TEMPLETON WORKSHOP

Causality in the quantum world: harnessing quantum effects in causal inference problems

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ABSTRACT BOOK

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Keynote & Invited talks

Quantum non-locality in Networks

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Quantum non-locality, i.e. the violation of some Bell inequality, has proven to be an extremely useful concept in analyzing entanglement, quantum randomness and cryptography, among others. In particular, it led to the fascinating field of device-independent quantum information processing.

Historically, the idea was that the particles emitted by various quantum sources carry additional variables, known as local hidden variables. The more modern view, strongly influenced by computer science, refers to these additional variables merely as shared randomness. This, however, leads to ambiguity when there is more than one source, as in quantum networks. Should the randomness produced by each source be considered as fully correlated, as in most common analyses, or should one analyze the situation assuming that each source produces independent randomness, closer to the historical spirit?

The latter is known, for the case of n independent sources, as n-locality. For example, in entanglement swapping there are two sources, hence "quantumness" should be analyzed using 2-locality (or, equivalently, bi-locality). The situation when the network has loops is especially interesting. Recent results for triangular networks will be presented.

Quantum Shannon Theory with Superposition of Causal Orders

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Shannon's theory of information was built on the assumption that the information carriers were classical systems. Its quantum counterpart, quantum Shannon theory, explores the new possibilities arising when the information carriers are quantum systems. In the standard framework, it is assumed that the communication takes place in a background spacetime with a fixed causal structure, known to all the communicating parties. An interesting question is how the communication is affected if the communicating parties are embedded in a spacetime where the causal relations are in a quantum superposition. In this talk I will describe an abstract communication model inspired by this scenario, exploring some of the curious features arising when communication channels are used in a superposition of alternative orders.

Communication through Quantum Causal Structures

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In a communication network, delays can produce uncertainty in the timing different nodes send and receive information. Traditional quantum information tools are not suited to such scenarios, as they typically presuppose sender and receiver to be in a fixed, known causal relation. An understanding of communication in general—uncertain or quantised—causal structures will be necessary for future quantum networks of increasing complexity. It is also instrumental for an information-theoretical approach of joint studies of quantum theory and relativity, where causal relations can be fundamentally undefined. Here I show how to describe communication scenarios for quantum processes without fixed causal structure. I present a generalised Holevo bound, limiting the amount of classical communication achievable through a general process. I further present bounds on both classical and quantum information capacity for causally separable processes, that is, with classical uncertainty of causal order.

Causal structures in higher-order quantum theory

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Conventional quantum processes are described by quantum circuits, that represent evolutions of states of systems from input to output. Here we move beyond this paradigm. We start introducing maps that represent transformation of input circuits to output circuits, axiomatically imposing minimal admissibility conditions. At this level, all the processes complying to such conditions have in principle a physical realisation scheme. In The construction of a hierarchy of maps then proceeds defining maps from transformations of a given type to transformations of another type and imposing admissibility conditions.
In these higher orders one encounters admissible maps that involve indefinite causal structures. Still, many of the maps in the hierarchy can be proved to have a realistic physical interpretation. In order to study the hierarchy, we classify all possible types of maps, introducing a simple rule for constructing new types from known ones. We use the hierarchy of types to introduce a partial order, which allows us to prove properties by indiction. We will then use induction proofs to discuss the characterisation of mathematically admissible maps of every level. We show an important structural result for a subclass of higher-order maps, and we conclude with the open question of their physical achievability.

Experimental Entanglement of Temporal Orders

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The study of causal relations, a cornerstone of physics, has recently been applied to the quantum realm, leading to the discovery that not all quantum processes have a definite causal structure. While such processes have previously been observed, these observations opened a 'loophole' whereby the observed process could be explained by an underlying theory with a definite causal structure. Here, we present the first experimental demonstration of entangled temporal orders, resulting in a process that is incompatible with a large class of generalized probabilistic theories which are local and have a definite temporal order. We experimentally invalidate this class by violating a Bell inequality. We thus conclude that nature is incompatible with the class of theories requiring a local definite temporal order.

Quantum superpositions of causal orders as an operational resource

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Causal nonseparability refers to processes where events take place in a coherent superposition of different causal orders. These may be the key resource for experimental violations of causal inequalities and have been recently identified as resources for concrete information-theoretic tasks. Here, we take a step forward by deriving a complete operational framework for causal nonseparability as a resource. Our first contribution is a formal definition for the specific notion of quantum control of causal orders, a stronger form of causal nonseparability – with the celebrated quantum switch as best-known example – where the causal orders of events for a target system are coherently controlled by a control system. We then build a resource theory – for both generic causal nonseparability as well as quantum control of causal orders – with a physically-motivated class of free operations, based on process-matrix concatenations. We present the framework explicitly in the mindset with a control register. However, our machinery is totally versatile, being directly applicable also to scenarios with a target register alone. Moreover, an important subclass of our operations is free not only with respect to causal nonseparability and quantum control of causal orders but it also preserves the very causal structure of causal processes. Hence, our treatment contains, as a built-in feature, the basis of a resource theory of quantum causal networks too. As applications, first, we establish a simple sufficient condition for pure-process free convertibility. This imposes a hierarchy of quantum control of causal orders with the quantum switch at the top. Second, we prove that causal-nonseparability distillation exists. More precisely, we show how to convert multiple copies of a process with arbitrarily little causal nonseparability into fewer copies of a quantum switch. Our findings reveal conceptually new, unexpected phenomena, with both fundamental and practical implications.
arXiv:1903.06180

Quantum Measurements of time

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We propose a time-of-arrival operator in quantum mechanics by conditioning on a quantum clock. This allows us to bypass the problems that afflict previous proposals, and to obtain a Hermitian time of arrival operator whose probability distribution arises from the Born rule and which has a clear physical interpretation. The same procedure can be employed to measure the ``time at which some event happens'' for arbitrary events (and not just specifically for the arrival time of a particle).

Textbook quantum mechanics cannot describe measurements of time, since time is a parameter and not a quantum observable. This is a clear fallacy of the theory, since time measurements are routinely carried out in laboratories using quantum systems that act as clocks. Clever and creative tricks were devised to overcome this fallacy. However, many of these proposals give conflicting predictions and none of them provides a prescription that applies to generic time measurements: they all focus on specific measurements, e.g. the time of arrival, at a given position, of a particle subject to a specific potential. In this talk we provide a general prescription for quantum measurements of the time at which an arbitrary event happens (the time of arrival being a specific instance). It entails quantizing the temporal reference frame, namely employing a quantum system as clock. Then, textbook quantum mechanics can be applied to describe time measurements through joint quantum observables of the system under analysis and the quantum clock. A simple Bayes conditioning of the Born rule probability of the joint state allows one to recover the full distribution of the time measurement.

This is work performed in collaboration with Krzysztof Sacha, and is based on the paper arXiv:1810.12869 .

Geometry from Order and Number: Causal Sets

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Among the various ideas put forward in the search for a theory of quantum gravity, the causal set hypothesis is distinguished by its logical simplicity and by the fact that it incorporates the assumption of an underlying spacetime discreteness organically and from the very beginning. After presenting the problem of quantum gravity in general, I will precis the causal set programme and touch on some old and some recent developments.

Quantifying Bell: the Resource Theory of Nonclassicality of Common-Cause Boxes

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We take a resource-theoretic approach to the problem of quantifying nonclassicality in Bell scenarios. We conceptualize the resource as a conditional probability distribution for outcome variables given setting variables that arises from a particular causal structure, namely, a common cause acting on the different wings of the Bell scenario. We term such resources "common-cause boxes". We define the distinction between classical and nonclassical in terms of what type of causal model is required to explain a common-cause box, and we quantify their relative nonclassicality by considering their interconvertibility relative to the set of operations that can be implemented using a classical common cause (these correspond to local operations and shared randomness). We prove that the set of free operations forms a polytope, which in turn allows us to derive an efficient algorithm for deciding whether one resource is more valuable than another. We moreover define two distinct monotones with simple closed-form expressions in the two-party binary-setting binary-outcome scenario, and use these to reveal various properties of the pre-order of resources, including a lower bound on the cardinality of any complete set of monotones. In particular, we show that the information contained in the degrees of violation of facet-defining Bell inequalities is not sufficient for quantifying nonclassicality, even though they are sufficient for witnessing nonclassicality. Finally, we show that the continuous set of convexly extremal quantumly realizable correlations are all at the top of the pre-order of quantumly realizable correlations. In addition to providing new insights on Bell nonclassicality, our work also sets the stage for quantifying nonclassicality in more general causal networks.

Joint work with Elie Wolfe, David Schmid, Ana Belen Sainz, and Ravi Kunjwal

Computational advantage from quantum control of multiple causal orders: an experimental demonstration of the N-switch

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Quantum control of causal orders is perhaps the most pristine form of causal nonseparability, and the only one known to date with a clear physical realisation. The best known example is the celebrated N-switch, originally introduced by Chiribella, D'Ariano, Perinotti, and Valiron. There, an N-dimensional control register coherently controls the causal order in which a target register undergoes events. From the practical point of view, the N-switch is promising as it has been identified as resource for a number of interesting information-theoretic problems. In particular, in 2014, Araújo, Costa, and Brukner found a decision problem involving commutation relations between black-box untaries for which the N-switch provides a quadratic speed-up in query complexity (to the black boxes) with respect to any circuit with fixed gate ordering, remarkably. However, the protocol requires the target-system dimension to be at least N, making it impractical for experiments already for moderate N. In fact, all experimental realisations of the N-switch so far reported are restricted to the simple case of N=2. Here, we present a modified decision problem for which the N-switch also provides a quadratic speed-up in query complexity but with no constraint on the target-system dimension. In particular, the target register can now be a single qubit while N still being arbitrarily large. We experimentally implement such protocol with a photonic interferometer based on multi-core fibres coupled to multiport beamsplitters, for N=4 different causal orders. The 4-dimensional control system is encoded in the spatial degree of freedom of a single photon and the target system is a qubit encoded in the polarisation degree of freedom. This experiment represents the first demonstration of quantum control of more than 2 causal orders.

