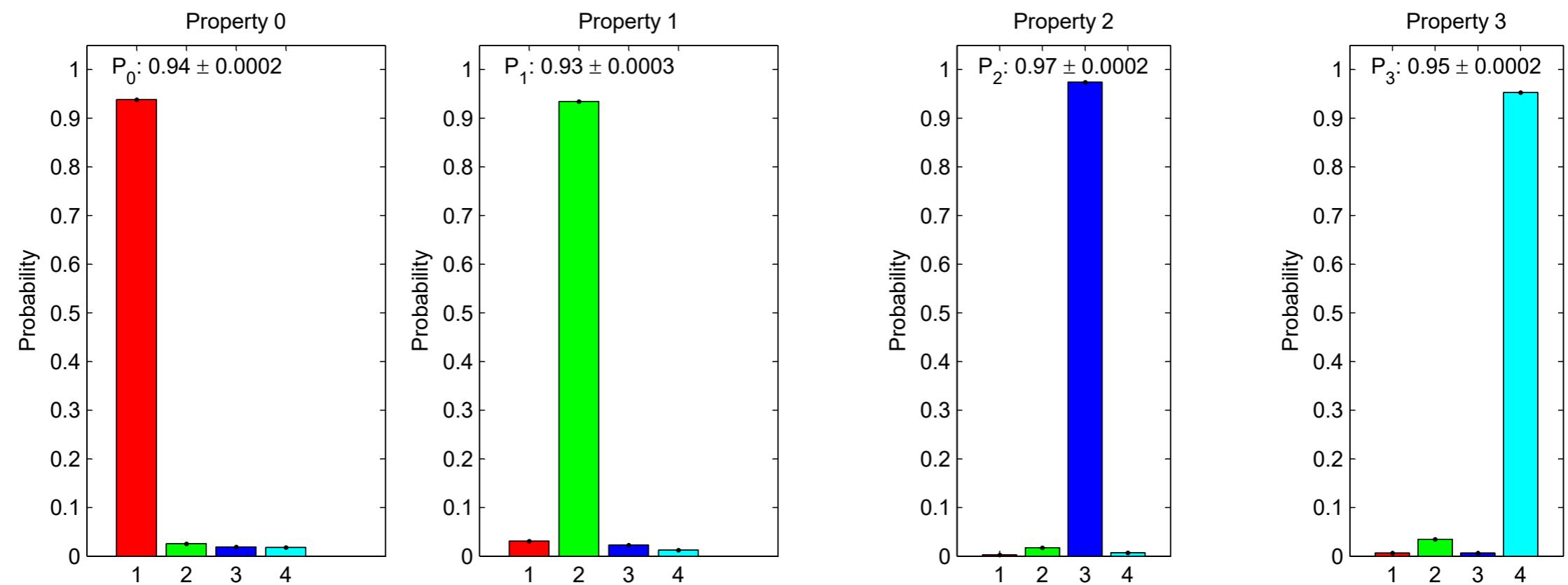


Experimental results

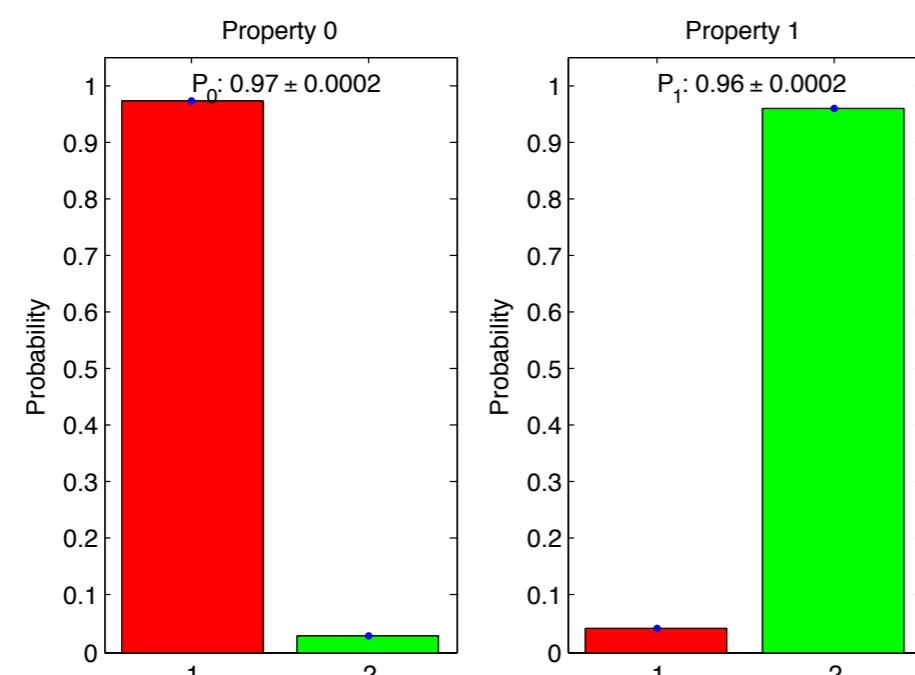
Experimentally deciding the Hadamard matrix row

Probabilities of measuring y as functions of y (approx. 10^6 measurements):

$N_p = 4$



$N_p = 2$



Average $p_{\text{success}} = 0.964$

(obtained by blocking 2 paths of
the same 4-router!!!)



However, it is possible to decide the problem without the N-switch....

Query complexity of the problem

Alternative approaches to find y :

Query Complexity:

1. Tomographically reconstruct each U_i and then multiply them:

(# of matrices to reconstruct)

$$O\left(\frac{N}{\left(\frac{\varepsilon}{N}\right)^2}\right) = O\left(\frac{N^3}{\varepsilon^2}\right)$$

(statistical error for each matrix)

(constant statistical error for the product)

2. Tomographically reconstruct each Π_x :

(# of permutations/phases to estimate)

$$O\left(\frac{N_p N}{\varepsilon^2}\right) \geq O\left(\frac{N^2}{\varepsilon^2}\right)$$

