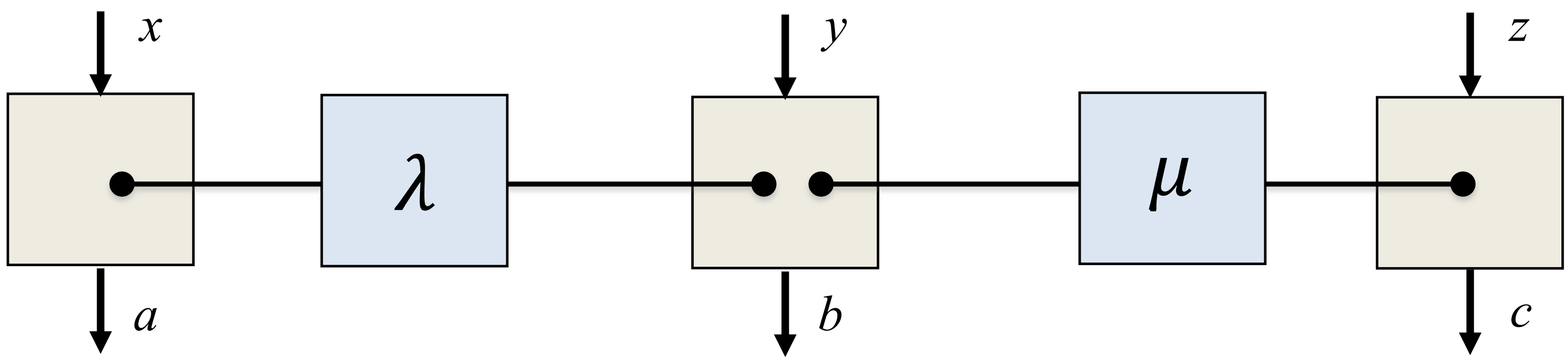
Bounding the sets of classical and quantum correlations in networks

A. Pozas-Kerstjens et al., to appear in PRL

Scalar extension

The method is only applicable to networks in which there are factorization constraints among observed variables, such as entanglement swapping.



$$\langle AC \rangle = \langle A \rangle \langle C \rangle$$

Scalar extension

Example: $S = \{1, A_0 A_1, C_0 C_1, \langle A_0 A_1 \rangle\}$

$$\Gamma = \begin{array}{c c} & \begin{array}{cccc} \mathbb{1} & A_0A_1 & C_0C_1 & \langle A_0A_1\rangle\mathbb{1} \end{array} \\ \begin{array}{c} \mathbb{1} \\ (A_0A_1)^\dagger \\ (C_0C_1)^\dagger \\ \langle A_0A_1\rangle^*\mathbb{1} \end{array} & \begin{pmatrix} 1 & v_1 & v_2 & v_3 \\ & 1 & v_4 & v_5 \\ & & 1 & v_6 \\ & & & v_7 \end{pmatrix} \end{array}$$

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$$v_1 = v_3$$
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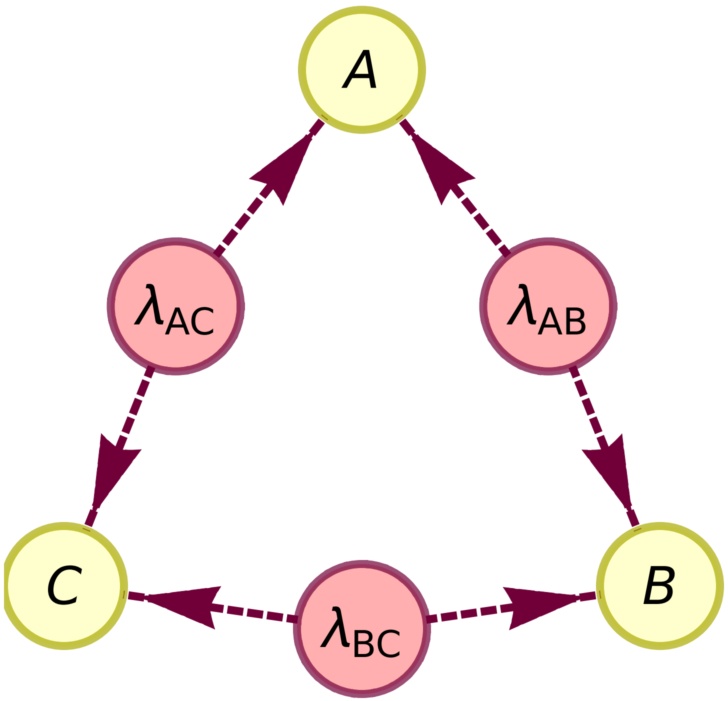
$$\boldsymbol{v_4 = v_6}$$

Quantum inflation: a general approach to quantum causal compatibility

E. Wolfe et al., to appear

Classical inflation



Wolfe, Spekkens & Fritz, arXiv:1609.00672

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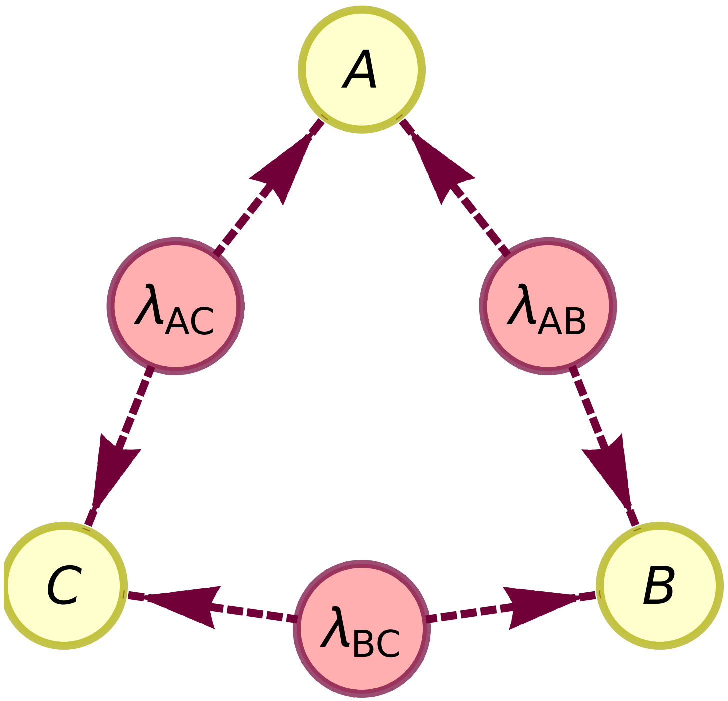