Fix points of quantum evolution on indefinite causal structures

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The process-matrix framework allows to describe attainable correlations on indefinite causal structures. Yet, in certain setups, it is unknown how to describe the underlying quantum states and their evolution that lead to such correlations; the description remains on the level of the process-matrix and the resulting correlations. In light of postselected closed time-like curves (PCTCs), which can be understood as a generalization of the process-matrix framework, this is not surprising: The impossibility to describe the quantum state that travels to the past is generally believed upon. In this talk we present preliminary results on an attempt to describe the underlying quantum states that undergo evolution on indefinite causal structures: The quantum state recursively describes the fix point of a quantum channel.

Measurement incompatibility and steering are necessary and sufficient for contextuality

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Quantum theory postulates properties of states and measurements that have no analogy in classical physics. Two paradigmatic features are quantum measurements that cannot be jointly measured and shared quantum states that allow stronger than classical correlations. Whereas such features are necessary for various quantum effects, a fundamental question is whether they also necessitate the failure of classical models of experimental statistics. Here, we show that incompatibility of measurements [1] and steering of entangled systems [2] are necessary and sufficient for producing genuinely quantum correlations that cannot be explained with a noncontextual model [3]. As contextuality is a signature of truly quantum effects, the result shows that nonclassicality on the level of theoretical quantum entities necessitates nonclassicality on the level of correlations.

The connection between measurement incompatibility, steering and contextuality allows us to use tools from the latter to tackle problems in the formers. We exploit this to present specific tests of contextuality and evidence that these are both necessary and sufficient for i) certifying every set of incompatible dichotomic qubit measurements, and ii) optimally certifying the steerability of a noisy singlet state under any given number of projective measurements.

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Experimental device-independent randomness generation within an instrumental scenario

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Random numbers generation is a task that finds applications in a great variety of fields, both in science, as well as in everyday life. Hence, the task of certifying the correct functioning of a randomness generator has a crucial relevance. In particular, when one implements, or adopts, a quantum random numbers generator, a wonder may be whether it is actually exploiting quantum correlations or not, but, at the same time, it could be impossible or extremely hard for the inner working of the device to be inspected.

In our work [1], we propose a quantum random number generation protocol, which certifies the presence of quantum correlations, i.e. of intrinsic randomness, through the violation of an instrumental process [2] (see Fig. 1's inset). In this scenario, A and B share a bipartite state, generated by Λ. A chooses what measurement to perform, according to a random variable X and then sends her outcome to B, which chooses its measurement, accordingly. If X can assume 3 values and all the measurements are binary, Bonet's inequality [3] holds in the classical case. Instead, when the state Bob and Alice are sharing is entangled, the upper bound (3) shifts, reaching its maximum $(1+2\sqrt{2})$ when the bipartite state is maximally entangled. We can quantify the lowest amount of randomness (min-entropy) which be certified by a violation I, with the following maximization:

$$H_\infty = -log_2(max_{a,b,x}\,(p(a,b|x))) \qquad (1)$$

given that:

$$\sum_{a,b,x} c_{a,b,x}\,p(a,b|x) = I$$

$$p(a,b|x) = tr(M_x^a \otimes M_y^b\,\rho)$$

This unfeasible task can be recast as a hierarchy of semidefinite programming problems, through the NPA hierarchy [4]. In Fig. 1, we show the min-entropy lower bound obtained at the second level of the hierarchy.

In the randomness generation protocol that we propose, after implementing an instrumental process on a photonic platform, we perform N experimental runs, evaluate the violation I and, through the bound in Fig. 2, the corresponding min-entropy. In the end, the string made of Alice's and Bob's outputs is given as input of a classical randomness extractor [5], to extract a smaller truly random string (its maximum size being N x H_∞(I)). In our work, we were able to reach a maximum extracted certified bits rate of 0.38 Hz, bringing a proof-of principle demonstration that the instrumental scenario can constitute a valid alternative to Bell-like scenarios, in the field of device-independent tasks.

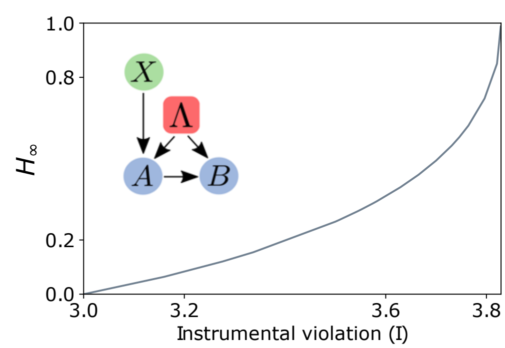

Figure 1: Certified Min-Entropy bound versus the extent of the quantum instrumental violation, obtained at the second level of the NPA hierarchy. The inset represents the causal graph of the instrumental scenario.

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Analysing causal structures in generalised probabilistic theories

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Causal structures give us a way to understand the origin of observed correlations. Given a set of observed variables, some of which may be correlated, a causal structure gives a more detailed picture of how these correlations come about. Depending on the situation, this causal structure may posit the existence of hidden common causes and the nature of these depends on the physical theory. For instance, the experimental violation of a Bell inequality can be explained either by adapting the causal structure within the realm of classical physics (at the expense of resorting to fine tuning [1]) or by allowing hidden systems to be non-classical. Causal structures also provide a suitable basis for analysing the features of different theories by allowing us to phrase communication and cryptographic protocols in terms of the dependencies among the involved systems. They help us predict the success of players engaged in a protocol when restricted according to different theories, for example, in random access coding and the related principle of Information Causality [2].

The differences between classical and quantum correlations within a given causal structure have been systematically studied [3]. However, less work has been dedicated to understanding the limitations of quantum systems [4,5] and of theories beyond. For the latter, there have been studies of the implications of the no-signalling principle in combination with causal structures [6,7] and there is an approach based on graph inflation [3]. In general, understanding the differences of generalised probabilistic theories (GPTs) with respect to various information processing tasks may inform the search for an information theoretic principle that singles out quantum correlations.

In this work, we introduce a technique for deriving constraints on the correlations that are achievable in different causal structures according to various GPTs. With this, we provide a tool to advance the systematic analysis of the differences between such theories in various scenarios. Our method is applicable to any causal structure and we illustrate its advantages with specific examples, extending and improving on previous results [6,7].

Our approach is based on the measurement entropy [8,9] and inspired by entropic approaches to analysing causal structures involving classical and quantum resources [10,5]. One key difference is that we explicitly include the conditional entropy in the analysis. We generate a series of entropic constraints that exclude certain causal explanations of observed correlations when restricted by various GPTs. It also allows us to show that the entropic constraints that hold for some causal structures with unobserved classical and quantum systems generalise to other GPTs including box-world. We apply our results to Information Causality [2], a candidate principle for singling out quantum theory, where we show that our method improves upon that of [7], yielding the stronger inequalities of [8]. Although box-world does not satisfy the notion of Information Causality, we identify minimal notions of causation that are satisfied. We further study the differences of the correlations generated by different GPTs (classical, quantum and box-world) in the triangle scenario.

In addition, we make technical contributions by showing that any set of achievable entropy vectors for a set of observed variables in a causal structure involving quantum or other GPT systems is a convex cone, which had previously only been shown for the entropy vectors of classical resources [11]. This insight allows for easy comparison of the entropic sets within different theories, and in some cases enables us to prove that a given characterisation is complete by showing that all extremal points are achievable. From our approach, we furthermore gain a better understanding of the quantum method proposed in [5], and show how that method can be simplified.

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Poster Presentations

A neural network oracle for classical correlations on causal structures

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Determining the set of classical correlations on a given causal structure is of primal importance for both foundational and applied quantum information theory. Once the classical set is established, we are granted the ability to state whether a probability distribution is genuinely quantum or not on the causal structure at hand. Such claims, apart from their theoretical significance, give rise to useful protocols such as quantum key distribution.

Whereas the set of classical correlations has linear boundaries in simple cases such as the Bell scenario, in even slightly more complex scenarios the boundary becomes, in general, nonlinear and non-convex. Though some progress has been made, we still lack a robust set of tools to investigate generic networks from an analytic and numerical approach.

In this work we explore the use of machine learning in these problems. In particular we model the response functions of the nodes in a given network with neural networks. Neural networks have proven to be useful as ansätze for generic nonlinear functions in terms of expressive power, ease of learning and robustness, both in- and outside the domain of physical sciences. They have also been used in the study of non-locality[1], however we note that our method is significantly different.

In our approach, for any given distribution on a causal structure, we train a neural network to learn the local responses of the parties to their inputs. If the distribution is inside the classical set, then a sufficiently expressive neural network should be able to learn the appropriate response functions and reproduce the target distribution. We can then compare what the machine managed to learn to the target distribution. For distributions outside the classical set, we should see that the machine can not approximate the given target. This gives us a criteria for deciding whether a target distribution is inside the classical set or not.