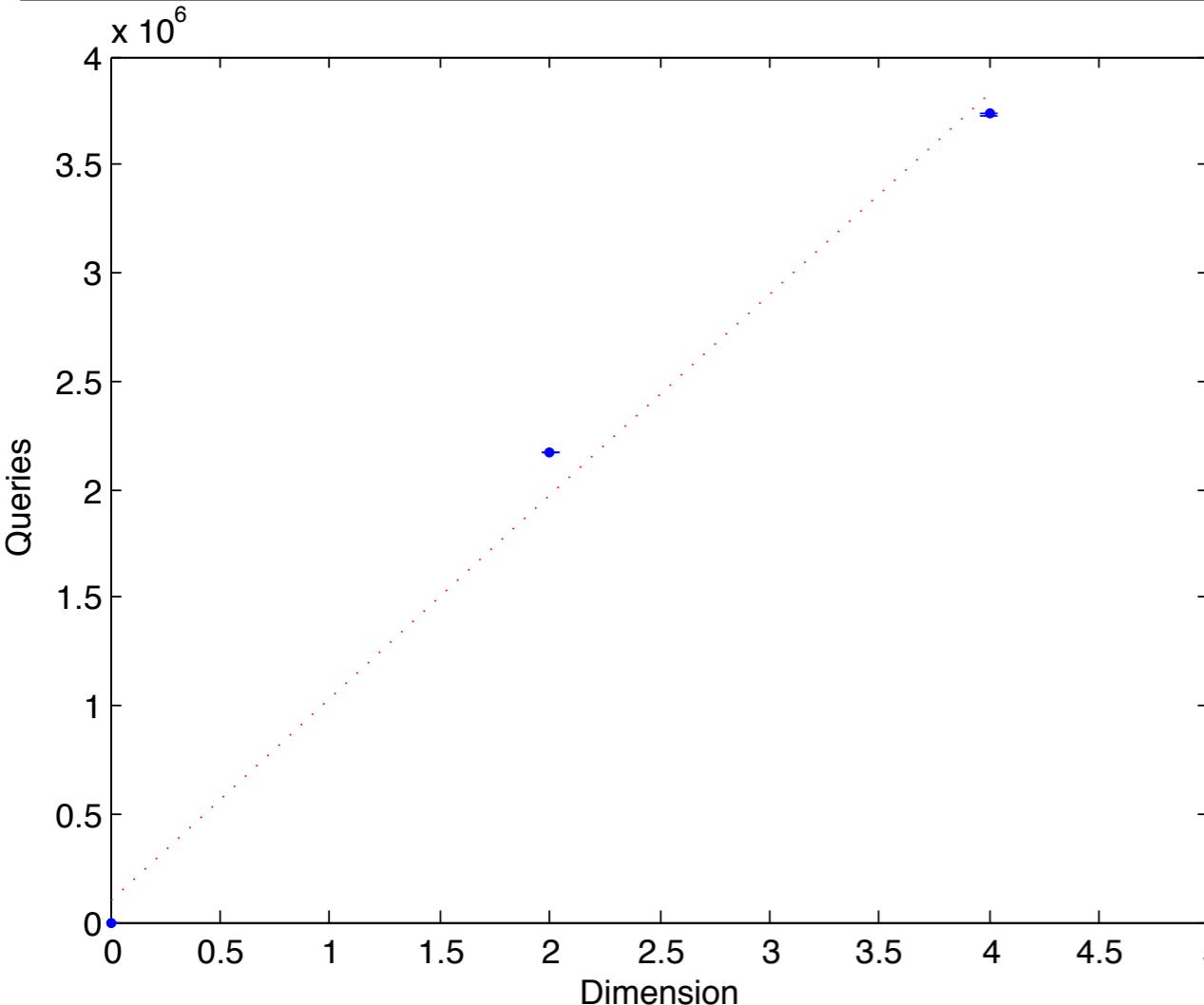
(query complexity per permutation/phase)