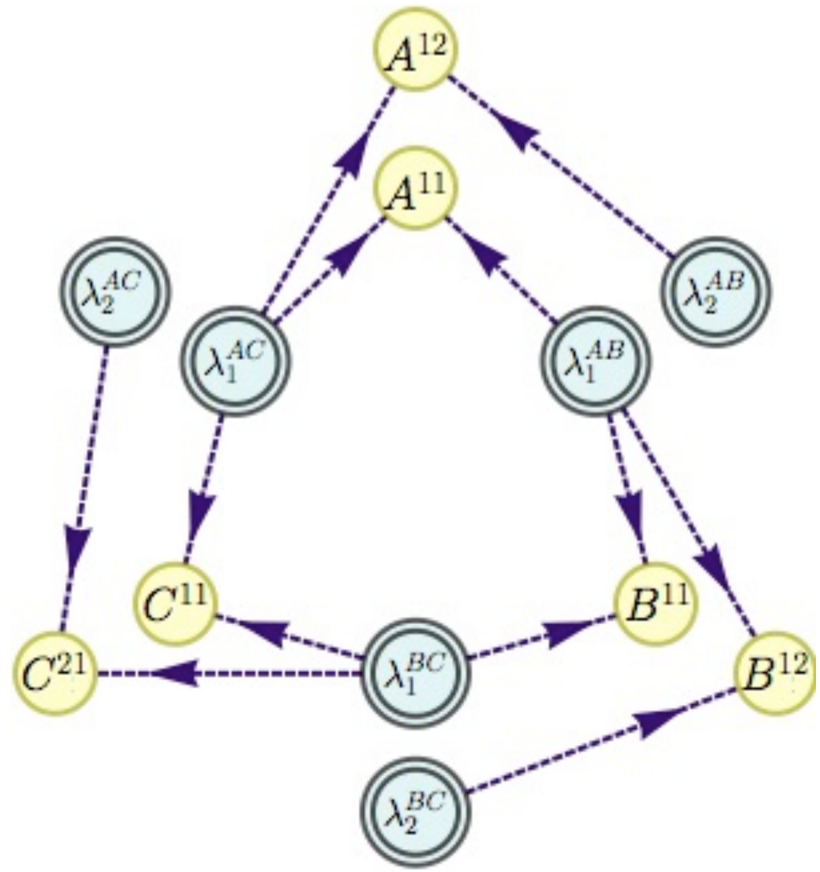

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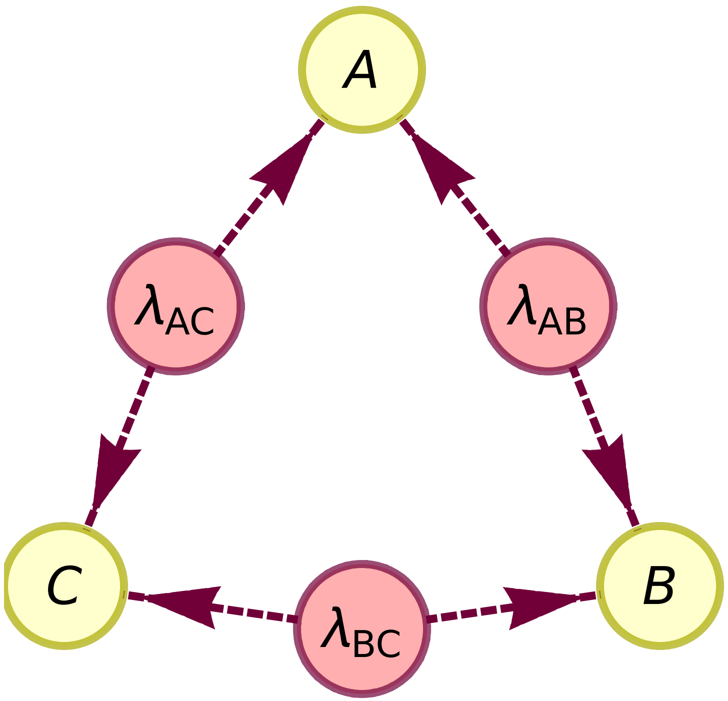