We explore the strength of this novel method by examining a notorious causal structure, the so-called 'triangle' network. In the classical triangle three parties, Alice, Bob and Charlie, share sources of randomness pairwise, as depicted in Fig. 1. In the quantum case one would exchange the classical random variables α, β, γ for quantum states distributed to the relevant parties. In each round of the game the parties output a, b and c, respectively, which are instances of the variables A, B and C, each with a finite alphabet size, usually 2 or 4.

The triangle configuration is among the simplest tripartite networks, yet it poses immense challenges theoretically and numerically. We use the triangle as a test-bed for our neural network algorithm. Apart from checking for the consistency of our method with known results, we examine two open questions.

1. Is the elegant quantum distribution proposed by N. Gisin [2] outside the classical set?
2. How noise-robust is the quantum distribution proposed by M. Renou *et al.* [3]?

For the elegant distribution, we add local noise to each of the parties, in which the measurement devices defect independently with probability $1 - \eta$, in which case they output a random number. If we plot the distance from the local set as perceived by the machine, we see a transition around $\eta^* = 0.85$. We can then compare what the curve should look like assuming this threshold value and that the classical set is locally flat. The coincidence is astounding.

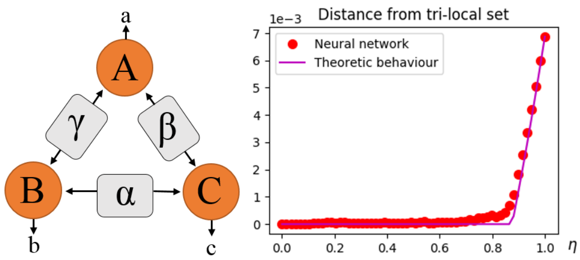

FIG. 1. (left) Triangle network. (right) The distance from the local set expressed in the Euclidean distance for the noisy elegant distribution.

For the second distribution, provided by M. Renou *et al.* (for a fixed parameter $u = 0.945$), we see that even for no added noise, there is a local distribution learned by the machine which is extremely close to the quantum distribution, exemplifying that the distribution is not very robust against local noise.

In conclusion, we explored the use of machine learning tools in a regime where analytic methods as well as optimization are limited. Our approach, in which we model the actions of actors in the network using neural networks, provides a new tool for causal inference on any directed acyclic graph.

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Integrated multiarm interferometers for quantum multiphase estimation protocols

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Generation of quantum resources and its application in inferring unknown physical quantities, is one of the most promising application of quantum information theory, defining the field of quantum metrology. Many fields, such as biological imaging, gravitational wave detection, quantum communication and microscopy, can benefit in exploiting quantum resources to the achieve optimal precision in the estimation of unknown parameters. Multiphase estimation context, where optical phases represent the unknown parameters, seem to be a convenient scenario to investigate multiparameter problems. While many theoretically investigation has been made in this direction, very few experimental studies have been still realized. Photonics devices realized by the advanced technique of femtosecond laser writing (FLW), seem to be natural candidates to implement these kinds of studies, given the capability of implementing complex circuits.

In [1] we identified a suitable platform to study experimental multiphase estimation problems. The platform (Fig.1) consists of a reconfigurable 3-mode interferometer, fabricated by FLW. With this device we have implemented an experimental two-phase estimation injecting input states of two indistinguishable photons. These states have shown the capability to achieve estimation precision better than any classical strategies, thus demonstrating the presence of quantum enhancement. Several investigations can be still made, exploiting multiphoton input states and the high reconfigurability of the chip: indeed, enlarging the number of input photons can improve the achievable quantum enhancement. Moreover, the adoption of adaptive protocols for multiphase estimation enhanced by machine learning techniques can be investigated, exploiting the large number of reconfigurable parameters.

Also, the generation of multiphoton states can be implemented throw the FLW technology: indeed, in [2] we realized a compact, robust and high quality on-chip source of entangled photons pair in telecom wavelength. The source design (Fig.2) is based on a Mach-Zehnder interferometer with two identical non linear waveguides along its two internal arms. The entangled output state can be changed by means of the chip reconfigurability and its hybrid structure.

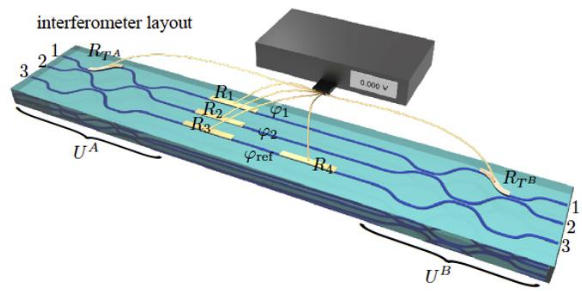

Figure 1: Reconfigurable 3-arm interferometer chip fabricated by femtosecond laser writing technique [1].

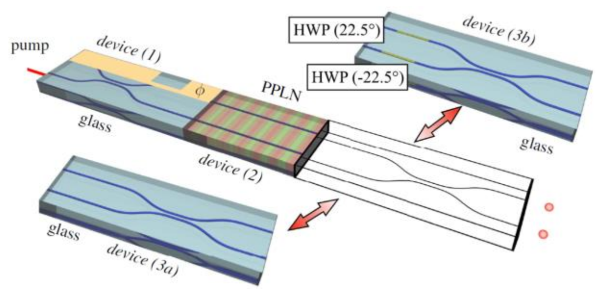

Figure 2: Reconfigurable on-chip source fabricated by femtosecond laser writing technique [2]. The source design is based on a 2-arm interferometer.

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Beyond the swap test: optimal estimation of quantum state overlap

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We study the estimation of the overlap between two Haar-random pure quantum states in a finite-dimensional Hilbert space, given M and N copies of them. The overlap estimation is a fundamental primitive in quantum computation and it is used in several quantum machine learning algorithms. We compute the statistics of the optimal measurement, which is a projection onto permutation invariant subspaces. This means that the initial states do not need to be labeled, and the task can be seen as an example of quantum unsupervised learning.

With a local-estimation approach, we calculate the quantum Fisher information for the family of average states at fixed overlap, and find an asymptotic limit which is saturated by a maximum likelihood estimator. With a global estimation approach, we obtain a simple exact formula for the optimal value of the average mean square error of the estimation. We compare with the standard swap test and two 1-LOCC strategies, relying on the estimation of one or both the states, showing that in general there is a finite asymptotic gap with respect to the optimal measurement, even if the scaling in M and N is the same. We comment on how this shows that the classicality of a bounded reference frame is a relative concept, depending on the task at hand. Moreover, we observe that the swap test is extremely inefficient to estimate small values of the overlap with respect to other strategies. Finally, we discuss the robustness of the optimal strategy with respect to depolarizing noise for qubit states, finding an increase of the mean square error as the square of the depolarizing parameter.

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Calibration of quantum sensors by neural networks

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Recent developments have shown significant advances in the realization of quantum technologies towards real-life applications. Among them, sensing with quantum resources promises to reach accuracy beyond what is permitted from classical counterparts [1]. This advantage is nonetheless conditioned the robustness of the sensors against noise as well as imperfections of the measuring instruments [2,3]. Furthermore in order to grant unbiased estimations, quantum sensors required to be thoroughly characterized by performing calibrations which are often demanding both computationally and in terms of resources.

In this work we report on a novel approach for the characterization of quantum sensors based on the employ of neural networks. We have explored the requirement and training operations for the proposed technique and we have tested it on a phase estimation protocol using N00N states.

Our technique offers two main advantages: it allows for the characterization of quantum sensors without the need of explicit modelling, and it poses no limitations to the metrological capabilities of the device despite the finite training needed for the network to perform the estimation. These are desirable traits for the characterization of emerging quantum technologies, as they allow to provide a reliable and scalable characterization technique, which is implicitly extremely robust to experimental imperfections.

The procedure followed is described in Figure 1: the network is trained by collecting a set of data N corresponding to different values of the parameter of interest; N can be either a number or a vector, since it can contain multiple measurements to obtain a correct normalization or to remove ambiguities. This is used as an input to a network. If the noise statistics on each measurement in N is known, a bootstrapping method can be employed to generate multiple fictitious runs of the experiment by means of a Monte Carlo routine. The training data will unavoidably be associated to an uncertainty, which will be accounted for during the training. For a fixed network size, the quality of the training will be influenced by the sampling resolution, and by the number of repetitions of M employed for each value of ϕ.

The training procedure will establish the weights between the neurons. Once the network has been trained, the device can be used for parameter estimation: the collected data N_0 corresponding to an unknown phase ϕ_0 are feed to the network as an input. As a result the network will deliver the outcome ϕ_0. By using the same bootstrapping method above on N_0, the uncertainty $\Delta\phi_0$ can also be evaluated.

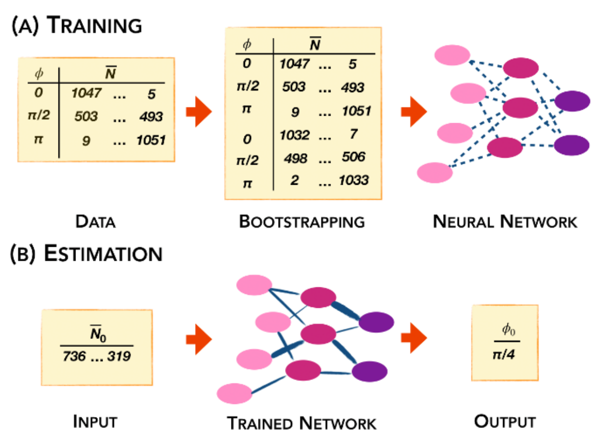

Figure 1: Schematics of the use of neural networks for parameter estimation. The first step (a) consists in training the network by inputting a set of test data using bootstrapping to account for uncertainties. Upon completion, the actual estimation (b) uses the trained network to extract an estimate of the parameter.