3. Directly estimate each $s(x|y)$ (e.g. with the Hadamard test):

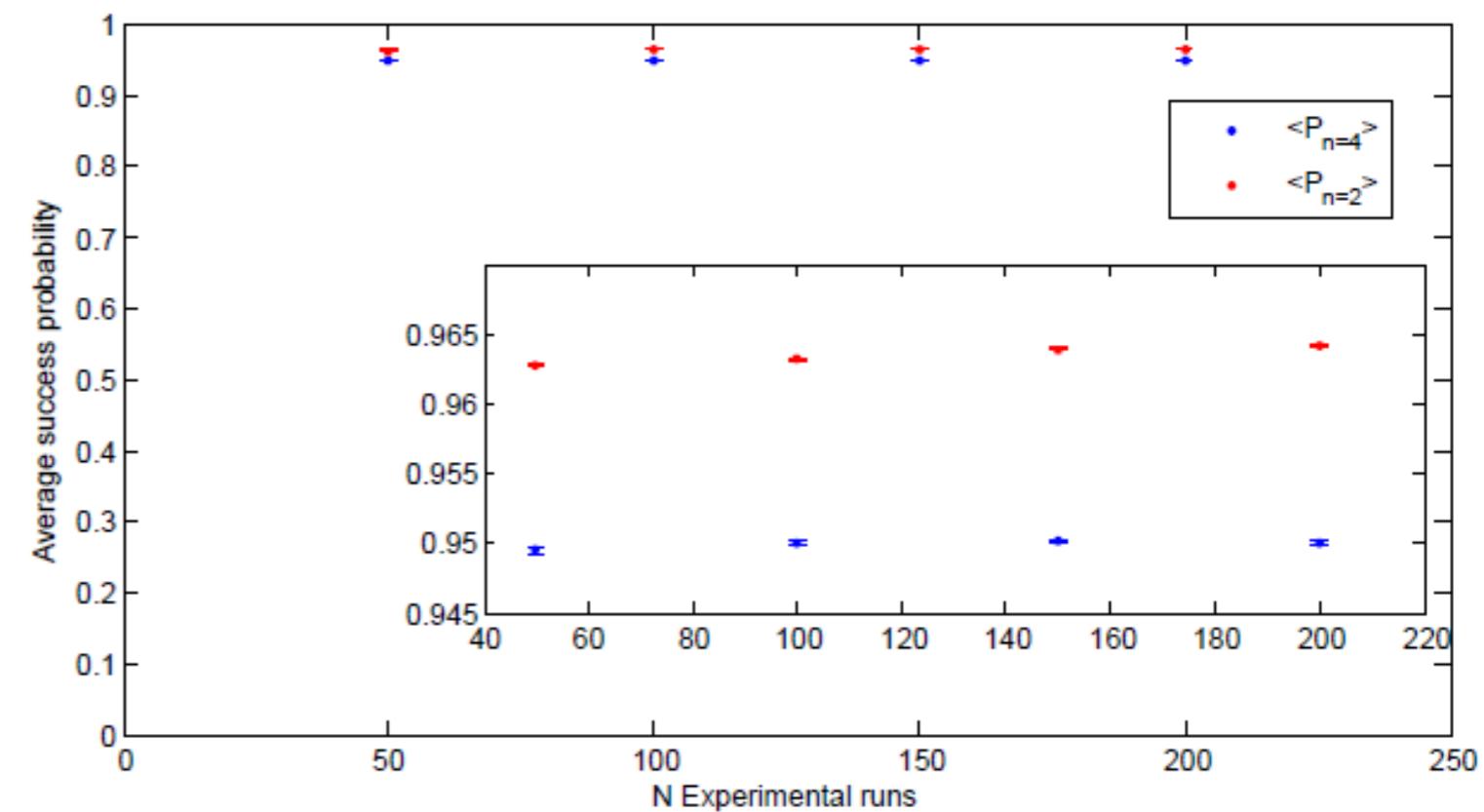
All scale worse than with the switch



Experimental query complexity



Average successful probabilities over N experimental runs accumulated.



Conclusions

- A decision problem with softer constraints on the target-system dimension.
- The N -switch solves it with query complexity quadratically better than other known algorithms.
- First implementation for $N=4$ different gate orders (modulo philosophical discussions).