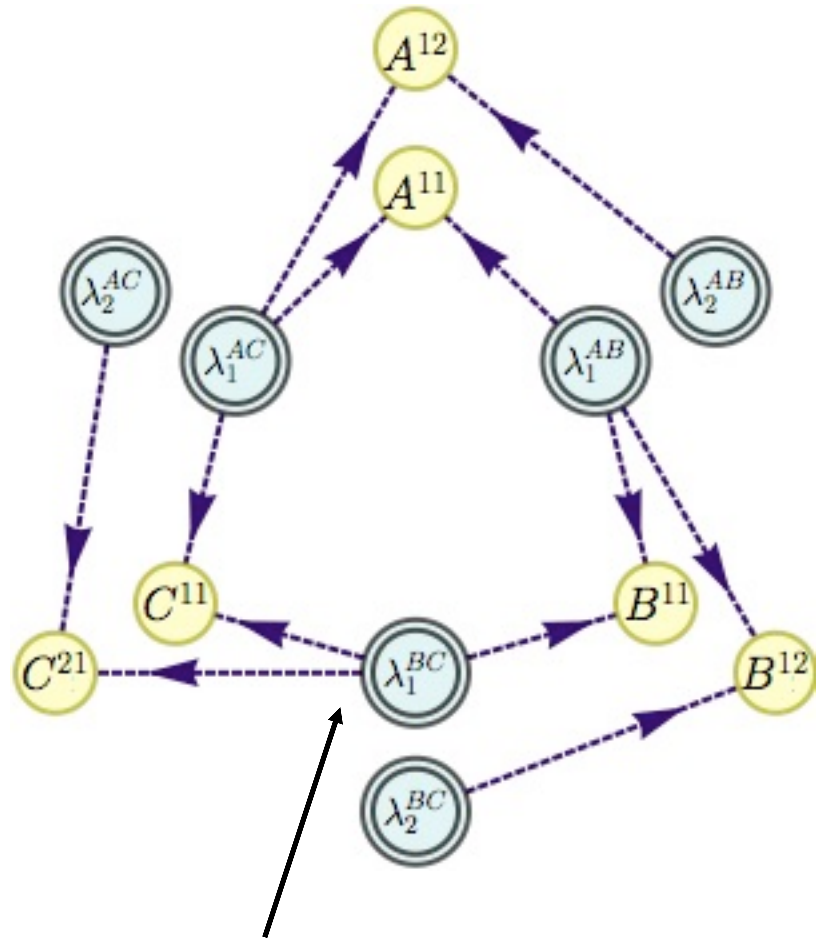

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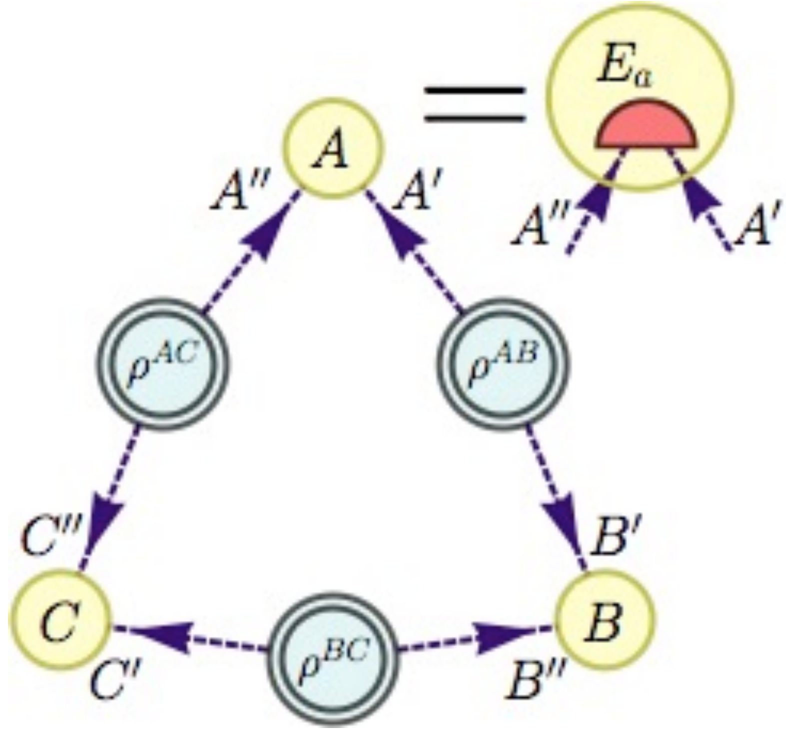
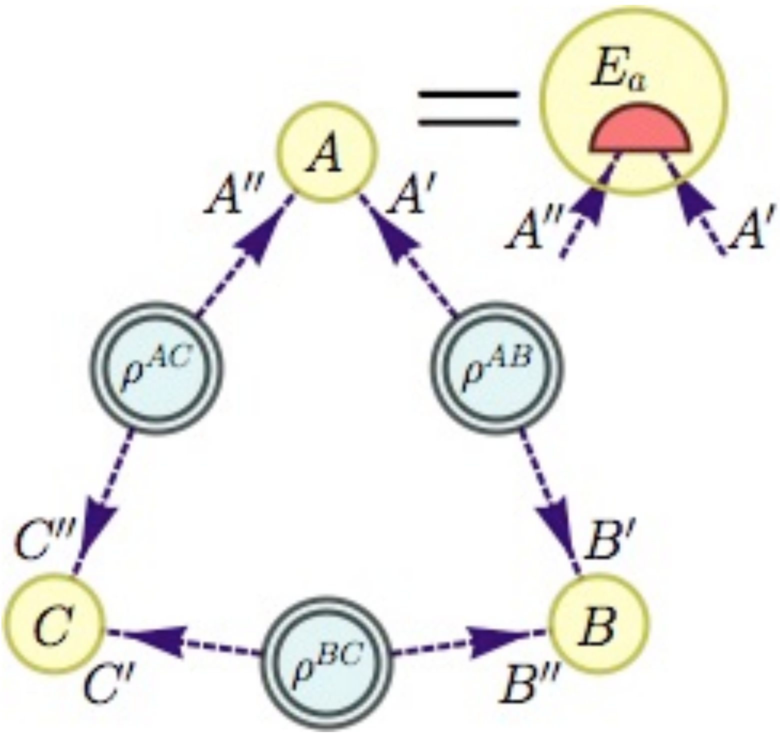
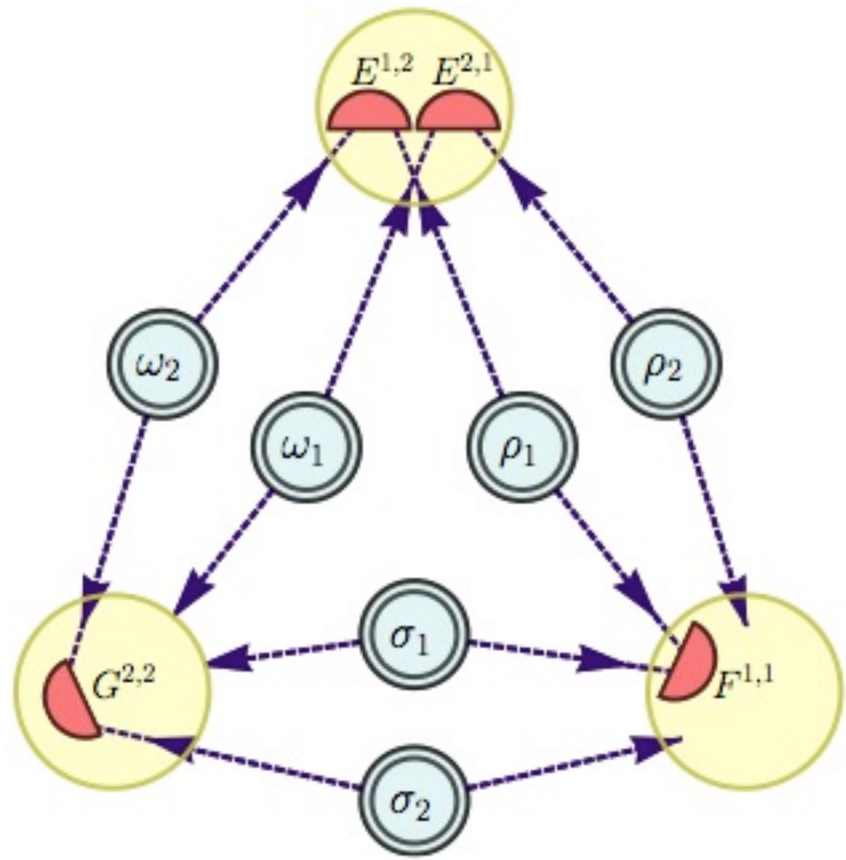
Problem: information broadcasting!!