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Genuine quantum nonlocality in the triangle network [11]

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(Dated: May 27, 2019)

Introduction.— Bell's theorem is arguably among the most important results in the foundations of quantum theory [1]. An interesting direction is to understand quantum nonlocality in scenarios involving more than two observers. The standard approach to this problem considers many distant observers sharing an entangled state distributed by a common source [2]: the concepts and tools developed for bipartite nonlocality can generally be extended here. Recently, a different approach to multipartite nonlocality was proposed [3, 4], focusing on quantum networks. Distant observers share entanglement distributed by several sources independent from each other. By performing joint entangled measurements, observers may correlate distant quantum systems and establish strong correlations across the entire network. Typically, each source connects only a strict subset of the observers. Quantum networks allow for completely novel forms of quantum correlations. It is possible to witness quantum nonlocality in experiments where all the observers perform a fixed measurement, i.e. they receive no input [4–8]. However, previous examples of this phenomenon all come from the usual form of quantum nonlocality, via the violation of a standard Bell inequality. **The problem of finding a completely novel forms of quantum nonlocality, genuine to the network configuration, was open. Here we address this question, by presenting the first genuine quantum nonlocal correlations in the triangle network** (see also [11]), which we argue is fundamentally different from previously known forms of quantum nonlocality. We generalize this result.

Scenario and main result.— We consider the triangle quantum network sketched in Fig. 1. Every pair of the observers A, B, C is connected by a (bipartite) source, providing a shared physical system (a classical variable or a quantum state). Based on the received physical resources, each observer provides an output. We focus on the correlations of the experiment. We consider the quantum correlations P_Q obtained when each source produces the same pure maximally entangled state of two qubits $|\phi^+\rangle = 1/\sqrt{2}(|00\rangle + |11\rangle)$ and each party performs a projective quantum measurement in the same basis $|01\rangle, |10\rangle, u|00\rangle + v|11\rangle, v|00\rangle - u|11\rangle$ with $u^2 + v^2 = 1$ and $0 < v < u < 1$. We show the following:

Theorem 1. *This quantum correlations P_Q cannot be reproduced by any classical trilocal model, for which the source distribute classical variables, when $0.89 \lesssim u < 1$.*

We provide several generalization of this result, notably for any $N-$cycle network, with N being odd.

Discussion.— We discuss why these examples should represent a form of quantum nonlocality that is genuine to the network configuration, in the sense that it is not a consequence of standard forms of Bell nonlocality. We first review the example presented by Fritz in [4], relying on the violation of a standard bipartite Bell inequality. Then, we analyze the significant differences between Fritz's construction and our example. We discuss the possibilities of "self-testing" from out correlations[9, 10], the possibilities of a noise tolerant result from the Finner inequality [8] or "inflation" technique [12] and the possibility of generating randomness from quantum nonlocality without inputs is a further interesting question.

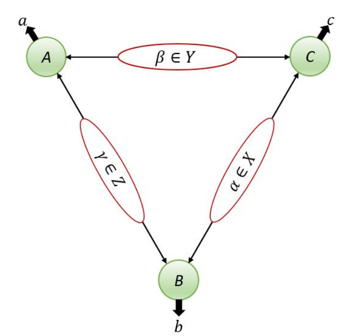

FIG. 1. The triangle network: three observers, connected by three independent bipartite sources (here classical variable).

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Experimental study of superdiffusive behaviour in all-optical Quantum Walks

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The movement of a quantum particle on a line can be reproduced by one-dimensional quantum random walks (QW). The behaviour of the depends on the interference effects between all possible trajectories that the walker can travel [1]. A lot of diffusion processes, occurring when a quantum system interacts with a sorrounding environment, can be replicated by mean of an appropriate QW [2]. A one-dimensional QW can be realized through a network of Beam Splitters (BSs), each of them representing a position site of the 1-D lattice. The walker moves along the network according to the state of another quantum system, the coin, living in a 2x2 Hilbert space, being encoded in the output ports of the BS. In this work, we present the experimental realization of a 1-D QW based on a novel bulk-optic scheme, presenting significant advantages with respect to other typical QW implementation, as integrated and bulk platforms [3]; moreover, we report the study of superdiffusion phenomenon, realized using the so-called P-diluted disorder. The setup realized in this work is based on two displaced-multipass Sagnac Interferometers (SIs), connected through a common central BS. The Sis reproduced a chain of Mach-Zehnder interferometers is reproduced (MZIs), although intrinsically stable. Exploring the vertical dimension of the BS as well as the horizontal one, a QW can be realized. This can be obtained by mean of suitable beam displacers (BDs) intercepting clockwise trajectories in SI1 and counterclockwise ones in SI2. When a beam passes through a BD, his height is increased realizing the entire network of BS. By mean of independently rotating thin glass plates (RPs) one can change phases of each QW mesh. Removable Mirrors (RMs) can be used for extracting and measuring each of the possible modes along which the walker can travel.

We reproduced experimentally an ordered QW, namely a QW with the same phase in each mesh, a disordered one, namely a QW with completely random phases and also P-diluted QWs, in which only a percentage P of the phases is chosen randomly. In Fig. 1 is reported the variance of the photon position during the evolution inside the QW. The graph shows the theoretical predictions of Var(n) (T), as a function of the number of steps, the predictions obtained taking into account the real parameters of our setup (TR) and the experimental values (E). Data agree with the theoretical simulations, showing that the setup is able to reproduce different kinds of 1-D QW. It is clear that the region between ordered and disordered QW, in which superdiffusion occurs, can be studied by the P-diluted disorder. Same results can be reproduced also for two interacting walkers, for which we reproduced experimentally the variance behavior for P=10%.

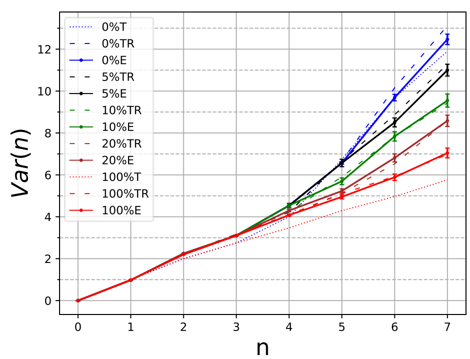

Figure 1: Single photon variance vs the number of step for different values of the parameter P. T: theoretical; TR: theoretical with real parameters; E: experimental.

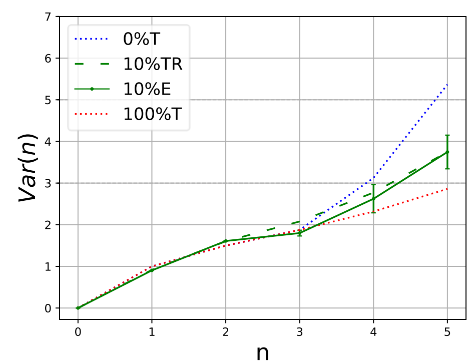

Figure 2: Two-photon variance vs the number of step.

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Effect of local dynamical processes on spin chain dynamics

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Quantum Information and communication aspect of quantum spin chains has been investigated over the last few years. From quantum information theory point of view, a quantum spin chain is a many-qubit system that can undergo various multi-party operations, both global and local along with background unitary evolution. A local operation that interrupts the background evolution can occur from a local quantum dynamical process (QDP): a local coherent operation or a decohering process. Any local quantum dynamical process occurring on a many qubit state changes the translation symmetry of the system and distribution of correlations and entanglement structure. The various sub parts of a multi-qubit system undergoing non unitary operations can be thought of as a simple model for decoherence in many body systems.

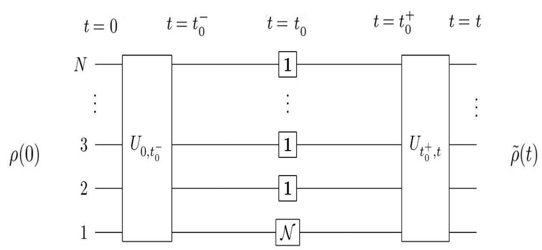

Figure 1: A schematic quantum circuit describing the sequence of operations: a global unitary from the Hamiltonian dynamics, the instantaneous local QDP $\mathcal{N}$,at time followed by another global unitary.

First, we will focus on the propagation of the signal of the intervening QDP in the chain. The possibility of detecting the occurrence of the QDP at farther sites from the dynamical evolution of the state after the epoch time of the QDP will be discussed for various models [1]. The signal propagation clearly distinguish the integrability or non integrability of the chain. Also, the local QDP can interfere with the quantum state propagation in the system and change the quantum state transfer fidelity. The effect of QDP on state transfer fidelity will be highlighted for integrable and non integrable models [2] .

The Loschmidt Echo is traditionally used as a measure for revival of a quantum state when imperfect time reversal procedures take place during closed Schroedinger evolution. We will show that multiple incoherent QDPs interrupting the dynamics can be thought of as if the system is interacting with an external decohering environment, while the action of multiple number of coherent QDPs though does not cause decoherence can generate non integrability in the system [3]. Results will be discussed to make a distinction between integrable and non integrable dynamics.

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Experimental quantification of genuine four-photon indistinguishability

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The quantum interference of two indistinguishable photons [1], known as Hong-Ou-Mandel (HOM) effect, is a practical tool for the characterization of the single photon sources, for quantum computing and for quantum communication. Furthermore, it represents a fundamental test for finding signatures of multiphoton interference.