- $N=2$ with essentially the same setup (just blocking two paths!).
- Optical-table: compact, programmable Mach-Zehnder interferometer with multi-core fibres and multi-port beam splitters.
- Outlook:
 1. More causal orders in the superposition;
 2. Multipartite communication complexity applications (EE games);
 3. Multipartite entanglement of temporal orders;
 4. Other applications?

Thank you for your attention!

(Postdoc positions soon)

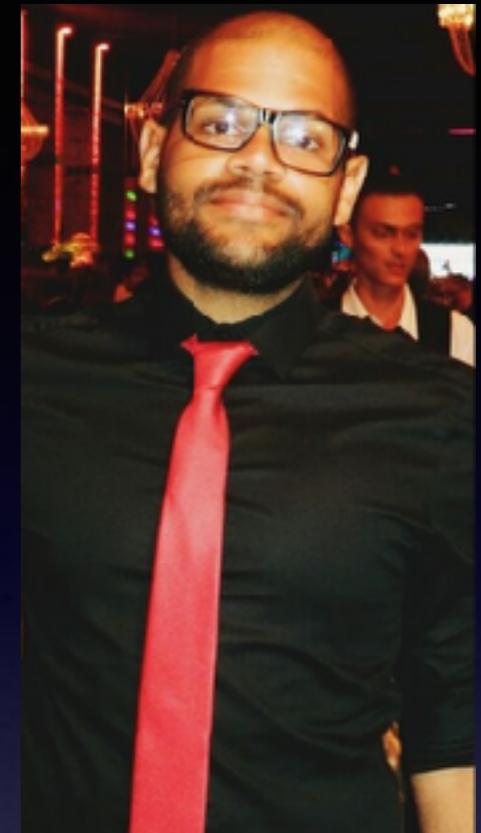
Group at Rio & São Paulo:



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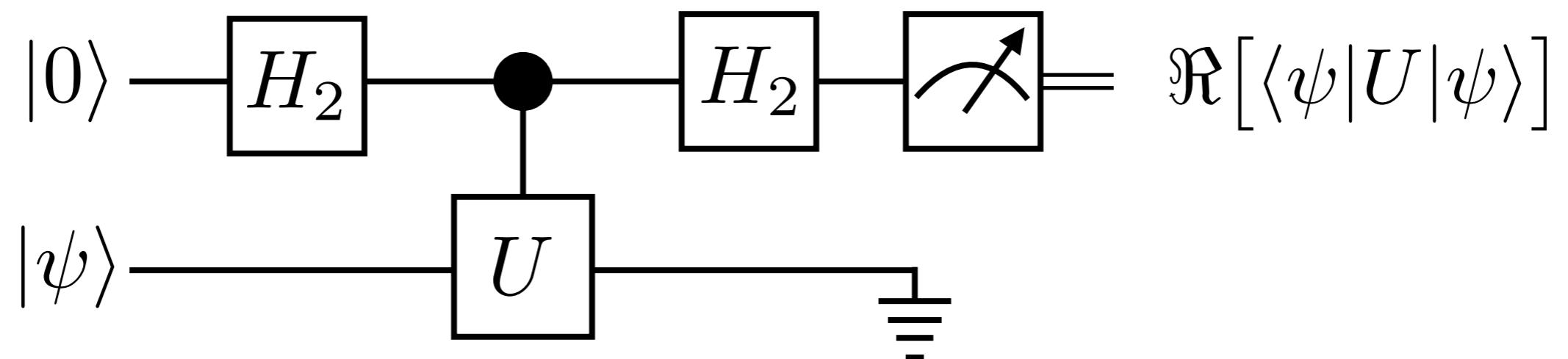