Quantum inflation



Quantum inflation





Quantum inflation

- Projection rules
- Commutation rules
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All these conditions can be imposed with non-commutative polynomial optimization.

$$
\begin{aligned}
p^* = & \min_{(\mathcal{H},X,\rho)} \langle p(X) \rangle_\rho \\
& \text{s.t.} \quad q_i(X) \succeq 0 \quad \forall i = 1 \ldots m_q
\end{aligned}
$$

Results

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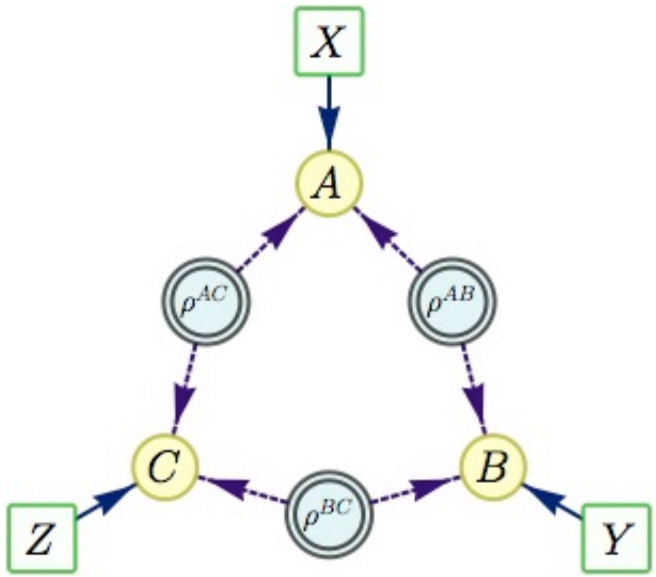

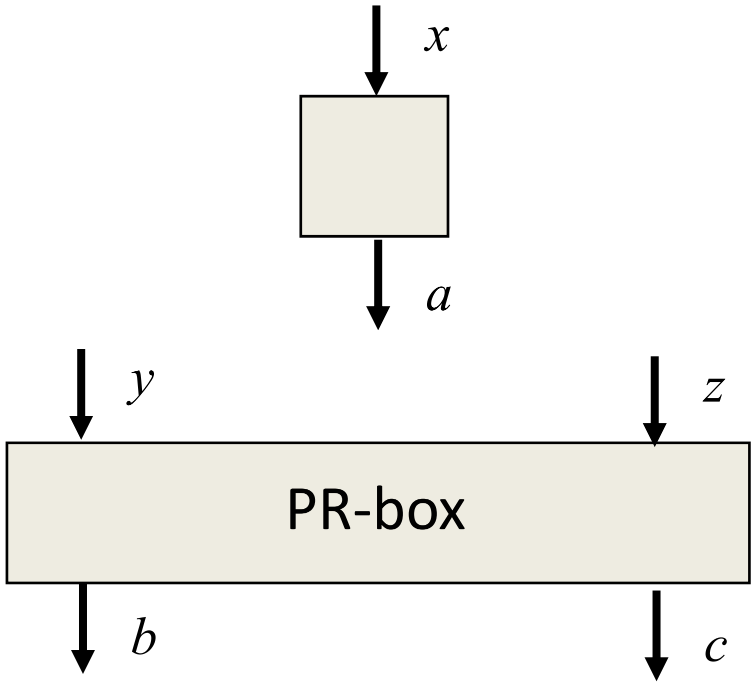

Results

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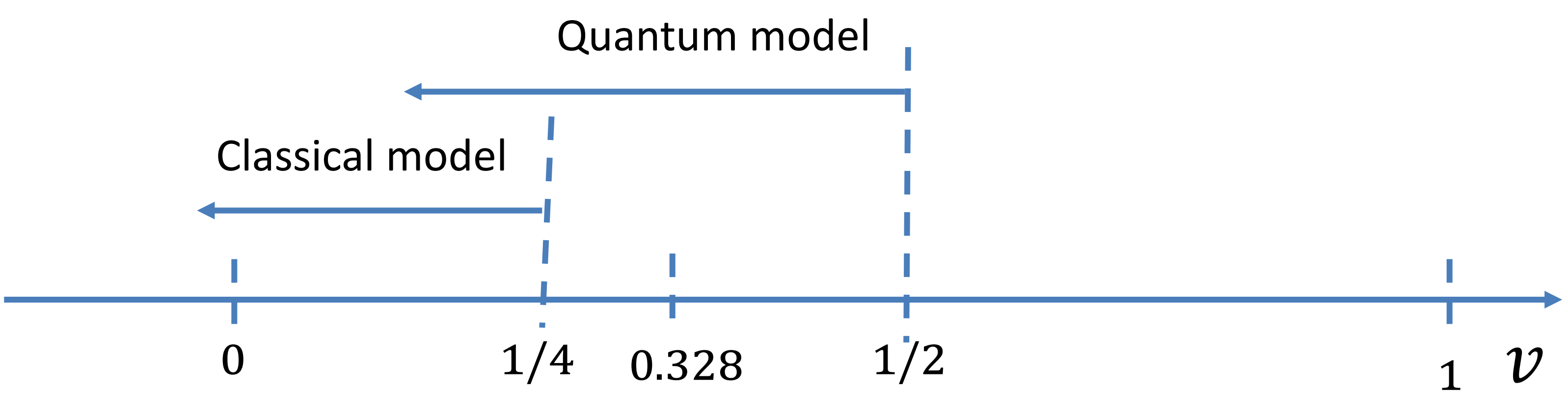
- Inflation + scalar: known quantum (both) and classical (scalar) violations for entanglement swapping..