We present an approach for the quantification of multiphoton indistinguishability based on two-photon HOM tests through a suitable interferometer designed for the purpose [2]. We can sketch the experiment with a linear graph in which the four nodes are the photons and the edges represent the HOM tests, as shown in Fig.1. The photon overlaps r_{ij} can be experimentally estimated in a HOM test via the bunching probabilities p^{ij}, i.e. the probabilities to find two photons in the same output port. We quantify the photon indistinguishability by modeling the state in two different forms, in according with Ref. [3] and Ref. [4], respectively. The first model state is given by formula:

$$\rho = c_1 \rho^1 + \sum_{s>1} c_s \rho_s^{\perp} \tag{1}$$

where ρ^1 is a state of 4 perfectly indistinguishable photons and $\rho_s^{\perp}$ are pure state in which at least 2 photons are orthogonal. The second model considers the 4 photons in a separable pure state.

One can quantify the degree of indistinguishability by obtaining no-trivial bound for c_1 from the measured bunching probability. Moreover, it is possible to infer precise bound for the unmeasured overlaps depending on the 4-photon state model.

In order to realize the indistinguishability test, the experimental set-up consists of a SPDC source that generates two photon pairs. Then, the 4 photons, labelled as (*A*,*B*,*C*,*D*), are injected in the interferometer with six input and output modes, sketched in Fig.1. Since the source is not deterministic, it generates probabilistically three input states, which correspond to (1, 1, 0, 0, 1, 1), (2, 2, 0, 0, 0, 0), (0, 0, 0, 0, 2, 2) expressed in terms of the occupation numbers of the input modes. However, the Hilbert spaces of the output states associated to each input are completely disjoint. In such way, we can post-select the output configuration corresponding to the input (1, 1, 0, 0, 1, 1) needed to perform the test. Hence, our interferometer allows to perform the experiment without the need for heralding.

We study in 8 different configurations of pairwise indistinguishability by tuning the temporal delays. The experimental data are used to quantify the indistinguishability degree and to find no-trivial bounds for the unmeasured overlaps r_{AC}, r_{BD} and r_{AD} according to the two models. Moreover, we verify directly the predictions on r_{AD} by swapping the input photons in the configuration (*C*,*B*,*A*,*D*) and its value is compatible with the estimated bounds of both models.

In this way, we show experimentally that our approach is a valid test for multiphoton indistinguishability and it represents a promising tool for the characterization of the deterministic or probabilistic future photon sources.

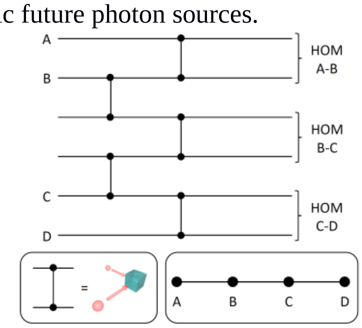

Figure 1: Sketch of the interferometer used for our indistinguishability test (the connecting lines represent beam splitters as shown in left box). The graph of the indistinguishability test is reported on the right box.

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Quantum Hypergraph states

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Quantum hypergraph states form a family of multiparticle quantum states that generalize the renowned class of graph states. We study the nonlocal properties of quantum hypergraph states. We demonstrate that the correlations in hypergraph states can be used to derive various types of nonlocality proofs, including Hardy-type arguments and Bell inequalities for genuine multiparticle nonlocality. Moreover, we show that hypergraph states allow for an exponentially increasing violation of local realism which is robust against loss of particles. Our results suggest that certain classes of hypergraph states are novel resources for quantum metrology [1].

We generalize the concept of local complementation from graph states to hypergraph states and derive graphical rules for more general unitary transformations too [2]. Using these rules, we characterize entanglement classes of hypergraph states under local operations, obtain tight entanglement witnesses, and calculate entanglement measures. Finally, we apply all the aforementioned analysis to endorse hypergraph states as powerful resource states for measurement-based quantum computation [3] and quantum error-correction.

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Unscrambling the Omelette of Causation and Inference in Operational and Ontological Theories

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A critical conceptual step in the development of the framework of classical causal modelling [1] was to cleanly differentiate between inference and influence. Doing so ensured that the relation between statistical correlations and causal relations could be explored formally, and thereby a calculus could be developed for inferring causal relations from observed correlations.

The problem of differentiating between inference and influence is even more vexing in quantum theory, where the interpretation of the formalism is notoriously controversial, and the standard formalism mixes together causal and inferential concepts into what is best described as an omelette that no one has yet seen how to unscramble (borrowing a phrase from E.T. Jaynes). The project of unscrambling this omelette has already led to interesting insights on foundational issues, such as a novel account of Bell's theorem [2] and of quantum evolution under initial system-environment correlations [3].

Rather than focusing on the formalism of quantum theory, as is usually done, in this work we consider the formalisms that have been proposed for operational and ontological theories. We argue that here too there is an omelette of causal and inferential concepts and that the failure of past works to correctly differentiate the two notions has led to some confusion and ambiguity. We then present a formalism that achieves the unscrambling in this sphere and in so doing resolves and clarifies various issues.

We define both operational theories and ontological theories as special instances of *causal-inferential theories*. As the name suggests, these have a causal component and an inferential component. The causal component is a process theory which encodes the types of systems that exist, which systems are causes of which others (with the overall causal structure constrained to be a directed acyclic graph, as in [1]), and the types of causal dependences that relate them. The inferential component is a process theory which specifies an agent's state of knowledge about the objects in the causal process theory, together with the rules by which an agent makes inferences. The causal and inferential process theories interact when an agent makes predictions and inferences, given knowledge about the causal structure. These interacting process theories together define a process theory which subsumes both, formalizing our notion of a causal-inferential theory.

An operational theory, then, is a causal-inferential theory in which the system types are general, the causal dependences which relate systems are lists of laboratory instructions, and the inferential process theory is given by classical probability theory and logic. An ontological theory is a causal-inferential theory in which the system types are classical, the physical interactions which relate systems are functional dynamics, and the inferential process theory is given by classical probability theory and logic. Thus, we formalize (as interacting process theories) the causal and inferential structures which have been (at best) implicit in the traditional definitions.

We show that the framework of generalized probabilistic theories can be recovered from an operational theory by quotienting with respect to inferential equivalence of states of knowledge about laboratory procedures. We also show that the traditional notion of an ontological model can be recovered by quotienting with respect to inferential equivalence of states of knowledge about ontological dynamics. So, our framework subsumes these other frameworks and clarifies the formal relationships between them.

We then define ontological modeling as a map from the operational to the ontological theory which preserves causal and inferential structure. Our approach resolves a number of ambiguities in past work, such as clarifying the role of convexity, Markovianity, the geometry of the space of effects, etc.

The two key notions of nonclassicality, generalized noncontextuality and causal incompatibility inequalities (such as Bell inequalities), emerge naturally in our framework as the preservation of inferential and causal structures, respectively. Noncontextuality holds when inferentially equivalent states of knowledge about laboratory procedures are represented by inferentially equivalent states of knowledge about functional dynamics. In contrast, causal incompatibility inequalities arise when the causal connectivity of the operational model is assumed to hold also in the ontological model.

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Exclusivity graph approach to instrumental inequality

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Inferring whether a variable A is the cause of another variable B is at the core of causal inference. However, unless interventions are available [1], one can cannot exclude that observed correlations between A and B are due to a latent common factor Λ, thus hindering any causal conclusions.

Instrumental variables [1, 2] allow the estimation of cause and effect relations even in presence of this unobserved latent factor, thus providing a fundamental tool for any science wherein causal inference plays an important role.

However, first, one has to guarantee that an appropriate instrument (fulfilling a set of causal constraints) has been employed, which is precisely the goal of the so-called instrumental tests [2]. Their violation, at least in classical physics, is an unambiguous proof that some of the causal assumptions underlying the instrumental causal structure are not fulfilled, that is, one should identify and use another instrumental variable.

More recently, the instrumental scenario has also attracted increasing attention in quantum physics, where, as recently shown, violations of the instrumental tests are possible even though the whole process is indeed subjected to an instrumental causal structure [3].

In the quantum case, instrumentality violations witness the presence of quantum entanglement as the latent factor and prove a stronger form of quantum non-locality compared to the standard Bell's scenario [3]. As a consequence, typical bounds on the causal influence of A into B have to be reevaluated and reinterpreted in the presence of quantum effects.

The purpose of our work is to provide a novel and complementary framework to the analysis of instrumental tests, as well as a broader class of causal scenarios, based on a graph theoretical approach introduced in the study of quantum contextuality to analyze the possible correlations obtained in quantum experiments [4]. This method allow us to reproduce the classical results by Bonet and to straightforwardly generalize them in the quantum scenario. It also offers an easy and general way to check for the incompatibility between the quantum and classical descriptions. Apart from the fundamental relevance of bridging the fields of quantum information and causal inference, our approach is also shown to be of practical use, being able to easily rederive and generalize previous results in the literature.

Given the fundamental importance of the instrumental scenario in causal inference and the increasing attention it has been receiving in quantum information (particularly in applications as randomness generation) we hope these results will strength the connections between both fields and motivate further applications of the graph-theoretical approach within causality.