Marcio M. Taddei
Postdoc



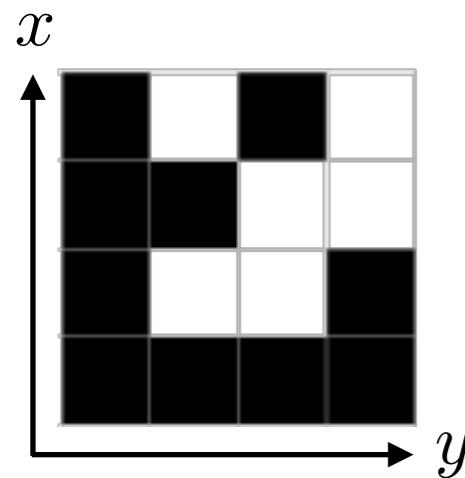
Eric G. A. Cavalcanti
Ph D student

The Hadamard test



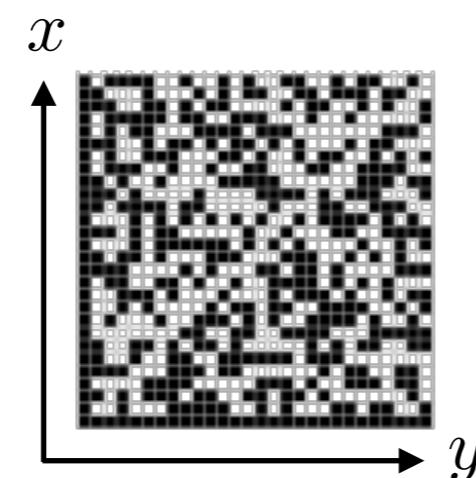
$d_c = 2, d_t$ arbitrary

$$s(x|y)$$



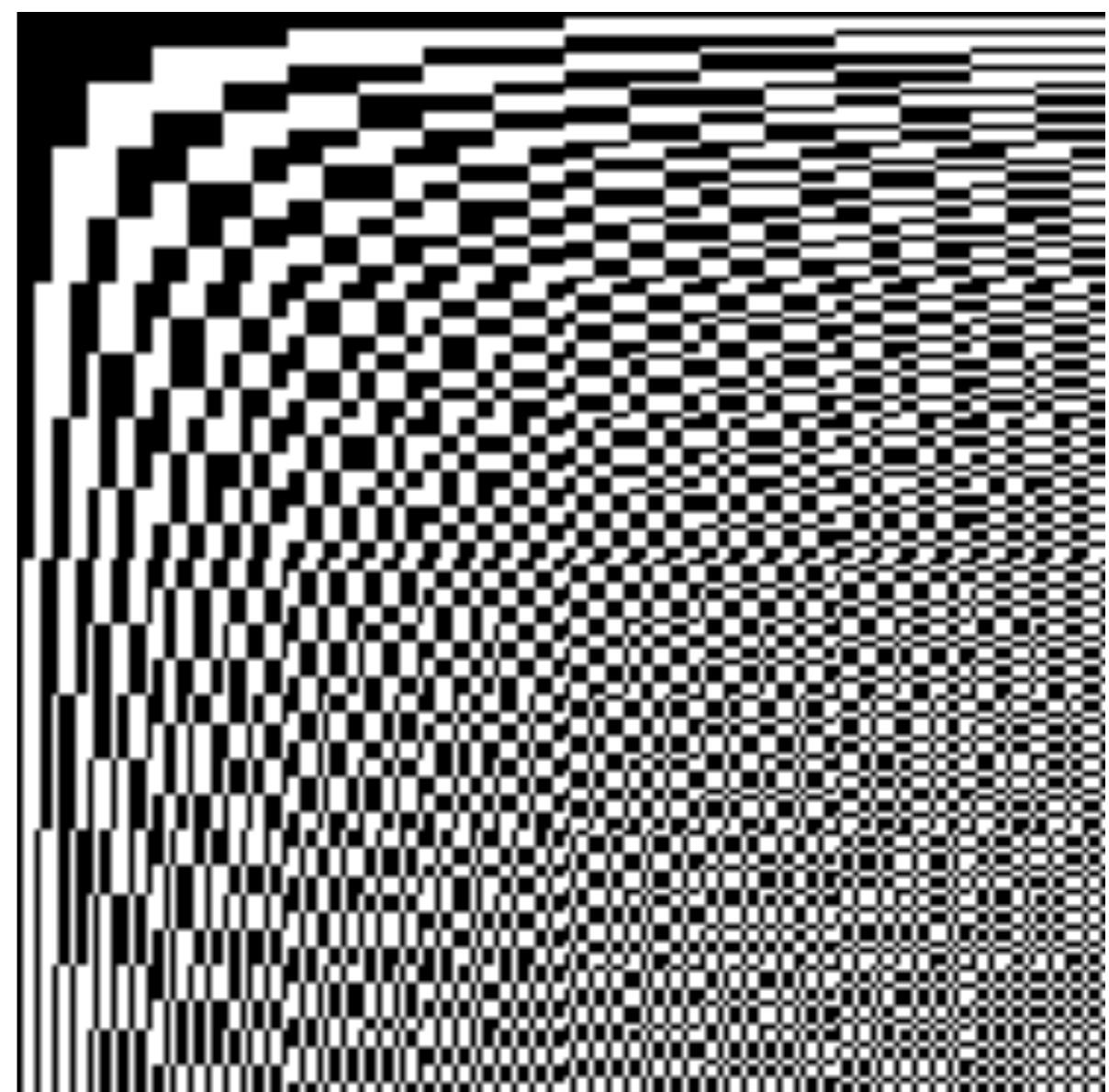
$$N_p = 4$$

$$s(x|y)$$



$$N_p = 28$$

The Hadamard's basis sequences of length 128:



The experimental 4-router and the unitaries