$$P_{2\mathrm{PR}}(a,b,c|x,y,z) := \left[1+(-1)^{a+b+c+xy+yz}\right]/8 \qquad P_{2\mathrm{PR},v} := vP_{2\mathrm{PR}} + (1-v)/8$$

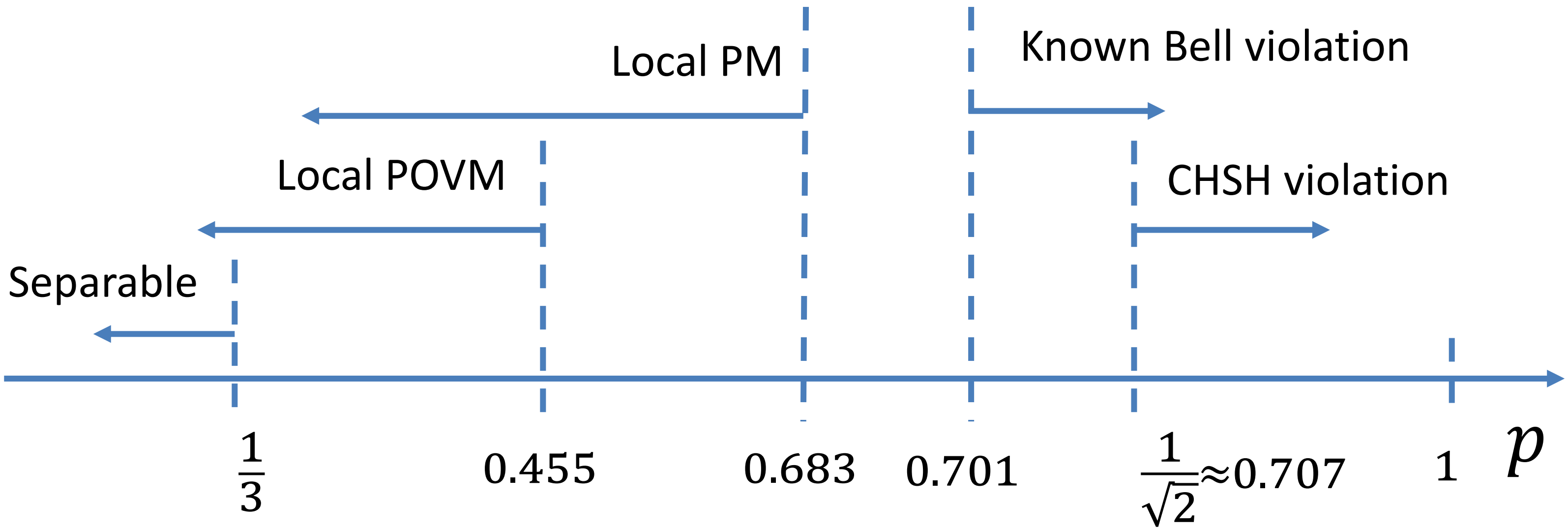


Activation phenomena in networks

Entanglement ≠ Nonlocality

Werner: there exist entangled states that do not violate any Bell inequality.

$$\rho(p) = p|\Phi\rangle\langle\Phi| + (1-p)\frac{1}{4}$$



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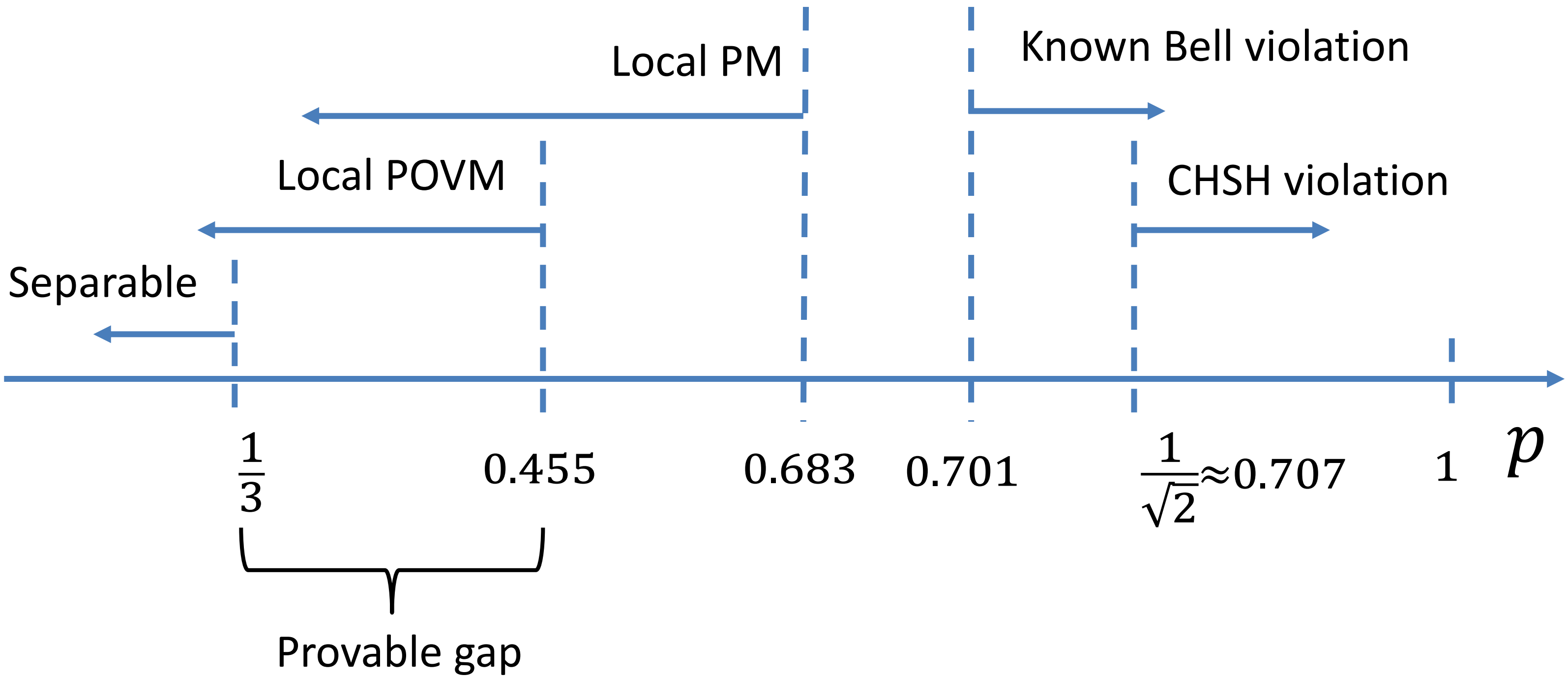
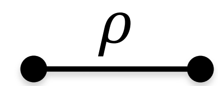


Networks reveal non-locality

- Consider a bipartite state ρ that does not violate any Bell inequality.