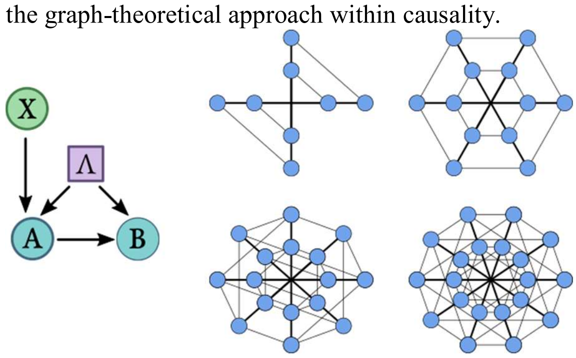

Figure 1: Directed acyclic graph (DAG) representation of the instrumental scenario, with its corresponding description in the exclusivity graph approach for different cardinality of the instrumental variable X.

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Analysing causal structures using Tsallis entropies

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Classical and quantum physics may impose different constraints on correlations arising from a causal structure. Exploiting this difference to certify non-classicality of observed correlations in a given causal structure not only strengthens our understanding of quantum and generalised causal structures, but also has several applications in quantum information processing e.g., in device independent quantum cryptography. Characterising the set of classical and quantum correlations that could arise from a causal structure is thus a central problem in the study of causality. The entropy vector method is often employed for tackling this problem and so far, the Shannon entropy has been used in this method for analyzing causal structures, but it has been found to have limitations in certifying non-classicality for certain causal structures [1]. Further, many known Shannon entropic inequalities such as those of [2,3] have no known quantum violations.

A natural question that arises is whether using other entropic measures can avoid these limitations. Here we discuss the use of Tsallis entropies for this task as these have the advantage of satisfying monotonicity, strong sub-additivity and the chain-rule [4], which are desirable properties for the entropy vector method. In comparison, other generalised entropies such as the Renyi, min and max entropies do not satisfy one or more of these properties. Exploring this question, we obtain the following results:

1) We derive constraints on classical Tsallis entropies of random variables that are implied by the causal structure between them. These encode conditional independences of the causal structure and upper bound the classical Tsallis conditional mutual information between the variables involved.

2) We generalize the classical causal constraints of Result 1) to analogous constraints on quantum Tsallis entropies implied by a quantum causal structure under certain assumptions.

3) While in the Shannon case, at most 1 constraint per variable is needed to characterise all the conditional independences of the causal structure, we find that in the Tsallis case more constraints may be needed, and for some causal structures, there are as many independent constraints as there are conditional independence/d-separation relations in the causal graph.

4) We use these new causal constraints in the entropy vector method to find certificates of non-classicality in terms of Tsallis entropic inequalities. Due to the nature of the new constraints mentioned in 3), we find that the computational procedure of the entropy vector method becomes too time consuming even for relatively small causal structures such as the bipartite Bell scenario.

5) Despite limitations encountered in 4), we find new Tsallis entropic inequalities for all q>=1 by using known Shannon entropic inequalities for the Triangle causal structure [2] and obtaining new, q-dependent lower bounds subject to our causal constraints of 1) and the Shannon constraints (monotonicity and strong-subadditivity) obeyed by (classical) Tsallis entropies for q>=1 [3]. These inequalities depend on the dimensions of the nodes (both observed and unobserved) and provide a way of comparing the dimensionality of classical resources needed for outperforming quantum resources with respect to these inequalities. However, we have not been able to find any quantum violation of these new inequalities neither through known non-local distributions in Triangle nor through randomly sampling quantum states and measurements.

Our results reveal new mathematical properties of classical and quantum Tsallis entropies and also indicate the significant drawbacks of using Tsallis entropies for analysing causal structures, while identifying possible reasons for these limitations.

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Device independent test of a delayed choice experiment

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Wave-particle duality is a fundamental signature of the non-classicality of quantum phenomena, and much debated still. In particular a striking contrast between classical and quantum physics was highlighted by Wheeler, in his famous Delayed Choice Experiment (DCE) [1]. In such experiment the setup revealing either the particle or wave nature of the system is decided after the system has entered the apparatus. For the case of a Mach-Zehnder interferometer, the second beam splitter is inserted or removed after the photon entered the interferometer.

However, usual DCEs, despite their counter-intuitive aspect, do have a simple classical explanation when studied in a causal framework [2,3]. This can be seen, using Causal Inference [2], where one can map the DCE into a prepare and measure (PAM) scenario (Fig.1). In such framework a local classical description can explain the quantum predictions of a DCE [3].

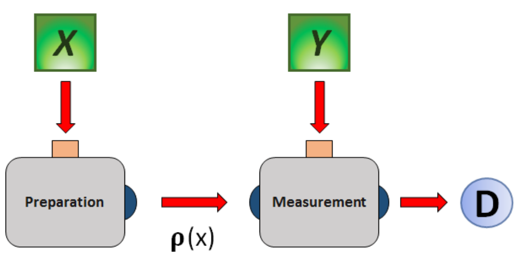

Figure 1: Structure of a prepare and measure scenario: upon receiving an input x, a state preparation device emits a quantum state that is then sent to a measurement device, where the measurement to be performed is selected by another input y, producing an output d.

Nevertheless the authors in [3] also propose a modification of the DCE (Fig.2), by which it can be rejected any classical model with the only assumption of its dimensionality. For this case the choice of measurement corresponds to the choice of an extra phase instead of the presence or absence of the second beam splitter (Fig.2).

In this work we experimentally implement such modified version of the DCE, using a photonic platform [4]. The Mach-Zehnder interferometer is realized in the polarization degree of freedom of single photons and the choice of measurement, done after the photon enters in the interferometer, is implemented by a Pockels cell [4].

By violating two dimension witnesses we experimentally rule out any classical model accounting of the modified DCE. Our conclusions are reached under a natural assumption about the dimensionality of the system under test and, notably, in a device-independent manner, that is, based solely on the observed data and without the need of special assumptions about the measurement apparatus.

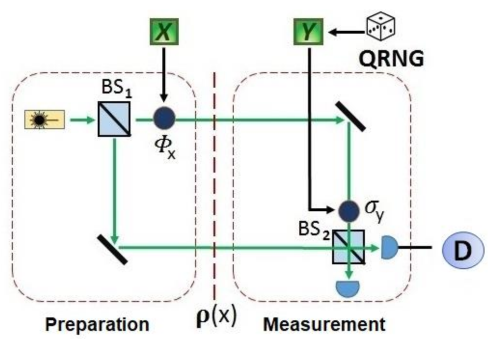

Figure 2: Modified Delayed Choice Experiment.

One of the two dimension witnesses is detection-loophole free, so without assuming fair sampling, and is violated by 41 standard deviations. The other one, violated by 70 standard deviations, permits to rule out also models in which correlations between the preparation and measurement devices are allowed.

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Page-Wootters conditional probability interpretation and the quantum measurement problem

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Quantum theory assigns probabilities to outcomes of experiments, by computing the modulus square of the state vector $|\psi\rangle$ projected onto an eigenstate of the observable that is to be measured. On the other hand, the time evolution of $|\psi\rangle$ is unitary, as given by the Schrödinger equation. Quantum physics thus exhibits two dynamics: unitary evolution and "collapse", and the tension between the application of either one or the other is usually referred to as the "quantum measurement problem". While in everyday practice this does not represent a problem, it becomes relevant when the distinction between the observer and the observed system becomes blurry, such as in the Schrödinger's cat *gedankenexperiment* [1], or in its more extreme version, the Wigner's friend *gedankenexperiment* [2]. The latter features one observer, the friend (F), performing a quantum measurement on a system (say a spin-1/2 particle). In turn, a second observer, Wigner (W), measures the joint system of F and the particle (located together in a sealed laboratory), which are thus treated as a quantum system. This scenario –and recent more sophisticated developments thereof [3-7]– leads to the paradox that F and W would assign different probabilities to the same measurement outcome, even though they correctly apply quantum mechanics. The reason is that while F applies the collapse dynamics and use its "collapsed" updated state to compute the subsequent probabilities, for W the lab is a closed quantum system and he applies unitary dynamics. Does this show a limit of validity of "textbook" quantum mechanics?

In this work we propose to apply the Page-Wootters conditional probability interpretation (CPI) of quantum mechanics [8] as a natural framework to deal with the ambiguous probability assignments in Wigner's friend-like scenarios. CPI is a formulation of quantum theory that promotes time to a dynamical variable, leading to a constrained Hamiltonian: $H|\psi\rangle\rangle = 0$. The physical joint state $|\psi\rangle\rangle$ is assigned to the clock system C and to the system S. This state assigns conditional probabilities of outcomes of measurements performed on S, conditioned on the clock C reading a particular time; as such, the CPI has a manifestly operational flavor. Intuitively, the CPI has the advantage of retrieving the time evolution as a feature emerging from the entanglement between different systems, since it treats time on the same footing of the other dynamical variables.

In the case of the Wigner's friend *gedankenexperiment*, the CPI allows to formally ask the two questions: (i) What is the probability that W measures outcome w at time t_2, given that F has measured outcome f at a previous time t_1? And (ii) what is the probability that F measures outcome f at time t_1, conditioned on the fact that W will measure outcome w at a later time t_2? We will show that there exists more than one way to consistently assign two-time conditional probabilities in the framework of CPI (*e.g.*, [9-10]), all of which are reduced to standard quantum mechanics for non-Wigner's friend scenarios. Depending on the choice of Born rule, however, the Wigner's friend gedankenexperiment leads to different probability assignments that resemble either "collapse" or full unitary evolution [11], thus resolving the paradox of the ambiguous probability assignments. Moreover, the formalism seems to impose a limit to the possible joint probabilities that can be meaningfully assigned, in accordance with previous no-go theorems [4-6].