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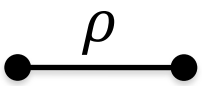
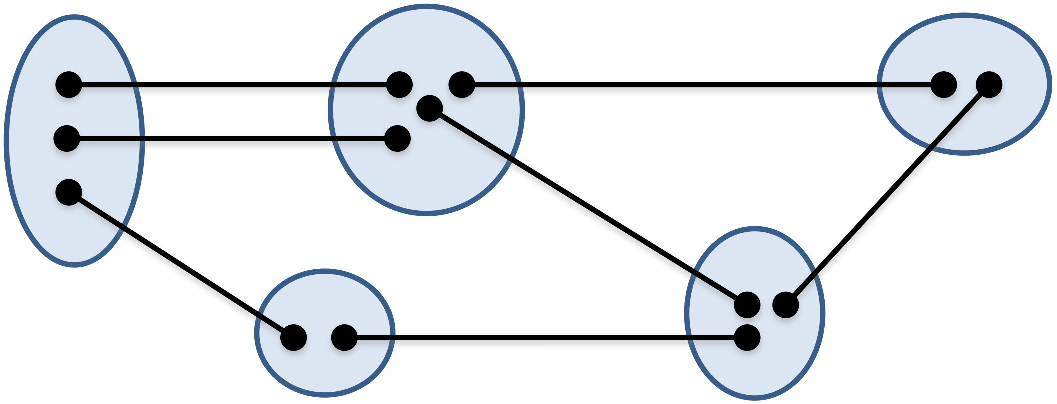


- Make a network out of many copies of the state.



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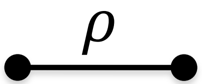

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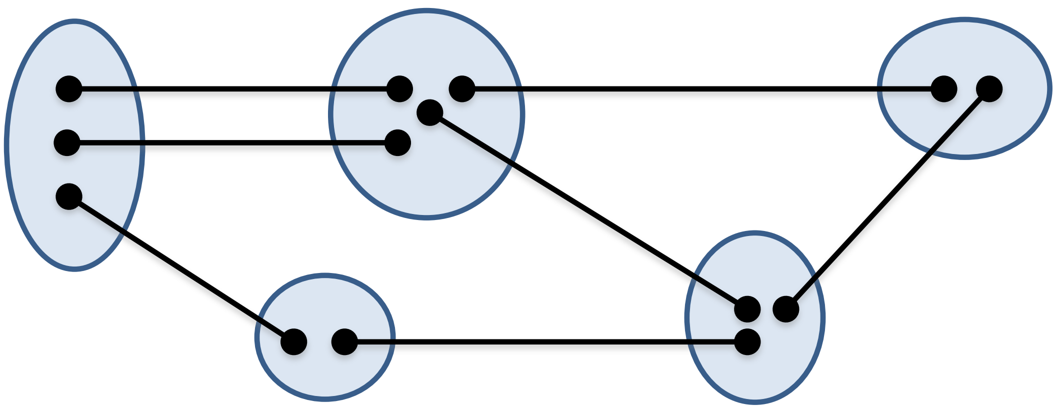

- If the network state ρ_N "=" $\rho^{\otimes N}$ violates an N-partite Bell inequality, the state ρ must be non-local.

Cavalcanti, Almeida, Scarani, Acin, Nature Comm. 2, 184 (2011)

Networks reveal non-locality

- All bipartite states with one-way distillable entanglement are non-local.

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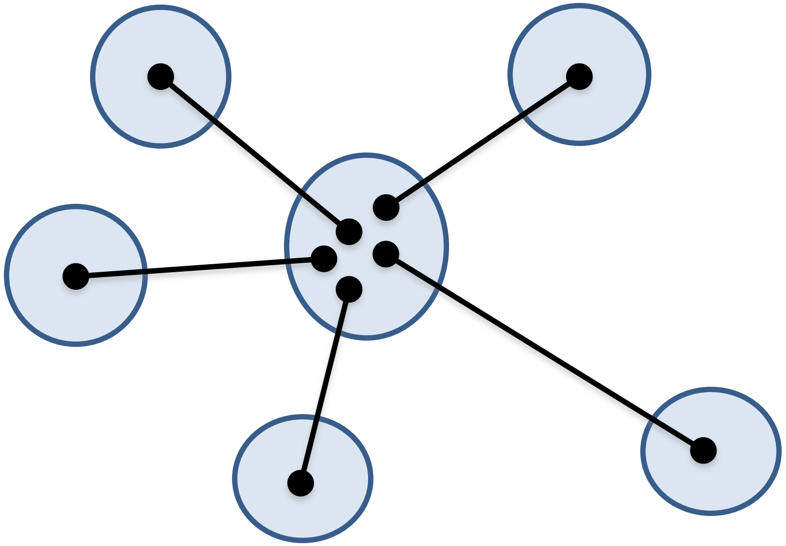
- Non-locality of Werner state for $p < 0.68$ in a star network with $N > 21$. In the limit when $N \to \infty$, one has $p \to 0.64$.



Cavalcanti, Almeida, Scarani, Acin, Nature Comm. 2, 184 (2011)

Activation of non-locality

- Palazuelos: there exist local entangled states ρ_{AB} such that $\rho_{AB}^{\otimes N}$ violates a Bell inequality for large N. Palazuelos, Phys. Rev. Lett. 109, 190401 (2012)

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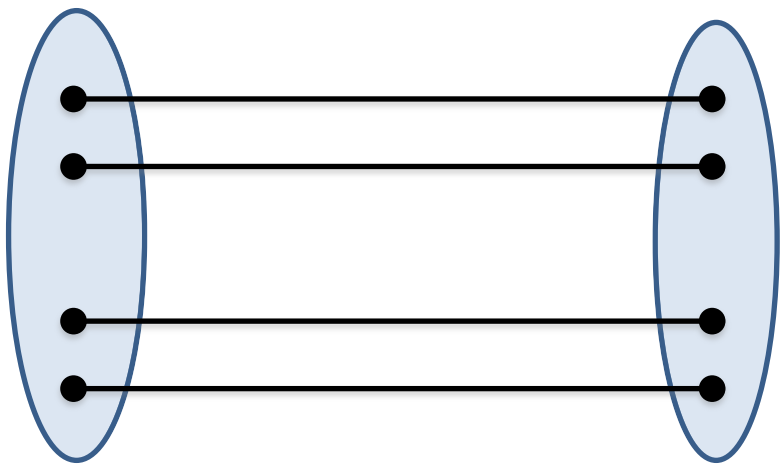

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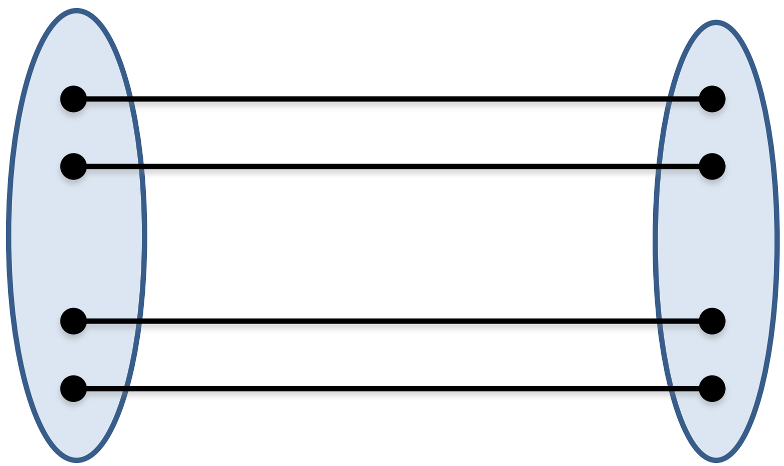
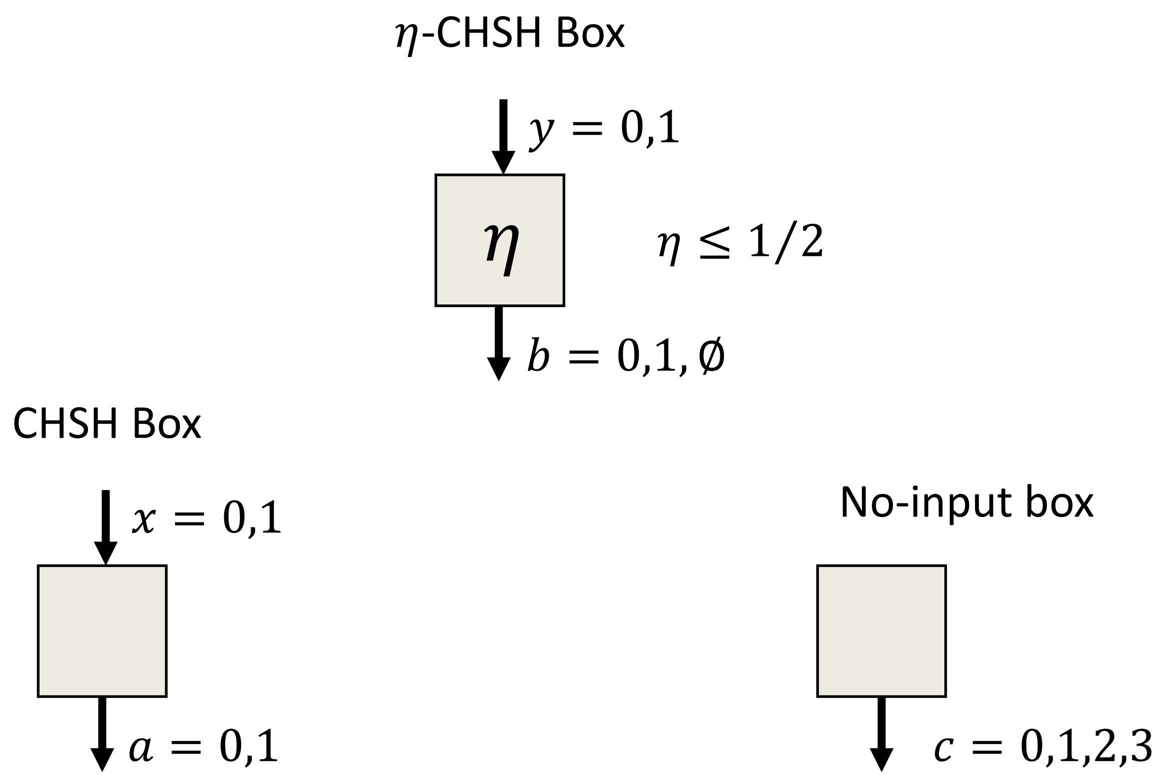


- We improved over Palazuelos' result showing that it applies to all states with singlet fidelity larger tan $1/d$. For isotropic states, this coincides with the separability bound. Cavalcanti, Acin, Brunner, Vertesi, PRA 87, 042104 (2013)