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Bounding correlations in quantum causal scenarios

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Causality is a seminal concept in science: any research discipline, from sociology and medicine to physics and chemistry, aims at understanding the causes that could explain the correlations observed among some measured variables. One of the consequences of Bell's theorem is that quantum causes can reproduce some correlations for which an analogue classical explanation is impossible. Furthermore, as demonstrated by Popescu and Rohrlich, it is known that there also exist correlations between variables that cannot be reproduced even by quantum causes.

The development of the Navascués-Pironio-Acín (NPA) hierarchy [1] was a cornerstone of quantum causality that provided numerically-efficient methods to determine whether a given correlation could be generated through performing measurements on quantum systems. Its development made possible the rise of the device-independent paradigm, where systems are considered as black boxes only characterized by the probabilities of obtaining outcomes given some inputs. As such, the NPA hierarchy has been applied in many scenarios, from characterizing the boundaries of the set of quantum correlations, to generating random numbers in a certifiable way or bounding the minimal Hilbert space dimension needed for achieving a correlation.

While incredibly successful, the NPA hierarchy is designed for networks where all variables are related to each other by a common quantum cause, and it cannot accommodate the restrictions of more complex causal structures. Even for simple cases as the scenario underlying entanglement swapping, where two independent hidden variables correlate each of two extreme variables with the same central one, there is a lack of tools to analyze the correlations that can be generated when the hidden variables are quantum.

We address such issue presenting two independent methods, quantum inflation and scalar extension, that allow to falsify whether a given quantum causal model can explain some correlations. On one hand, quantum inflation generalizes the inflation technique for classical causal inference [2,3] and can be used for analyzing quantum correlations in arbitrary causal structures. On the other hand, scalar extension [4] modifies the NPA hierarchy to allow imposing relaxations of factorization constraints in semidefinite programs, so one can study the set of quantum correlations in networks with causally-independent nodes, such as that underlying entanglement swapping. While being applicable to a subset of the networks where quantum inflation can be applied, scalar extension needs of much less computational resources to identify supra-quantum network correlations.

In the case of scalar extension, we report on the phenomenon of measurement non-locality activation, by which measurement devices that are not able to certify nonlocality in a bipartite scenario can be used for certifying nonclassical correlations in more complex causal networks. In the case of quantum inflation, we use it to demonstrate that well-known tripartite probability distributions cannot be obtained by sharing bipartite quantum states between every pair of parties in a triangular structure, and compute upper bounds to the maximum quantum value of multipartite Bell inequalitites attainable in different causal scenarios.

Given the success the NPA hierarchy achieved in bipartite quantum causality, we expect our results to find applications in many fields: from the characterization of correlations in complex quantum networks such as the sought-after quantum internet, to the study of quantum effects in thermodynamic and biological processes.

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A new Primitive of Indefinite Causal Order.

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Coherent controlling of noisy quantum circuits in space as well as time has been shown to be a resource for sending quantum information. In this work we present a new primitive of coherent control over causal orders, namely, genuine-quantum-n-SWITCH, which turns out to be useful for perfectly accomplishing a communication task called delayed choice quantum communication (DCQC) in a resource-efficient way. The task involves a sender and several spatially separated probable receivers. However the sender has access to noisy quantum channels for limited time and the final receiver is announced at a later time when the noisy channels are no longer available. We show that genuine-quantum-n-SWITCH is resource-efficient than 'n' numbers of quantum switches.

Experimental Protocol for Quantum State Engineering through one-dimensional Quantum Walk

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In quantum information, the engineering of high dimensional quantum states has great importance. A suitable system is the quantum counterpart of the classical random walk, the Quantum Walk (QW). In Ref [1] has been proposed a platform-independent scheme for quantum state engineering based on the QW dynamics [2]. In the discrete-time quantum walk the walker space is infinite dimensional and the coin space is two-dimensional. In this scheme the walker dynamics is controlled by a suitable choice of step-dependent coin operators with the final goal to prepare arbitary qudit states in the walker space.

We demonstrated exprimentally a state-engineering protocol based on the controlled dynamics generated by a 5-steps QW encoding the walker state in the orbital angular momentum (OAM) degree of freedom $\{|m\rangle_w\}$ and the coin state in the spin angular momentum (SAM) $\{|\uparrow\rangle_c, |\downarrow\rangle_c\}$ [3].

For each step, the step-dependent coin operator is implemented by a set of waveplates that can perform an arbitrary unitary operator in the polarization space. The shift operator acting on walker position is implemented using an anisotropic and birefringent material (q-plate) that can conditionally change the values of the OAM according to the polarization state [4].

The initial state of the QW is:

$$|\Psi\rangle_0 = |0\rangle_w \otimes (|\uparrow\rangle_c + |\downarrow\rangle_c)/\sqrt{2}.$$

Using a numerical optimization [1], we calculated the parameters of the coin operators for each target state. At the end of the QW, to divede the two degrees of freedom, we projected the coin state onto $(|\uparrow\rangle_c + |\downarrow\rangle_c)/\sqrt{2}$. The residual OAM state is analysed through a spatial-light modulator (SLM) followed by coupling into a single-mode fiber. Projecting the OAM state onto an orthonormal basis that contains the target state, the fidelity between the QW output state $|\Psi\rangle$ and the target state $|\Psi_T\rangle$ $(|\langle\Psi_T|\Psi\rangle|^2)$ is estimated.

To demonstrate the effectiveness of this technique we concentrated on classes of physically relevant states. Preliminary, we engineered the states of the computational basis. Then we focused on interesting states as cat-like states, which are coherent superpositions of extremal walker positions, spin-coherent states, which are the spin-like counterpart of coherent states of a quantum harmonic oscillator and Fourier basis states, which are balanced states.

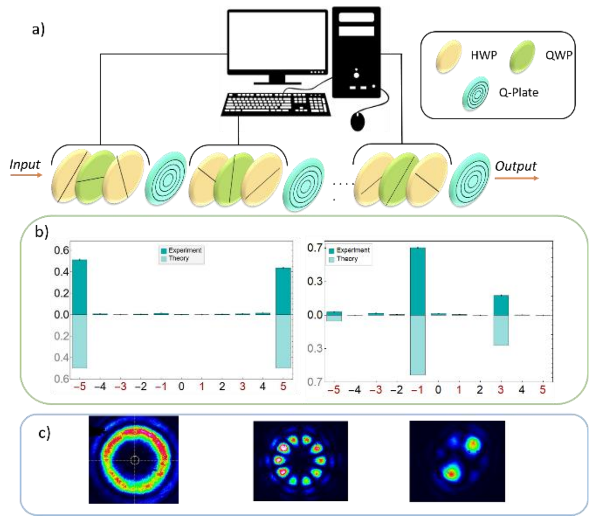

Figure 1 a) Conceptual scheme of the engineering protocol: Controlling the walker dynamics through suitable step-dependent coin operation, arbitrary states in the orbital angular momentum space can be engineered. b) Examples of probability distribution on the computational basis: a cat-like state and a spin-coherent state, respectively. c) Three intensity distributions obtained with coherent light: the state of the computational basis $|5\rangle$, a cat-like state and a state of the Fourier basis.

Finally, we generated randomly chosen qudits to verify the flexibility of the scheme. Totally, we synthesized 32 six-dimensional states, reaching an average fidelity of $\bar{F} = 0.954 \pm 0.001$.

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Causal inequalities from variable elimination methods

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The celebrated result of John Stewart Bell [1] rules out the assumption that all the observed correlations in nature should have an underlined causal explanation. Although it is now widely accepted that "correlations do not imply causation" some parts of the "classical" causal reasoning still remain in the standard formulation of the laws of quantum mechanics. In particular, it is commonly assumed that the order in which events take place is definite.

In the recent work of [2] the authors consider a relaxation of the assumption of existence of the global causal order to a less restrictive requirement on local order of events inside a closed laboratory. As a consequence, correlations in two frameworks, one with the global causal order and the one without, can be distinguished by linear inequalities satisfied by all correlations in the former. These linear constraints are known as *causal inequalities*.

In a later work of [3] all causal inequalities were obtained for the simplest scenario of two parties performing binary input and binary outcome operations in their local laboratories. These inequalities were obtained by solving the facet enumeration problem: given a polytope represented by its extremal points find its representation in terms of its supporting hyperplanes. This problem is know to be hard, which was the reason why the same method could not be used to find all tripartite causal inequalities even in the simplest scenario [4].

In this work we propose a new method to obtain causal inequalities. This method is based on variable elimination techniques of Fourier and Motzkin (see e.g. [5]). The main idea of the method is to start with the full description of the set of correlations in causal scenarios in terms of axioms of probabilities for the extended set of random variables. This extended set of variables contains, among others, a hidden variable that determines the global causal order of the events occurring in separated laboratories. Later, all the terms in the system of inequalities for the extended set containing the unknown hidden variable are eliminated one by one as prescribed in [5].

We demonstrate the effectiveness of the new method by finding *all* the causal inequalities for bipartite scenarios with two settings and three outcomes and three settings and two outcomes per party. We classify the obtained inequalities and show that in the first scenario there are seven classes of causal inequalities and in the second scenario – 24 unique classes. We discuss violations of the inequalities in each of these classes by the extreme two-way signaling boxes as well as quantum correlations with no causal order, introduced in [2].

Unlike the method of [3], the new method does not require generation of all extremal causal correlations, which can be problematic for scenarios beyond the simplest one. This fact provides an advantage in solving membership problems for the cases when finding all causal inequalities for a given scenario is still out of reach. In particular, we show that this advantage is exponential in the number of settings and outcomes.

We conclude by discussing interesting types of acausal quantum processes that display violation of the newly obtained inequalities.

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Toy theory for quantum space-time: possible futures

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In the quest for quantum gravity Hardy [1] suggests the radical aspects of quantum physics (probabilistic nature) and relativity (dynamic causality) would manifest together. In the recent years, this has led to an active study of causally neutral formulations of quantum theory through many approaches including the causaloid [1], quantum combs [2], process matrices [3] and quantum conditional states [4]. In particular it has paved the way for the study of indefinite causal orders, that exhibit new phenomenon such as the quantum switch [2] and the violation of causal inequality [3].

At the heart of this emerging field of indefinite causal structures live higher order objects such as combs or the process matrix that can be difficult to interpret from a space-time perspective. In the past we learnt that there is value in constructing toy theories to shed light on foundational questions [5]. Can we have a theory for indefinite causal structures without eluding to higher order objects? To tackle this question we construct a quantum space-time toy theory that is composed solely of local laboratories, that exhibits locally definite but globally indefinite causal structures and which reproduces the quantum switch.

Setup The toy theory consists of closed labs that have a one input - one output restriction and locally obey quantum theory, so far this is similar to the process matrix framework [3]. Further, the toy theory lives in some 1+1 D space-time and the labs locally perceive Minkowski space-time. Lastly, the labs can only communicate with each other using photon(s).

Tessellation In 1+1 D Minkowski space-time there are only four null lines, if the local laboratories are agnostic to the past null line that sends them the input and have an operational choice to send as output the photon(s) to either of the two future null lines then the local laboratory can be represented by an equilateral triangle with the three edges depicting input (past) and the two possible outputs (futures). Composing the labs such that each lab must have an input and output and that the output of a lab is the input of the nearest lab in its future leads to a triangular tessellation that represents space-time.

Coloring Once we have the tessellation, we can impose a local coloring law with four colours never repeated in adjacent triangles. The colors represented by {0,1,2,3} obey the algebra over Z_4. Each time one goes from a lab to its future/past lab the assigned coloring is +1/-1. And the third edge possibly resembles a space-like separation with +2/-2 color.

No Closed Time-like curves Given the coloring law, the one input one output restriction on labs and that every triangle on the tessellation is a lab we prove that the toy theory does not admit closed time-like curves

Switch and Indefiniteness The operational choice to send photons was introduced as a classical degree of freedom. Instead a lab can use beamsplitters to elevate the path information into a quantum degree of freedom which can help construct a quantum switch.

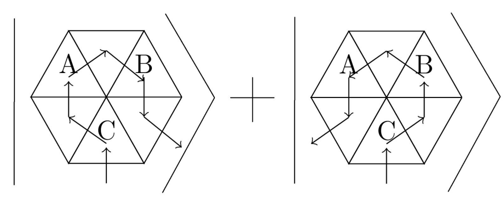

Figure 1: Switch with controller C choosing signalling relations between A and B

Discussion Further details / examples omitted here incorporate entanglement between paths and thus Bell inequality, discussion on causal violations and codes solving for possible causal structure solutions. The toy theory is an apt playground to study known and discover new indefinite causal order phenomenon.

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Superposition of causal orders for quantum discrimination of quantum processes

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We address the superposition of causal orders in the quantum switch as a convenient framework for quantum process discrimination in the presence of noise in qubit systems, using Bayes strategy. We show that, for different kinds of qubit noises, the indefinite causal order between the unitary to be discriminated and noise, gives enhancement compared to the definite causal order case, without reaching the ultimate bound of discrimination in general. Whereas, for entanglement breaking channels, the enhancement is significant, where the quantum switch allows for the attainability of the ultimate bound for discrimination posed by quantum mechanics. Memory effects escorting the superposition of causal orders are discussed, where we point out that processes describing an indefinite causal order, violate the notion of Markov-locality. Accordingly, a suggestion for the simulation of indefinite causal orders in more generic scenarios beyond the quantum switch is given.

Keywords: Quantum discrimination; Indefinite causal order; Markov-locality

The dot-formalism – from causal to compositional structure

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We explore a two-fold question: First, what compositional structure of a unitary is implied by its causal structure? Second, how can that compositional structure be represented graphically so as to make the causal structure evident in a faithful way.

Definition: *Given a unitary* $U : \mathcal{H}_A \otimes \mathcal{H}_C \to \mathcal{H}_B \otimes \mathcal{H}_D$, *'A does not influence D', written* $A \nrightarrow D$, *if and only if there exists a channel* $\mathcal{M} : \mathcal{L}(\mathcal{H}_C) \to \mathcal{L}(\mathcal{H}_D)$ *such that* $\mathrm{Tr}_B[U\rho U^\dagger] = \mathcal{M}(\mathrm{Tr}_A[\rho])$ *for all* $\rho \in \mathcal{L}(\mathcal{H}_A \otimes \mathcal{H}_C)$.

Suppose $A_1 \nrightarrow B_3$ holds for $U : \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \mathcal{H}_{A_3} \to \mathcal{H}_{B_1} \otimes \mathcal{H}_{B_2} \otimes \mathcal{H}_{B_3}$, Schumacher & Westmoreland showed [1] that there then exist unitaries V and W such that $U = (V \otimes \mathbb{1}_{B_3})(\mathbb{1}_{A_1} \otimes W)$, which can be expressed graphically as in Fig. 1.

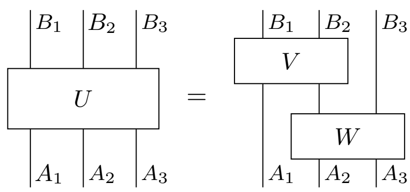

Figure 1

Here, $A_1 \nrightarrow B_3$ is evident since there is no path from A_1 to B_3, reading the diagram bottom up. Similarly, if $A_3 \nrightarrow B_1$ holds, there exist unitaries V' and W' such that $U = (\mathbb{1}_{B_1} \otimes V')(W' \otimes \mathbb{1}_{A_3})$. Clearly, a unitary defined by a circuit diagram as in Fig. 2 satisfies both constraints. However, not all unitaries satisfying both constraints are of that form. Hence, what compositional structure is implied if both conditions hold simultaneously? The answer is given by this result:

Theorem [J.Barrett, RL]: *Given a unitary* $U : \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \mathcal{H}_{A_3} \to \mathcal{H}_{B_1} \otimes \mathcal{H}_{B_2} \otimes \mathcal{H}_{B_3}$, *if* $A_1 \nrightarrow B_3$ *and* $A_3 \nrightarrow B_1$ *hold, then*
$U = (\mathbb{1}_{B_1} \otimes T \otimes \mathbb{1}_{B_3}) \left(\bigoplus_i V_i \otimes W_i\right) (\mathbb{1}_{A_1} \otimes S \otimes \mathbb{1}_{A_3})$
for some unitaries,

$$T : \quad \oplus_i \mathcal{H}_{G_i^L} \otimes \mathcal{H}_{G_i^R} \to \mathcal{H}_{B_2} \ ,$$
$$S : \quad \mathcal{H}_{A_2} \to \oplus_i \mathcal{H}_{F_i^L} \otimes \mathcal{H}_{F_i^R} \ ,$$

and families of unitaries

$$V_i : \quad \mathcal{H}_{A_1} \otimes \mathcal{H}_{F_i^L} \to \mathcal{H}_{B_1} \otimes \mathcal{H}_{G_i^L} \ ,$$
$$W_i : \quad \mathcal{H}_{F_i^R} \otimes \mathcal{H}_{A_3} \to \mathcal{H}_{G_i^R} \otimes \mathcal{H}_{B_3} \ .$$

We represent this structure graphically as in Fig. 3 and refer to it as a 'dot-diagram'. The basic idea of a 'dot', in Fig. 3 labelled S, first appeared in [2] and what it essentially does is making the structure $\oplus_i \mathcal{H}_{F_i^L} \otimes \mathcal{H}_{F_i^R}$ expressible through indexed parallel wires. This is to be contrasted with $(\oplus_i \mathcal{H}_{F_i^L}) \otimes (\oplus_j \mathcal{H}_{F_j^R})$, which contains all cross terms. Dot-diagrams thereby help overcome what circuit diagrams fail to do – simultaneously expressing all causal constraints of a unitary.

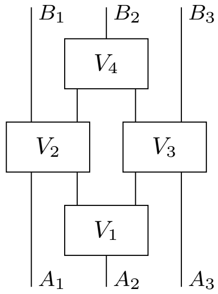

Figure 2

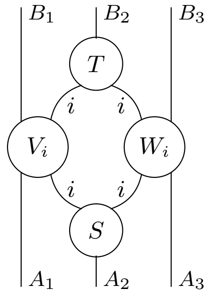

Figure 3

More generally, given a unitary from n input systems $A_1, ..., A_n$ to k output systems $B_1, ..., B_k$, its *causal structure* is given by, for each output system B_i the subset of input systems that can influence B_i. The open question is whether any such causal structure implies a 'dot-decomposition' making all causal constraints evident.

We have shown many more 'dot-decompositions' of unitaries and found the need of more general dot-diagrams associating wires with indices in an involved way to label the relevant subspaces. This lead to a whole new 'dot-formalism' and is a programme that sits neatly with the work on quantum causal models in Refs. [2,3], strengthening the insight that quantum causal structure is inseparable from the structure of direct sums over tensor products. A completion of this programme would amount to, for each possible causal structure of a unitary, to know what the compositional structure is that mediates precisely the required causal influences and no others. Such a completion would constitute a substantially better handle on quantum causal structure – both conceptually, as well as, mathematically for proving conjectures concerning causal structure.

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