New activation phenomena



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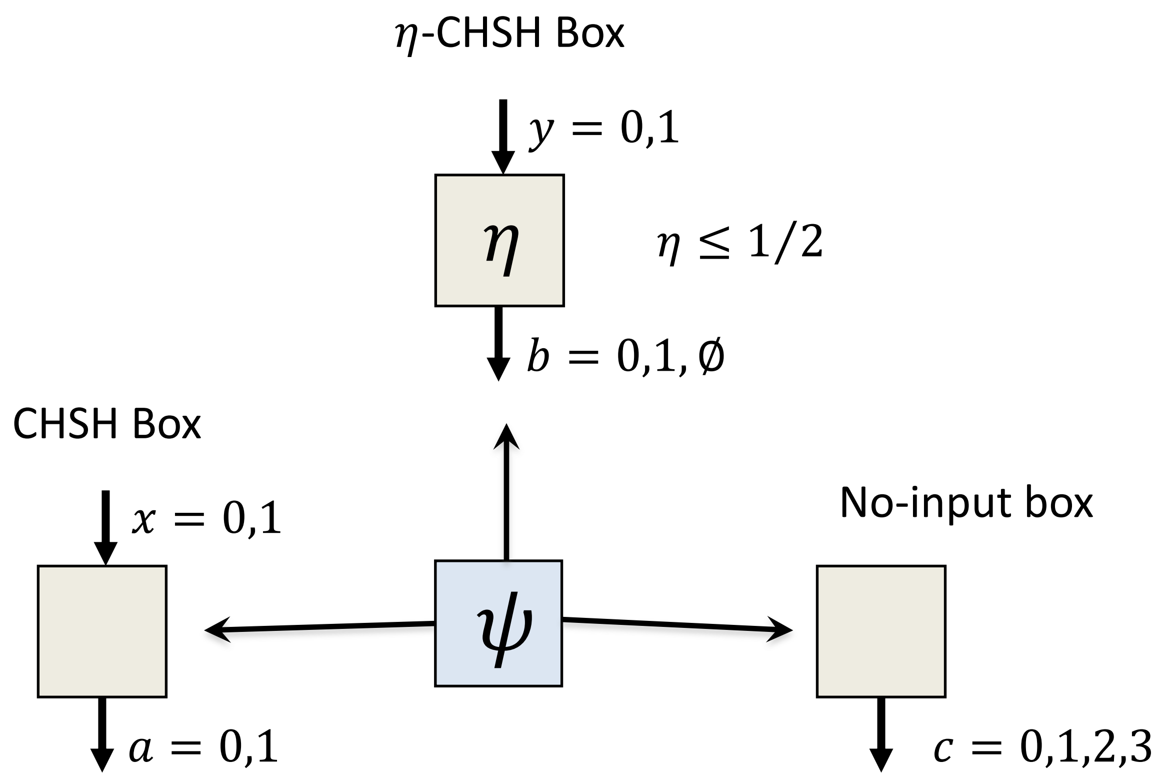
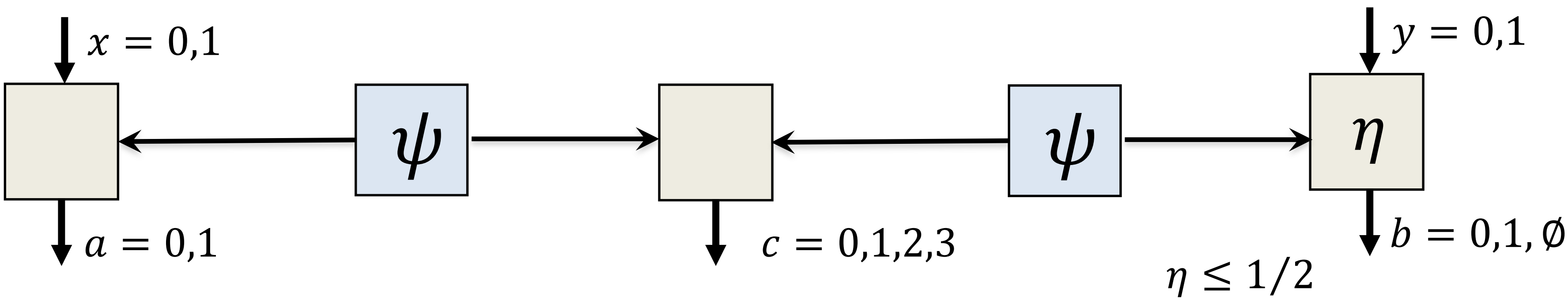


No Bell violation is posible with these measuring devices.

New activation phenomena

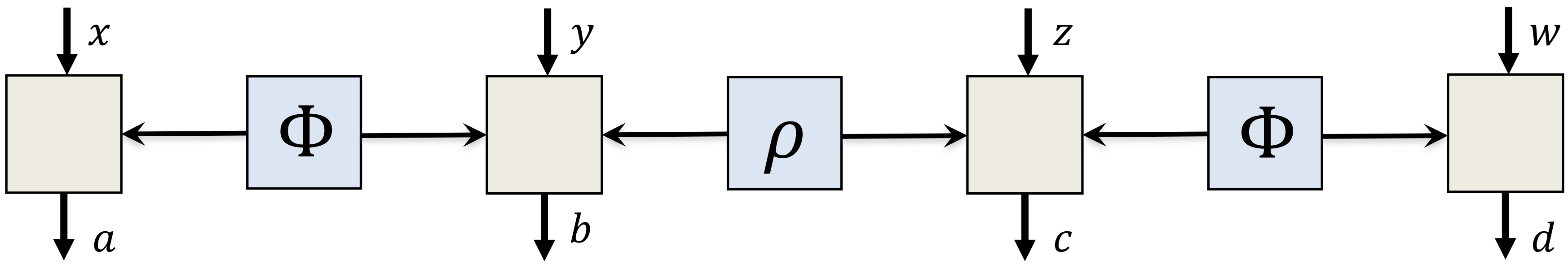


It is possible to violate bilocality with these boxes. Conjecture: valid for all $\eta > 0$.

To do so, we derive a method to bound the set of classical and quantum correlations in these networks through semi-definite programming.

A. Pozas-Kerstjens *et al.*, arXiv:1904.08943

New DI entanglement detection



For any entangled state ρ one one can construct a Bell inequality

$\beta_\rho = \sum W_{abcdxyzw} p(abcd|xyzw)$ such that:

- $\beta_\rho < 0$ for some $p(abcd|xyzw)=\mathrm{tr}\left(\Pi_{a|x}\otimes\Pi_{b|y}\otimes\Pi_{c|z}\otimes\Pi_{d|w}\Phi\otimes\rho\otimes\Phi\right)$
- $\beta_\rho \geq 0$ for all $p(abcd|xyzw)=\mathrm{tr}\left(\Pi_{a|x}\otimes\Pi_{b|y}\otimes\Pi_{c|z}\otimes\Pi_{d|w}\Phi\otimes\rho_S\otimes\Phi\right)$

Bowles, Supic, Cavalcanti, Acin, Phys. Rev. Lett. 121, 180503 (2018)

Conclusions

- Scalar method: simple and applicable to networks with independence constraints among observed variables.
- Inflation method: systematic method to study quantum causality.
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- Problem 2: identify networks with a large classical vs quantum separation.
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- Important: small quantum networks are within reach with present technology.

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